Energy Maps for Mobile Wireless Networks: Coherence Time versus Spreading Period
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Abstract—We show that even though mobile networks are highly unpredictable when viewed at the individual node scale, the end-to-end quality-of-service (QoS) metrics can be stationary when the mobile network is viewed in the aggregate. We define the coherence time as the maximum duration for which the end-to-end QoS metric remains roughly constant, and the spreading period as the minimum duration required to spread QoS information to all the nodes. We show that if the coherence time is greater than the spreading period, the end-to-end QoS metric can be tracked. We focus on the energy consumption as the end-to-end QoS metric, and describe a novel method by which an energy map can be constructed and refined in the joint memory of the mobile nodes. Finally, we show how energy maps can be utilized by an application that aims to minimize a node’s total energy consumption over its near-future trajectory.

Index Terms—wireless, mobile, network, QoS, energy

1 INTRODUCTION
In mobile networks, providing end-to-end Quality of Service (QoS) guarantees, such as a guaranteed end-to-end throughput, delay, bit-error-rate, or energy consumption, has been an extremely challenging problem. The main difficulty stems from the fact that the path from the source to the destination can change extremely fast. As a result, neither proactive nor reactive routing protocols, which require that the paths that are maintained or discovered remain stable, can provide end-to-end QoS guarantees. In highly mobile networks, local routing protocols such as geographic routing are able to route the data in the right direction, however, are unable to provide end-to-end QoS guarantees because the end-to-end paths are still viewed as highly unreliable, and only dynamically and locally determinable. The main assumption that underlies all of these approaches is that a path is a fixed sequence of nodes. An alternative approach to routing was proposed in [1], where a path is viewed as a physical path through space, and a physical trajectory is embedded in the packet to be routed. However, again, there, no claims can be made about how to provide end-to-end QoS guarantees from sources to destinations. Under the current view that focuses on mobility on the node scale, such guarantees appear indeed very difficult to provide when the nodes are highly mobile.

This paper proposes a new approach with the goal of providing end-to-end QoS guarantees in mobile networks. Since QoS spans a very large topic, with many metrics that fall under its definition, in this paper, we choose to focus on a single QoS metric that has recently become important: end-to-end per-packet energy consumption. This metric is important in mobile networks in which the mechanical energy that is used for propulsion is not converted to electrical energy used for RF communication: For example, when a pedestrian uses a cell phone, or a pager, the mechanical energy that he uses to walk is not typically transferable to his electronic device. In fact, today, much of the communication has this underlying assumption. In this paper, we focus on such mobile networks with a very large number of (mobile) pedestrians, each of which has an energy-limited device. Similar scenarios also arise on the digital battlefield where the pedestrians are the soldiers.

Our main idea is illustrated in Fig. 1. In Fig. 1(a), assume that our aim is to transfer information from location \((x, y)\) to a fixed base station \(BS\) with minimum energy, through a mobile network in which the node

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buffers are not congested. Assume that in snapshot $t = 0$, the intermediate nodes are $A$ and $B$, and in snapshot $t = 1$, Node $C$ moves in $A$’s place. In such a case, proactive and reactive routing protocols would say that the original path has been broken. Geographic routing protocols would welcome this, but be unaware of its implications for end-to-end QoS metrics. Here, our main idea is that the relay nodes in between are interchangeable. When sending information from $(x, y)$ to $BS$, the source node would not care who relays the information, but rather care that the end-to-end QoS metric remain the same. In Fig. 1(b), we generalize this idea: The figure first shows the instantaneous path constructed from $(x, y)$ to $BS$ at snapshot $t = 0$. At snapshot $t = 1$, the flock of nodes between $(x, y)$ and $BS$ has moved away, but it has been replaced by another flock of nodes that moved roughly in its place, such that the density of the nodes in between has remained the same. In this case, a different, instantaneous minimum energy path is constructed; however, its end-to-end energy metric is expected to be close to the one at snapshot $t = 0$. This arises because in this regime of no congestion, the end-to-end minimum energy consumption per packet depends on the sum of the transmit and receive energy consumptions over the hops, which depends on the presence of mobile nodes at intermediate locations, but not the identity of these nodes.

Hence, even though a network may be highly mobile on the individual node scale, its average end-to-end QoS metrics between fixed physical locations in space may still be stationary. This view has the potential to reduce much of the uncertainty in mobile networks and to help provide end-to-end QoS guarantees. However, the nodes themselves, who are also the sources of the traffic, are mobile; hence, how can they utilize this stationarity of QoS metrics between fixed locations in space? Our main idea is to focus on the energy metric, and construct an "energy map" in space between physical locations. This energy map will remain stationary for durations that are much longer than those we observe on the individual node mobility scale. We define the “coherence time” of this map as the maximum duration for which the map remains roughly constant. If the nodes can jointly construct this energy map and spread it among themselves within this coherence time, then the energy map can be used to give each node, per-packet end-to-end energy consumption guarantees that are valid. We define the “spreading period” as the minimum duration required to jointly construct and spread the energy map to all of the nodes. Hence, if the coherence time of the map is larger than the spreading period, then we have a mobile network for which the energy metrics can be tracked. This idea generalizes to other QoS metrics, such as end-to-end delay and throughput, although the construction of the map of each QoS metric is different.

The rest of this paper is organized as follows: In Section 2, we discuss the related work. In Section 3, we state our assumptions. In Section 4, we present our general mathematical framework. In Section 5, we present a method to construct the energy potential. In Section 6, we display our simulation results. In Section 7, we apply our energy maps to the problem of energy minimization over a trajectory. In Section 8, we present our conclusions and future directions.

2 Related Work

The challenge of achieving quality-of-service (QoS) in mobile networks has attracted much attention in the past. The QoS issues were discussed extensively in [3]. The QoS metrics (average delay, and average number of hops) were compared by simulations in [4] for on-demand routing, and hierarchical routing methods. In [5], a QoS routing protocol was developed, which can establish QoS routes with reserved bandwidth on a per flow basis in a TDMA network. Distributed QoS routing was developed in [6] to select a path with sufficient resources to satisfy QoS requirements in the presence of imprecise state information. In all of these past approaches, the underlying assumption was that a path was constituted by a fixed sequence of nodes, and if the nodes changed, then the path was no longer there (and a path update was required). In contrast, in this work, we focus on paths through space, and the nodes are taken to be interchangeable, and we aim to associate a QoS metric with the physical path through space.

Much of the literature on mobility has focused on how to track the nodes in mobile networks in order to estimate node locations [7]. Both the proactive [8], [9], [10], [11], [12] and reactive [13], [14], [15] routing protocols conceive paths as fixed sequences of nodes. The localized routing protocols such as directed diffusion [16] and gradient routing [17] rely on the establishment of a gradient, but usually assume that the sensors have fixed positions in space for such gradients to be established. Geographic routing [18], [19] approaches use the position information to route towards the destination; however, geographic routing by itself usually sidesteps the consideration of QoS metrics (such as end-to-end energy consumption and end-to-end delay). Recent papers [20] have proposed to combine geographic and energy-aware routing approaches by a weighted combination of the two metrics and demonstrated high performance. In addition, TTDD [21] and SEAD [22] have emerged as two promising approaches to address routing in the presence of sink mobility in large-scale sensor networks.

In the last six years, the long-term node position densities over rectangular simulation areas were derived [23], [24], [25], [26] for commonly-used simulation models such as the Random Waypoint Model and others [27], [28]. The key result of these papers is that there is an underlying invariant probability density function for the position of nodes over space, which is useful for simulation purposes. Real networks do not admit such simple descriptions; however, recent work [29], [30], [31] has shown the spatial regularity patterns in mobile networks, which can partially be modeled using
gravitational and fluid models. These recent studies and associated spatial models are different from previous investigations into time-domain mobility prediction [32], [33], [34], [35], [36], [37], [7] in that the spatial models focus on the invariant, spatial characteristics of mobile networks as a whole, rather than attempting to predict the mobility patterns of individual nodes or groups of nodes. We are motivated by these characterizations of mobility in the spatial domain, and seek to identify invariants such as the energy consumption and delay between fixed locations in space, over a time duration during which this information can be spread to the network and utilized.

3 ASSUMPTIONS

Because the energy maps between arbitrarily locations in space are difficult to visualize, we shall constrain ourselves in this paper to a mobile network with a single, stationary destination node, which we call the “base station” BS. Each pedestrian user in this network carries an energy-limited device which represents a “node”, and the nodes move within a bounded, connected deployment region \( D \subset \mathbb{R}^2 \). Each user wants to send information to the base station, possibly by using the other nodes in the mobile network as relays. Examples of application scenarios for this setting are: (1) Soldiers on a battlefield who need to send critical information packets to the military headquarters, (2) Scientists in the forest who make measurements to be relayed quickly to the laboratory collection site for analysis, (3) Emergency rescue teams in a disaster area who send critical information to the disaster management headquarters.

We assume an energy consumption model that has a transmit and a receive energy consumption component. The average transmit energy over a distance \( d \) goes as \( d^{-q} \), where \( q \geq 2 \) is the path loss exponent for outdoor transmission, and we assume that the receive energy (spent by the receiver) per bit is a constant \( c \) for all of the nodes.

Mobile, energy-limited networks have a very large design space, where optimal designs differ depending on the application in question. For example, if the delay constraint of the application is very large such that the nodes move appreciably within this duration, the optimal methods to minimize energy consumption will utilize the physical transport of the packet with the node (a.k.a. message ferrying [38]). If the delay constraints are too tight, then multi-hop transmission cannot be utilized and direct transmission to the base station must be used, albeit at a very high energy consumption.

In this paper, we restrict this very large design space to a manageable size by making the following assumptions: (1) The delay constraint from any source node to the base station BS is much less than the time scale over which the nodes move appreciably. Hence, an optimal routing solution is unable to use the physical transport of the packet (as used in message ferrying). (2) The nodes have limited battery supplies; hence, a multi-hop transmission scheme to the base station is used to conserve energy. (3) The delay constraint can be met via multi-hop transmission. (4) The average rate of data generation at each node is small enough such that there is no network congestion at the relay nodes.

An example which satisfies all of the above specifications is as follows: Assume that there are 100 mobile nodes deployed over a square \( D \), and each node generates messages at an average rate of 100 kilobytes per hour. Each packet is 1 kilobyte, and must be transferred to the base station within 5 s. Assume that the data rate of the RF modem is 500 kbps. Assume that every node has a battery energy of about 5 kJ (roughly that of a AAA battery.) Assume that each node moves with a speed of 0.3 m/s (speed of a pedestrian). Say that in a worst-case scenario, a node will need to carry the traffic of 20 of the 100 nodes, which is 4.44 kbps. This is only 0.009 of the data rate. In this regime, there will be negligible congestion at the nodes; hence Assumption (4) is satisfied. The delay constraint is short enough to satisfy Assumption (1) and still long enough to satisfy Assumption (3). Finally, the battery energy is small enough to satisfy Assumption (2).

Consider a packet that leaves a mobile node, when the mobile node is at location \((x, y)\) of the deployment region \( D \). By Assumptions (1) and (4) above, a packet is able to complete its transmission from this source node, currently located at \((x, y)\), to the base station at \((0, 0)\), without the relay nodes’ having moved appreciably in that duration. For example, if the node velocities are on the order of 10 meters per second, and it takes a maximum of 10 hops to reach the base station, the maximum distance a node on this path moves, by the time the packet reaches the base station, is 10 centimeters, for link propagation and processing delays on the order of 1 ms, which is reasonable for RF communications and silicon-based technology. At pedestrian speeds of 0.5 meters per second, the maximum distance a pedestrian on the utilized path moves is 5 millimeters, in the same scenario. Hence, because the RF communication and silicon processing speeds at the nodes are much faster than the time scales over which the nodes move appreciably, the path over which the packet is transferred from the source node to the base station can be visualized as instantaneous with respect to the time scale over which nodes move.

In the next section, we present a mathematical framework based on the above assumptions, in order to investigate the trackability of energy metrics in mobile networks.

4 MATHEMATICAL FRAMEWORK

Let \( \mathbb{N} \) denote the set of nodes over the deployment region \( D \). Let \( N \) denote the number of nodes in this node set. The mobile network is characterized by the trajectories of the nodes in the network: Let \( \{X_i(t)\}_{i \in \mathbb{N}} \) denote the
positions of the nodes 1 through \( N \) as a function of time \( t \). Note that this collection is a vector random process. Further, we choose our coordinate system such that the location of the base station coincides with \( 0 \stackrel{\text{def}}{=} (0, 0) \).

The first subsection below is devoted to the definitions of the energy density, energy potential, and the energy variance function. These definitions are specific to the QoS metric of interest in this paper. The second subsection below defines the concepts of coherence time and spreading period. These two concepts are general, and are applicable to any QoS metric.

### 4.1 Energy Density, Potential, and Variance

A minimum energy path from location \( x \) to the base station at \( 0 \) is defined as a path that consumes the least amount of energy, over all of the possible paths, to send information from a node currently located at \( x \) to the base station at \( 0 \) through the mobile network in between. We show that for mobility models for which a limiting node density function exists over the deployment region, the probability density of the minimum energy path, instantaneously constructed at time \( t \), from \( x \) to a base station at \( 0 \) also has a limiting distribution. To this end, define the probability density function \( f(\mathbf{x}_1(t), \ldots, \mathbf{x}_N(t)) = \mathbf{P}(\mathbf{x}_1(t) \in dx_1, \ldots, \mathbf{x}_N(t) \in dx_N) \), where \( \mathbf{x}_1, \ldots, \mathbf{x}_N \in \mathcal{D}^N \). Let \( \mathcal{E}(x, t) \) denote the minimum energy per bit required to send information from \( x \) to \( 0 \), along the minimum energy path constructed at time \( t \). Note that because the positions of the nodes are random processes, \( \mathcal{E}(x, t) \) is a random process.

**Theorem 1:** Assume that (1) there exists an \( M > 0 \) such that, for all \( \mathbf{x}_1, \ldots, \mathbf{x}_N \in \mathcal{D}^N \), \( f(\mathbf{x}_1(t), \ldots, \mathbf{x}_N(t))(x_1, \ldots, x_N) < M \), and that (2), for all \( \mathbf{x}_1, \ldots, \mathbf{x}_N \in \mathcal{D}^N \), \( f(\mathbf{x}_1, \ldots, \mathbf{x}_N) \stackrel{\text{def}}{=} \lim_{t \to \infty} f(\mathbf{x}_1(t), \ldots, \mathbf{x}_N(t))(x_1, \ldots, x_N) \) exists. Then, \( \mathcal{E}(x, t) \stackrel{\text{def}}{=} \lim_{t \to \infty} P(\mathcal{E}(x, t) \in dx) \) exists for every \( x \in \mathcal{D} \), for every \( e \in \mathcal{R}_+ \).

The proof of this theorem, and all others in this paper, appear in the Appendix. In most of this paper, we shall use the expected value of this end-to-end minimum energy consumption as our main metric:

**Definition 1 (Energy Potential):** The energy potential \( \Psi(x, t) \) of location \( x \in \mathcal{D} \), at time \( t \), with respect to location \( 0 \), is defined as the expected value of the minimum energy required to send information from \( x \) to \( 0 \), at time \( t \); namely, \( \Psi(x, t) \stackrel{\text{def}}{=} E[\mathcal{E}(x, t)] \) for every \( x \in \mathcal{D} \).

Now, similar to the result in Theorem 1, if there is a limiting node density function for that mobility model, then the energy potential also has a limiting distribution.

**Theorem 2:** Under the same two conditions (1)-(2) as in Theorem 1, \( \Psi(x) \stackrel{\text{def}}{=} \lim_{t \to \infty} \Psi(x, t) \) exists for every \( x \in \mathcal{D} \).

Although Theorem 2 might appear to be a corollary of Theorem 1, it is not, due to the order of the integrals and the limiting operations, as shown in the Appendix.

Later in this paper, we shall find that the measurement of the energy potential itself would require global, end-to-end measurements. The development of a practical scheme that is based on only local measurements will utilize the following concept of an energy field, which is local.

**Definition 2 (Energy Field):** The energy field \( F(x, t) \) is defined as the negative of the gradient of the energy potential; namely, \( F(x, t) \stackrel{\text{def}}{=} -\nabla \Psi(x, t) \).

Giving end-to-end guarantees of per-packet energy consumption requires a characterization of not only the average end-to-end energy but also its variance:

**Definition 3 (Variance Function):** The variance function \( \Phi(x, t) \) of location \( x \in \mathcal{D} \), at time \( t \), with respect to location \( 0 \), is defined as the variance of the minimum energy required to send information from \( x \) to \( 0 \), at time \( t \); namely, \( \Phi(x, t) \stackrel{\text{def}}{=} \text{Var}[\mathcal{E}(x, t)] \) for every \( x \in \mathcal{D} \), for every \( t \in [0, +\infty) \).

The following theorem shows that the variance function also has a limiting distribution under the same assumptions as for the energy density and the energy potential:

**Theorem 3:** Under the same conditions (1)-(2) as in Theorem 1, \( V(x) \stackrel{\text{def}}{=} \lim_{t \to \infty} V(x, t) \) exists for every \( x \in \mathcal{D} \).

### 4.2 Coherence Time and Spreading Period

The coherence time and the spreading period are two fundamental concepts that are applicable to any QoS metric, not only energy. Roughly speaking, the “coherence time” is the maximum duration for which the QoS potential function over space remains roughly constant. The coherence time is important because the trackability of QoS metrics in mobile networks will depend critically on the coherence time of the QoS metric.

**Definition 4 (Coherence Time):** Let \( \{\Psi(x, t) \mid x \in \mathcal{D}, -\infty < t < +\infty\} \) be a real-valued, stochastic process over \( \mathcal{D} \). Let \( \delta > 0 \), and \( \mu(x, t) \stackrel{\text{def}}{=} E[\Psi(x, t)] \). Then, the coherence time of \( \Psi(x, t) \) is defined as: \( T_c(\delta) = \min_{x \in \mathcal{D}, t \in \mathbb{R}_+} \max_{\tau} \max_{t' : t - \tau \leq t' \leq t} \{\tau : |\mu(x, t) - \mu(x, t')| \leq \delta\} \).

For simulation models that involve abrupt changes of state, we replace \( T_c(\delta) \) by \( T_c(\delta)(\hat{s}) \), where \( \hat{s} \) denotes the current state, and \( t \in \mathbb{R} \) by \( t \mid \Psi(x, t) \) is in state \( \hat{s} \). The “spreading period” is defined as the minimum duration that is required to spread the measurements of the QoS potential function so that it can be constructed with small error. This is a measure of how fast information can be spread in a mobile network for the purpose of the construction of the QoS potential function.

**Definition 5 (Spreading Period):** Let \( \Psi(x, t) \) be a real-valued, stochastic process over \( \mathcal{D} \), where \( x \in \mathcal{D}, -\infty < t < +\infty \). Let \( \mu(x, t) \stackrel{\text{def}}{=} E[\Psi(x, t)] \). Let \( M \) be a finite set of
distinct positions \( \{x^{(m)} \in D \} \). Let \( V^{(m)} \) denote the set of positions in \( D \) that fall in the Voronoi cell of \( x^{(m)} \in M \). Let \( S_k[t - \tau, t] \) be the set of all samples \( \Psi(x_n, t_n) \) available to node \( k \) during \( [t - \tau, t] \) via its own samples of \( \Psi(x, t) \) or from other nodes via the exchange of samples. Let

\[
Q_k^{(m)}[t - \tau, t] \overset{\text{def}}{=} \{(x_n, t_n) | x_n \in V^{(m)}, (x_n, t_n) \in S_k[t - \tau, t]\}.
\]

Let \( N_k^{(m)} \overset{\text{def}}{=} |Q_k^{(m)}[t - \tau, t]| \). Let \( \epsilon > 0 \), and \( \delta_{spr} > 0 \). Define \( \tilde{\mu}_k^{(\tau)}(x^{(m)}, t) \overset{\text{def}}{=} \frac{1}{N_k^{(m)}} \sum_{(x_n, t_n) \in Q_k^{(m)}[t - \tau, t]} \Psi(x_n, t_n), \forall x^{(m)} \in M \), and let \( \tilde{\mu}_k^{(\tau)}(x, t), \forall x \in V^{(m)} \). (Arbitrarily assign each boundary to either center, and fix this assignment.) Define \( T_{\text{spr}}^{(\delta_{spr}; \epsilon; M)}(t) \overset{\text{def}}{=} \arg \min_{\tau} \left\{ \left[ \frac{1}{N} \sum_{k \in \mathbb{R}} \tilde{\mu}_k^{(\tau)}(x, t) - \mu(x, t) \right] > \delta_{spr} \right\} \leq \epsilon, \forall x \in D \). Let \( T^{(\delta_{spr}; \epsilon; \tau; M)}(t) \overset{\text{def}}{=} \{ t | T_{\text{spr}}^{(\delta_{spr}; \epsilon; M)}(t) \exists \} \). Then, the spreading period is defined as:

\[
T^{(\delta_{spr}; \epsilon; \tau; M)}(t) \overset{\text{def}}{=} \max_{t \in T^{(\delta_{spr}; \epsilon; \tau; M)}} \left( T_{\text{spr}}^{(\delta_{spr}; \epsilon; M)}(t) \right).
\]

In the definition of \( T_{\text{spr}}^{(\delta_{spr}; \epsilon; M)}(t) \), \( \forall x \in \mathbb{D} \) may be replaced by \( \forall x \in \mathbb{D}_r \), where \( \mathbb{D}_r \) is the relevant part of the deployment region over which we would like to construct an approximation of \( \mu(x, t) \) for the application at hand.

The main theorem of this paper is that if the coherence time is larger than the spreading period, then we have a network in which the end-to-end energy metrics are tractable:

**Theorem 4:** Let \( \{\Psi(x, t) | x \in D, -\infty < t < +\infty\} \) be a real-valued, stochastic process over \( D \). Let \( \mu(x, t) \overset{\text{def}}{=} E[\Psi(x, t)] \). Let \( \epsilon > 0 \), and \( \delta_{spr} > 0 \). Assume that for this process \( \Psi(x, t) \), there exists an \( M \) such that \( T_{\text{spr}}^{(\delta_{spr}; \epsilon; M)} \) exists and is a bounded function of \( \delta_{spr}, \epsilon \). Let \( \delta_c > 0 \). Let \( T_{\text{spr}} \overset{\text{def}}{=} T_{\text{spr}}^{(\delta_{spr}; \epsilon)} \) and \( T_c \overset{\text{def}}{=} T_c^{(\delta_c)} \). Let \( T^{(\delta_{spr}; \epsilon; \tau; M)} \) be defined as in Definition 5. If \( T_c > T_{\text{spr}} \), then \( \forall x \in D, \forall t \in T^{(\delta_{spr}; \epsilon; \tau; M)} \),

\[
P \left\{ \left| \frac{1}{N} \sum_{k \in \mathbb{R}} \tilde{\mu}_k^{(T_{\text{spr}})}(x, t) - \mu(x, t) \right| > \delta_{spr} + \delta_c \right\} \leq \epsilon.
\]

This establishes an analogy with the point-to-point wireless channel whose state is trackable if the coherence time of the channel is much larger than the symbol duration. In a sense, our framework characterizes the end-to-end channel from one physical location to another through a mobile network, as if it were a single channel, and defines the coherence time of the QoS metric on this set of end-to-end channels from all the locations. In the point-to-point channel case, the symbol duration is the fundamental time unit with which training sequences can be sent to the receiver in order to estimate the channel. In that case, if the coherence time of the channel is much larger than the symbol duration, then we have a point-to-point channel that can be estimated accurately. Similarly, in the network setting, if the coherence time of the QoS metric is larger than the spreading period, then we have a mobile network that is trackable with respect to this QoS metric.

The following theorem establishes sufficient conditions for the existence and boundedness of the spreading period, which we shall use later in the paper:

**Theorem 5:** Let \( \{\Psi(x, t) | x \in D, -\infty < t < +\infty\} \) be a real-valued, stochastic process over \( D \). Let \( \mu(x, t) \overset{\text{def}}{=} E[\Psi(x, t)] \). Assume that \( \mu(x, t) \) is continuous in \( x \), and that \( \mu(x) \overset{\text{def}}{=} \lim_{\nu \to -\infty} \mu(x, t) \) exists \( \forall x \in D \). Assume that \( \exists \nu > 0 \) such that \( \text{Var}(\Psi(x, t)) \leq \nu \forall x \in D \). Assume that for every finite set \( M' = \{x^{(m)} \in D\} \) of distinct positions in \( D \), for every \( m \in M' \), every node visits some point in \( V^{(m)} \) with positive probability. Let \( \delta > 0 \), and \( \epsilon > 0 \). Then, \( \exists \) a finite set of distinct positions \( M \) in \( D \), and a \( \tilde{t} > 0 \) such that \( T_{\text{spr}}^{(\delta_{spr}; \epsilon; M)}(t) \) exists, and is a bounded function of \( (\delta, \epsilon) \), \( \forall t \geq \tilde{t} \).

### 5 Construction of the Energy Potential

In this section, we establish methods for the construction of the energy potential function \( \Phi(x, t) \), under the assumption that the energy potential has a limiting distribution \( \Phi(x) = \lim_{t \to -\infty} \Phi(x, t) \). Our construction method will be based on such energy potentials that have a limiting distribution. In Section 6.2, we examine such potentials in the transient regime.

In this section, first, as a preliminary step, we describe a centralized construction of the energy potential. We shall later use the results of this centralized construction as a benchmark against which to measure the performance of practical construction methods. Next, we describe a localized, distributed construction of the energy potential by the nodes, via the local estimation of the energy field. This divides into two conceptual parts: (1) The local estimation of the energy field, assuming that all of the local measurements can be exchanged instantaneously by the nodes, (2) The actual construction of the energy field, in which the local measurements obtained by the nodes are exchanged upon encounters with other nodes.

We assume that each mobile node has access to an accurate measurement of its own instantaneous position as well as the current global time (e.g. via a GPS receiver). Further, assume that topology control is in place, such that whenever a node would like to transfer a packet to the base station, a minimum energy path can be found within the delay constraint of the application.
When the energy potential has a limiting distribution, there exists a \( \delta > 0 \) such that the coherence time \( T_c^{(1)} \) of the energy density is unbounded. If the spreading period exists and is bounded, then Theorem 4 shows that the energy potential can be constructed with small error within the spreading period. We now address the techniques for this construction under the assumption that the spreading period exists and is bounded (Note that Theorem 5 gives sufficient conditions for this.)

For a finite set \( \mathcal{M} \) of distinct positions on \( \mathcal{D} \), the Voronoi tessellation which takes these points in \( \mathcal{M} \) as centers, partitions \( \mathcal{D} \) into \( |\mathcal{M}| \) cells. Pick \( \mathcal{M} \) such that the cell sizes are small enough that the construction of the energy potential \( \Phi(\mathbf{x}) \), is approximated closely by its samples which are collected as in Definition 5. The particular method in which these samples are taken differ, as we specify in the subsections below.

5.1 Centralized Construction of the Energy Potential

As a preliminary step, we shall describe a centralized construction of the energy potential, which can be used in simulations, but cannot be used in practice. This will serve as a benchmark against which to compare the performance of the distributed algorithms we shall consider in the following subsections.

Assume that in this centralized construction, we are able to compute the instantaneous minimum energy path from a mobile node \( i \) to \( BS \) at any time. We record such a measurement every time the mobile node enters a new cell (of the Voronoi tessellation) in the simulation. We attribute each measurement as a “sample” of the energy potential, to the center of the cell as described in Definition 5. In general, over the coherence time of the energy density function, we collect these measurements, and for each cell center, average the measurements that are attributed to that cell center. This gives us the centralized construction of the energy potential.

5.2 Distributed Construction with Instantaneous Exchange

In practice, the mobile nodes cannot “sample” the end-to-end energy potential because an end-to-end measurement from each mobile node \( i \) to \( BS \) would be needed. Such measurements, even if initiated, can only be collected by \( BS \). However, the nodes themselves would need these measurements to construct the energy potential.

5.3 Distributed Construction with Dissemination

In the previous subsection, we assumed that the local measurements of the energy field were instantaneously available to all of the nodes. In practice, the local measurements of the energy field can be exchanged only via local encounters with the other nodes.
Fig. 4. Dissemination Algorithm

Given that the local measurement method of the energy field has been fixed, the “dissemination method” that does not lose any information would be to have each node \(i\) tell, every other node \(j\) it encounters, the entire set of local measurements that node \(i\) has made and heard from others. When a node \(j\) hears a set of local measurements from another node \(i\), it first checks whether any of these is a replica of a measurement that it already knows. If so, it is filtered out. Otherwise, the measurement is added to \(j\)’s database. At each time \(t\), based on all of these measurements that a node knows, the node constructs the energy potential as its best estimate of this function over \(D\).

The main drawback of this scheme is that the number of measurements grows linearly with time, and linearly with the number of nodes \(N\). Hence, the overhead per encounter also grows, and at some point, will dominate the data transfers. However, this overhead can be easily avoided as follows: If the spreading period exists and is bounded, then after a bounded time interval, we expect every node to have heard the old measurements. Hence, these old measurements need not be distributed. Hence, there exists a “window size” \(W\) into the past such that any measurement that occurred more than \(W\) seconds ago need not be exchanged upon encounters. The window size \(W\) will decrease as the tolerable distortion in the construction increases. We explore the practical trade-off between these quantities empirically in Section 6.

Fig. 4 displays the dissemination algorithm. As shown on the upper left hand side of this figure, each node \(i\) reports to all of its neighbors all of the samples of the energy field that it knows of and that have generation time stamps of the last \(W\) seconds. (Here, we show \(W\) as a discrete-time step, assuming a fixed sampling interval of 1 second.) At the same time, node \(i\) receives from each of its neighbors \(j\), the samples of the energy field that its neighbor knows of and that were generated in the last \(W\) seconds. In the Filtering block, these received samples are filtered; that is, any replicas that node \(i\) already knows are removed. The remaining measurements, which are new information, are used to update the average energy field in the database of this node at each cell. We define the parameter \(S\) (not shown) as the number of the most recently collected samples for each cell, that the Update Field Information block uses to compute the running average. As the coherence time of the energy density function increases, the parameter \(S\) should increase as well. We address the empirical determination of \(S\) in Section 6.2. Finally, in the Update Next Report block in the figure, the node prepares its next report by shifting the \(W\)-window by one, and adding in the new energy field measurements both heard from neighbors in the last iteration, as well as the measurement that node \(i\) made in this iteration.

6 Simulation Results

In this section, we set up two simulations to apply the methods developed in the preceding sections: First, we set up a simulation to examine the construction of an energy potential under a single limiting distribution. Second, we set up a simulation to examine the performance of our construction method under abrupt changes of the limiting distribution, and when the potential is in its transient regime. Our main goal in this section is to examine whether the end-to-end energy metrics through mobile networks can be tracked via the methods that we developed.

In both simulations, the deployment region \(D\) is 1 km by 1 km. The base station is located in the middle of \(D\). The set \(M\) is a 20 by 20 bounded lattice, made up of a total of 400 centers. Each Voronoi cell is thus a square cell. There are 100 mobile nodes whose mobility model we shall specify in each subsection.

6.1 Energy Potential in Stationary Regime

In this subsection, we set up a simulation that exhibits an energy potential with a limiting distribution. Further, the observation interval is long enough such that the potential can be observed in its stationary regime. When energy is the QoS metric, any mobility model that has a limiting distribution for the node density has an energy potential with a limiting distribution (Theorem 2). In order to exhibit an example, we choose the Random Waypoint (RWP) Model whose limiting distribution was derived in [23], [24] and shown to be independent of node velocity. In our simulation, each node moves with a velocity of 10 m/s in each travel interval, and the pause time is 0 seconds. We choose a “sampling interval” \(T_s\) of 5 seconds, with which each node samples the energy field. Since the cell size is 50 m by 50 m, on average, a node will have moved to a new cell before it makes a new measurement. Throughout our simulations, we assume that the transmit energy is dominant over receiver energy, and that the path loss exponent is \(q = 2\). The transmission range of each node is 120 meters.\(^2\)

\(^2\) The MAC layer dynamics are not modeled in our simulations.
First, for the centralized construction, at each sampling interval, we measure the minimum energy consumption of the instantaneous minimum-energy path from each node to the base station $BS$. First, in order to find the limiting distribution of the energy potential, we let the simulation interval $T = 50,000$ seconds (10,000 iterations). For each cell, we average the measurements that are collected for that cell, over this simulation interval. In addition, for each cell, we also measure the average node density. Fig. 5(a) displays the average node density, and Fig. 5(b) displays the energy potential, both over 10,000 iterations. The first part of this figure concurs with the limiting distribution of the node density analytically obtained in [23]. The second part of this figure is new: It shows the limiting distribution of the energy potential, under the RWP model. Each point in this graph shows the minimum energy required to send a packet from location $(x, y)$ to $(0, 0)$ through the mobile network in between. The key point here is that even though the network in between is highly mobile, and even though the instantaneous minimum energy consumption from $(x, y)$ to $(0, 0)$ changes over time, its average converges to the limiting distribution shown. (The receive energy $c = 0$ in this graph; if $c$ increased, this graph would be shifted upwards.)

Second, in order to allow the nodes to estimate this energy potential, we first apply our idealized, instantaneous exchange scheme from Section 5.2, as a preliminary step toward the actual distributed algorithm. Here, we must supply a method for the local measurement of the energy field, suitable for the topological properties of this deployment region. Because this deployment region has no holes, we propose to use in this case, the following “Joules-per-bit-meter” heuristic: Every node estimates the local energy field by measuring the Joules per bit required per meter of progress (headway) made toward the base station $BS$, minimized over all of the node’s neighbors. Fig. 6(b) displays a node, labeled as the “source”, with three neighbors in the direction of $BS$, which is located far toward the right (not shown) on the $+x$ axis. All of these neighbors are assumed to be within the transmission range of the source. We say that a neighbor $k$ “dominates” neighbor $j$ iff the Joules-per-bit-meter cost for $k$ is less than that for $j$. For the path loss model with path loss exponent $q$, the set of positions of $j$ that are dominated by a fixed node $k$ can be geometrically specified and is called the “dominated region” of node $k$. The figure shows the dominated region of Node 1 when $q = 4$. (For $q = 2$, the non-dominated region is circular.) In the figure, intuitively, Node 3 is dominated by Node 1 because even though the source requires small transmit energy to reach Node 3, this move does not make much progress toward the base station. Node 2 is dominated by Node 1 because even though it makes much headway toward the base station, this comes at a high energy cost.

At each sampling time, every node sends out a beacon to its current set of neighbors. This beacon states the (1) node ID, and (2) node position of the source, and (3) the energy that it takes to send the beacon packet. When a neighbor $j$ receives the beacon, it measures the energy at which it receives this packet, from which it computes the channel gain from the source to $j$. Node $j$ immediately returns an ACK to the source. In its ACK, it places (1) its node ID, (2) its current position, and (3) the measured channel gain. After the source receives this ACK, it computes the dominated region of node $j$ to quickly exclude much of the geometric search area from consideration to find the minimum Joules-per-bit-meter neighbor. This will most likely be in the area where the beacon was already sent; hence, typically, no further search will be necessary. Then, for the minimum Joules-per-bit-meter neighbor, denoted by $j^*$, the source node creates the local measurement packet that contains (1) the source node’s node ID, (2) the position at which this minimum Joules-per-bit-meter measurement was initiated, (3) the time at which the measurement was initiated (the “generation time stamp”), (4) the channel gain measurement, and (5) the “transmission vector”, which is the vector from the position of the source to the position of $j^*$. The source is able to compute this transmission vector from its own position which it knows, and $j^*$’s position which appeared in the ACK packet from $j^*$. Next, Node $i$ computes its best estimate of the energy field, for each cell, as follows: It vectorially averages the transmission vectors, and it performs a scalar average of the (minimum Joules-
per-bit-meter) channel gain samples. The average energy field computed this way, over 50,000 seconds, assuming instantaneous-exchange of local measurements among the nodes is shown in Fig. 6(a).

Next, each node constructs its best estimate of the energy potential by the Path Integration Algorithm. Fig. 3(b) shows the virtual paths constructed in this simulation via this algorithm. The virtual paths are smoother and more directed toward the base station than the actual paths because the vector average of the transmission vectors removes most of the variation in the tangential direction. However, since the channel gains are scalar, the magnitudes of energy consumption in the transmission are still preserved.

Third, we use the dissemination algorithm to spread the local measurements of the energy field. Fig. 7 shows the percentage error of this distributed construction against the centralized construction, as a function of the absolute time in seconds, parameterized by the window size $W$, which is discrete-time steps at each sampling interval of 5 seconds. This plot is the average over all of the nodes. As $W$ increases from 0 to 5, the minimum duration required to achieve a target percentage error decreases. The performance is not much improved beyond $W = 5$. In the graph, initially, the percentage error is 100% or above because none of the nodes have any measurements of the energy potential. However, for $W = 5$, when the nodes report the measurements with generation times 5 time steps into the past, the percentage error decreases quickly to 20% in 65 seconds, and to about 10% in 100 seconds. In 65 seconds, each node in this RWP model moves about 1.3 average lengths from a point to its next point in the RWP model. Whether this minimum duration required for an accurate construction is long or short depends on the application requirements, which we shall treat in Section 7. The error floor of 10% arises from (1) the non-adaptive uniform cell resolution, (2) the heuristic Joules-per-bit-meter method used for energy field estimation, (3) the path integration method itself, which is also a heuristic.

Fig. 8 shows the overhead of broadcasting measurements of the energy field, per node per sampling interval, averaged over all of the nodes. In the beginning, no node has any measurements; however, via exchanges, the number of broadcast measurements grows quickly despite the elimination of replicas. After about 100-200 seconds, the overhead settles to a steady-state value. This empirical value is much lower than the theoretical upper bound, which is $NW + 1$. For example, for $W = 5$, the steady-state value is 75, which is much less than 501. As $W$ increases, the overhead increases. The following calculation gives the numerical value of the expected overhead in a network of this size: Each packet that reports a measurement has the node ID, which is 7 bits to encode 100 nodes. There are 400 cells, which are encoded by 9 bits, which is used to specify the source position. The transmission vector can be quantized to 18 angular displacements (of 10 degrees) in the half plane toward the base station. This requires 5 bits. The channel gain can be quantized to 8 bits. For the generation time stamp, using a wrap-around after 5 spreading periods, we need 7 bits. Hence, in total, 36 bits are required for each measurement packet. This implies that for $W = 5$, 75 measurements at steady-state require 338 bytes on average. If the link speeds are 100 kbps, 338 bytes are transmitted in roughly 30 ms, which is only 0.6% of the time $T_s = 5$ seconds, which is the interval at which these samples are broadcast. Hence, $W = 5$ represents a good trade-off between the average percentage error and the average overhead.

## 6.2 Energy Potential in Transient Regime

In this section, we present simulation results in a more realistic regime where the energy potential does not have enough time to approach a limiting distribution; that is, we analyze the network in the transient regime. Our main goal is to examine whether our methods can track the energy metrics in this scenario.

Our simulation set-up is shown in Fig. 9. All of the simulation set-up is the same as in Section 6.1 except the following: There are now 4 attractors, with an attraction...
area of 150 m by 150 m around each attractor. Each attractor can be a lecture hall on campus, or a monument in a tourist area, toward which the pedestrians flock and then from which they may disperse. Each node exhibits the following mobility pattern independent of others: There are two “modes” of our simulation: (1) Attractors are ON, and (2) Attractors are OFF. If the attractors are ON, each node acts as follows: If the node is still outside of the attraction areas, it follows the RWP model over the entire deployment region. If its destination ever falls within an attraction area, then it follows the modified contraction model \[30\] from that point onwards, until it converges to an attractor. If the attractors are OFF, then, every node follows the RWP model over the entire deployment region. As before, in order to be able to use a regular sampling interval, we have each node move with a constant velocity of \(10\) m/s during travel intervals. All of the pause times are 0. The sampling interval used is \(T_s = 5\) seconds.

In our simulation, we shall switch from the OFF mode, to the ON mode, and back to the OFF mode. This model is able to emulate the following scenarios: The students converge to different lecture halls on campus. After the lectures are over at the end of the hour, they disperse from the lecture halls. Similarly, it can simulate the tourists converging to certain monuments at peak times, and their dispersing from them.

In this simulation, both the ON and OFF modes have limiting distributions. However, there is not enough time to settle down to either of these. Hence, the energy potential estimation operates in the transient regime.

Fig. 10 shows the variations in node density for the following simulation: The OFF mode lasts 1000 seconds, followed by 500 seconds of the ON mode, which is followed by 1000 seconds of the OFF mode. In the figure, at \(t = 1000\) seconds, the node density is that of the RWP model, albeit taken over only the 200 available samples. At \(t = 1050\), about 1/4 of the nodes fall within the attraction areas, and at \(t = 1500\), about 1/2 of the nodes are at the attractors and the rest outside. (Since the remainder of the nodes is very sparse, the node density outside looks as if it were zero, but it is not.) At \(t = 1500\), we switch to the OFF mode, and at \(t = 1600\), the node density approximates again that of the ideal RWP model density.

Fig. 11 shows the concurrent variations in the energy potential, constructed via the centralized algorithm. The cup-shaped energy potential ripples much and gets distorted in the transient regime between \(t = 1000\) and \(t = 1500\); however, its shape is still discernible. Quantitatively, the magnitude of the energies have increased significantly because there are now large spaces with only a sparse distribution of nodes over which the bits need to hop to reach the base station. Most of the nodes are concentrated at the attractors and thus do not provide a good distribution of relay nodes in between. However, the cup shape of the energy potential has remarkably been retained. The reason is that even though the energy potential changes with variations in node density, ordinably whether a cell is better or worse in the average energy consumption with respect to another
Fig. 12. The percentage error of the distributed, localized construction with respect to the instantaneous energy potential of the centralized construction.

cell has not changed too much: The cells that are closer to the base station are, on average, still better positions than the ones further away. This turns out to be important for the application that we shall consider in the next section.

Because the network is in the transient regime, we measure the performance of our distributed construction against the instantaneous energy potential \( \Phi(x, t) \) rather than the limiting distribution of the energy potential \( \Phi(x) \) that we used in the previous section. In addition, in contrast with the stationary regime, here, it may not pay off to take too many samples into the past in computing an estimate of the average energy field. Recall from Section 5.3 that we use the parameter \( S \) denote the number of samples into the past that will be used (in the Update Field Information block in Fig. 4) to compute the average energy field.

Fig. 12 shows the percentage error, averaged over all of the nodes, of the constructed energy potential, against the instantaneous energy potential. (We have used \( W = 5 \).) The percentage error starts high in the beginning since measurements of the energy field are not initially available, and goes down to about 17% by \( t = 1000 \) seconds for both \( S = 1 \) and \( S = 50 \). (Fig. 7 used \( S = t/T_s \), where \( t \) is the simulation time; that is, it used all of the samples available and was able to achieve better performance.) We expect that as the energy potential begins approaching its limiting distribution, using a higher \( S \) perform better. The node density changes less, and the samples collected become good approximations of the limiting energy field. Indeed, towards \( t = 1000 \), \( S = 50 \) outperforms \( S = 1 \). However, when the attractors are turned ON at \( t = 1000 \), the quick changes that follow in the node density function render the old energy field data obsolete, and we see that \( S = 1 \) outperforms \( S = 50 \) toward \( t = 1500 \). If we waited long enough in this ON mode, the nodes would conglomerate at the attractors, and the trend would reverse. However, at \( t = 1500 \), we return to the OFF mode. Upon quickly changing node densities due to dispersal, \( S = 1 \) briefly continues to outperform \( S = 50 \), but the trend reverses quickly as the network begins approaching the limiting distribution of the OFF mode.

7 Applications

Giving end-to-end guarantees of the per-packet energy consumption in a network requires not only that the end-to-end average energy \( \Phi(x, t) \) can be accurately estimated but that the variance \( V(x, t) \) (Definition 3) around that average be relatively small. A key measure of the “fluctuation” is the ratio of the standard deviation to the average, namely \( \sigma(x, t)/\Phi(x, t) \), where \( \sigma(x, t) = \sqrt{V(x, t)} \). Fig. 13(a) shows the limiting distribution of this measure over the deployment region, for the simulation scenario of Section 6.1. We see that this ratio peaks at the base station, however, decreases rapidly down to less than 5% far from the base station. The peak around the base station, which occurs due to the uniform cell size, is unimportant since when the nodes are near the base station, they communicate directly and do not use the estimates. In our simulations, we let each node estimate this fluctuation, and found that the empirical distribution of the energy density function is roughly Gaussian. This result was consistent across almost all of the cells which had sufficient data. Hence, energy maps can provide the nodes with such statements of the form “the average energy consumption from this location to the base station is \( X \) Joules per packet with a 5% fluctuation”. The variation of the fluctuation as a function of \( N \) is a topic of future work on the scaling laws of QoS metrics in mobile networks, and cannot be addressed within the scope of this paper.

In our construction of energy maps, we assumed that the application delay constraints were in the mid-regime such that the relay nodes had to deliver the packet to the base station in a multi-hop fashion right away. In a more sophisticated application scenario, assume that we have another “overlay” application, which operates on top of the native routing architecture, with much looser delay constraints. We assume that all of the other assumptions in Section 3 are maintained. This overlay application can then take advantage of the energy maps to decide when to release its traffic into the network. Fig. 13(b) shows the contours of the energy potential obtained for the scenario in Section 6.1, and the near-future trajectory of a mobile node, truncated by a delay deadline \( D \).
stage: (1) cell #: $k_j$

Fig. 14. The trellis diagram for the energy minimization dynamic program.

Our aim will be to minimize the total energy consumption of sending $B$ bits until the delay deadline $D$. This requires knowledge of not only the energy potential, but also the distribution of the energy density function over space. However, since every node builds a database of the local measurements of the energy field, it can also compute the empirical distribution of the energy density function for each cell from this database. For simplicity of implementation, this distribution at each cell can be quantized, e.g. to 3 “states” (labeled “good”, “medium”, “bad”) at each cell. Then, a node computes a trellis diagram, as shown in Fig. 14, that shows the possible sequence of states through which the node will travel until it hits its delay deadline $D$ for those $B$ bits. For each cell that the near-future trajectory of this node intersects, a (vertical) “stage” in the trellis diagram is ascribed. In the figure, there are 3 stages until the delay deadline. The arcs are labeled with the transition probabilities, which are computed empirically by the node from its database of local measurements of the energy field.

Let $Q$ be the probability transition matrix of the trellis diagram. Let $M$ be the total number of stages until the delay deadline, and $m: 1 \leq m \leq M$ be the stage index. Let $S'$ be the total number of states, and $s: 1 \leq s \leq S'$ be the state index. Let $\hat{\mu}^{(m)}$ be this node’s best estimate of the expected value of the end-to-end energy cost for stage $m$, if the node decides to send the $B$ bits at stage $m$. Let $\mu^{(m)}$ be the expected value of the end-to-end energy cost in stage $m$. Then, $\nu^{(M)} = \hat{\mu}^{(M)}$, and $\forall m : 1 \leq m \leq M - 1$, $\nu^{(m)} = \min\{\hat{\mu}^{(m)}, E(s)[\nu^{(m+1)}]\}$.

Let $m^* = \arg\min_m \nu^{(m)}$. Then, the optimal decision of the node is to send out the $B$ bits at the current stage that it is in, iff the current stage is $m^*$. (Note that the new local measurements of the energy field that are collected while on this trajectory are automatically integrated into the database and thus into this decision dynamically.)

We use the same simulation set-ups as in Sections 6.1 and 6.2, for the “time-invariant” (stationary) and “time-varying” (transient) cases, respectively. We use $W = 5$ for both. Let a “session” denote an interval in which a node has a delay deadline of $D$ to send $B$ bits. In our simulations, each session is $MT_s$ seconds, corresponding to $M$ stages, and $B = 1$ kbits for each session. Each node schedules subsequent sessions without gaps until the end of the simulation. We obtain the results in 1 simulation run, and average over the 100 nodes. We let each node use only $L$ measurements into the past, for each cell (separately). The first row of Fig. 15 displays the average energy consumption incurred per session as function of the number of stages $M$, for the time-invariant and time-varying cases. The energy consumption decreases as $M$, the delay deadline, increases since the node is able to minimize the energy consumption over a longer time horizon over which it can attain better positions to send its bits. The graph is parameterized by the number of states $S'$, and by $L$. Quantizing with 1 state instead of 3 results in a performance loss, which however is small. The reason is that for many of the cells that are far from the base station, the ratio of the standard deviation to the average energy is small. For the same reason, varying $L$ in the range shown has little effect on performance. The second row of Fig. 15 displays the significant energy savings over a constant traffic allocation scheme that allocates the $B$ bits uniformly over the stages.

8 Conclusions and Future Work

We presented a novel framework by which network mobility can be viewed in the aggregate from the perspective of end-to-end QoS metrics. The concepts of coherence time and spreading period, that we defined, are applicable not only to energy but also to other QoS metrics such as end-to-end delay. Forming delay maps must take into account congestion and interference, which can be done by forming congestion, interference and delay maps over the spatial domain. In our future work, we plan to pursue this approach, explore the scaling of this methodology as a function of the number of nodes, and investigate the coherence time and spreading period under increasingly realistic mobility models.
9 APPENDIX

Proof of Theorem 1: Note that for any vector of fixed positions \((x_1, \ldots, x_N) \in D^N\) of the \(N\) nodes, there exists a function \(g : \mathbb{R}^{2N} \rightarrow \mathbb{R}\), which we call the “minimum energy function”, that returns the minimum energy \(g(x_1, \ldots, x_N)\) required to send information from position \(x\) to 0 via the \(N\) nodes. The same function \(g\) maps the random process of positions to the corresponding instantaneous end-to-end energy by: \(E(x, t) = g(X_1(t), \ldots, X_N(t))\). Then,

\[
\lim_{t \to \infty} P\{E(x, t) \leq e\} = \lim_{t \to \infty} \int \cdots \int f(x_1(t), \ldots, x_N(t)) \, dx_1 \cdots dx_N
\]

where the last step above follows by Lebesgue’s Dominated Convergence Theorem, from the fact that \(f(x_1(t), \ldots, x_N(t))\) is bounded above by \(M\). Now, because the limit on the right hand side exists, the limit on the left hand side does as well. This proves the theorem.

Proof of Theorem 2: Note that for every fixed set of positions \((x_1, \ldots, x_N)\) of the nodes, the minimum energy \(g(x_1, \ldots, x_N)\) required to send information from \(x\) to 0, is a bounded function. Then, by steps similar to those in the proof of Theorem 1, the result follows.

Proof of Theorem 3: By steps similar to those in Theorem 2, it follows that \(\lim_{t \to \infty} \mathbb{E}[E^2(x, t)] = \lim_{t \to \infty} \mathbb{E}[\mathbb{E}[E^2(x, t)]^2]\) exists.

Proof of Theorem 4: Let \(r(x, t') = E(x, t') - E(x, t), \forall t' \in \mathcal{R}, \forall x \in D, \text{ and } \hat{n}(x, t') = E(x, t') - E(x, t'), \forall t' \in \mathcal{R}, \forall x \in D\). Let \(m\) be such that \(x \in \mathcal{V}(m)\). Then,

\[
\mathbb{LH} = P\left\{ \left( \frac{1}{N} \sum_{k \in \mathcal{R}} \text{\textbar} \frac{1}{N} \sum_{(x_n, t_n) \in Q_k^{(m)}} \mathbb{E}(t - t_{spr}, t) \right) \right\}
\]

Above, \((a)\) follows from the facts that \(T_{spr} < T_c\), and that \(|r(x, t')| \leq \delta_c\) on \([t - T_c, t]\), and \((b)\) follows by the definition of \(T_{spr}\).

Proof of Theorem 5: Let \(r(x, t) = E(x, t) - E(x, t), \forall t \in \mathcal{R}, \forall x \in D\). Let \(\delta > 0\). Then, \(\mathbb{E}[E^2(x, t)] = \lim_{t \to \infty} \mathbb{E}[E^2(x, t)]^2\) exists. Choose the finite set \(\mathcal{M}\) as a square lattice on \(D\) with spacing \(\sqrt{2}d\). Then, \(\forall x \in D, \forall t \in [t' + \tau, +\infty)\),

\[
P\left\{ \left( \frac{1}{N} \sum_{k \in \mathcal{R}} \mathbb{E}(x_n, t_n) \right) - E(x, t) \right\} > \delta\}
\]

Above, \((a)\) follows from the following facts: The first term in (1) is less than \(\delta/4\) due to the choice of \(\mathcal{M}\); each of the third and fourth terms is less than \(\delta/4\) due to the choice of \(\tau\). Above, \((b)\) follows from the following facts: Even if every node has access to its own samples of \(E(x, t)\) (i.e., no samples are exchanged), due to the hypothesis on the samples, Chebyshev’s inequality applies, with \(T_{spr} = \frac{2\tau}{\gamma}\) samples collected by a node in \(\tau\) seconds. Hence, we have shown that \(\exists \tilde{t} = t' + \frac{2\tau}{\gamma}\), and a set \(\mathcal{M}\) such that \(\forall t \in [\tilde{t} + \infty)\), \(T_{spr}(x, t)\) exists. Further, \(T_{spr}(x, t) < +\infty\); hence, it is bounded in \((\delta, \epsilon)\).

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