

Position Based CDMA with Multiuser Detection (P-CDMA/MUD) for Wireless Ad Hoc Networks

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Abstract — In this paper, we propose a novel position-based MAC protocol based on multiuser detection for wireless ad hoc networks. We solve the duplex scheduling problem in ad hoc networks and allocate user signatures dynamically based on position. Our protocol leverages recent advances in low-power GPS receivers and low-complexity multiuser detectors. By using multiuser detection, we dispense with the notion of collisions at the MAC sublayer and give a throughput comparison between P-CDMA/MUD and 1-persistent CSMA. We show that P-CDMA/MUD can achieve a throughput one to two orders of magnitude over that of 1-persistent CSMA.

I. INTRODUCTION

In this paper, we propose a novel MAC protocol for wireless ad hoc networks that is motivated by two recent advances in low-power RF circuit technology: (1) low-power implementation of GPS receivers that provide accurate position and absolute timing information [2][4], (2) low-complexity multiuser detectors that provide robustness against the near-far problem in DS-CDMA communications [1][6][7][8].

Even though multiuser detection provides near-far resistance [7] (a measure of the detector's robustness in multiple access interference), the near-far resistance of a multiuser detector is degraded in highly asynchronous channels mainly due to the difficulty of designing user signatures with low partial crosscorrelations.

In our protocol, the absolute timing information from each node's GPS receiver is used to set up a network of nodes that is asynchronous by only a maximum fixed number of chips at the CDMA receiver to control the degradation in the BER performance of the multiuser detector to acceptable levels.

Even though simultaneous detection of many users is possible in wireless CDMA systems, a user's transmission and reception (possibly from multiple users) must be separated either in time or frequency. In a wireless ad hoc network, it is a challenge to allocate transmit and receive slots to different nodes, especially for efficient neighbor search and for information dissemination in broadcast and multicast protocols. This challenge due to the duplexing constraint in wireless systems will be called the "duplex scheduling" problem in ad hoc networks.

Another major challenge in the design of a CDMA-based system for ad hoc networks is the dynamic allocation of user signatures to active users in the system. In our protocol, we use the absolute timing information provided by the GPS receiver to solve the duplex scheduling problem and use the

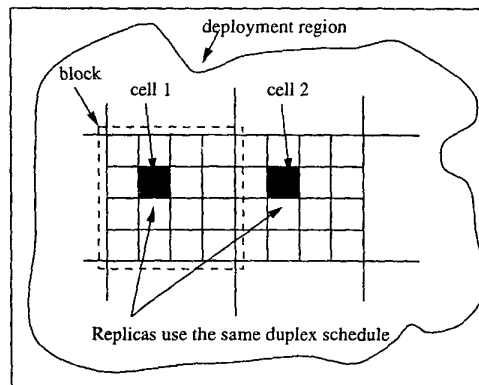


Fig. 1: Concatenation of two blocks on the deployment region

position information to solve the problem of dynamic assignment of duplex schedules and user signatures to nodes in the network.

II. POSITION BASED DUPLEX SCHEDULING

In our model of the wireless ad hoc network, nodes are distributed randomly over a two-dimensional bounded area called the *deployment region*. Each node is assumed to know its own position by virtue of its GPS receiver but not the position of any other node in the network.

In the discussion of the protocol, we will assume that the duplexing scheme is in time (Time Division Duplexing: TDD) rather than in frequency (Frequency Division Duplexing: FDD) to make the discussion more concrete.

Our solution is to assign different duplex schedules to different areas on the deployment region. Under such an assignment, a node would obtain its position from its GPS receiver module and use the duplex schedule associated with the particular area it is in to schedule its transmissions.

In our design, we first take a square block out of the deployment region and divide this block into a grid of square cells, as shown in Figure 1. Then, we assign a duplex schedule to each cell in this block. The duplex schedules must be assigned such that nodes that fall in different cells can communicate with each other. We call this type of communication *inter-cell communication*.

In order to meet this constraint for inter-cell communication, we demand that the duplex schedule assignment to a

Duplex Schedules for $N = 16$		
Cell number	Binary code	Duplex schedule
1	0000	RRRRTTTT
2	0001	RRRTTTTR
3	0010	RRTRTTRT
4	0011	RRTTTTRR
5	0100	RTRRTTTR
6	0101	RTRTRTRR
7	0110	RTTRRRRT
8	0111	RTTTTRRR
9	1000	TRRRRTTT
10	1001	TRRTTRTR
11	1010	TRTRTRTR
12	1011	TRTTTRRR
13	1100	TTRRRRTT
14	1101	TTRRTRTR
15	1110	TTTRRRRT
16	1111	TTTTRRRR

Tab. 1: Duplex schedules for $N = 16$. The second half of each duplex schedule is the reverse of its first half.

block be such that each cell in the block has a chance to communicate with all the other cells within the block at least once during the neighbor update period. The length of the duplex schedule needed to achieve this result within a block is called the “duplex period” for neighbor search. The duplex period must be made as short as possible while still meeting the above constraint.

For N cells, assuming that $\log_2 N$ is an integer, we can attain a duplex period of $2 \log_2 N$ by assigning to a cell the binary representation of its number within the block concatenated with its bit-flipped version to achieve a 2-way communication between every cell by the end of the duplex period. Table II gives the binary code and the duplex schedule for each cell for a design with $N = 16$. In the table, “T” stands for a transmit slot, and “R” stands for a receive slot in time.

If we concatenate these blocks for which we have designed duplex schedules to cover the whole deployment region, we get an efficient way to allow communication between any given cell and almost the rest of the deployment region. This idea of concatenation is illustrated in Figure 1. Under the optimal duplex schedule, cell 1 has a chance to communicate with all the other cells in its block. Since the adjacent block uses the same set of duplex schedules, cell 1 has a chance to communicate with all the cells in the adjacent block except for the cell which uses the same duplex schedule as its own.

III. DYNAMIC ASSIGNMENT OF USER SIGNATURES

We will now discuss the assignment of user signatures to different cells. The two types of interference that must be controlled by the allocation of user signatures over the deployment region are the multiple access interference (MAI) and the co-channel interference (CCI) at each node’s receiver. The MAI at a receiver is the interference caused by other users that employ user signatures which exhibit a partial crosscorrelation with the desired user’s signal. The CCI at a receiver is the interference caused by a user that employs the same user signature as the desired user. The challenge is to allocate user

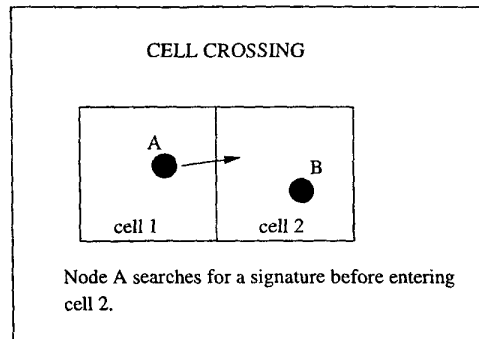


Fig. 2: Node A searches for a new signature before switching cells and uses the new signature after crossing the cell boundary.

signatures dynamically to achieve good performance at every node’s receiver in the network.

In our design, we accommodate up to L users per cell where L is a design parameter. In order to make sure that the inter-cell transmissions of these (at most) L users can be distinguished by users in other cells, we allocate a family of L distinct user signatures to each cell. We call this family of user signatures the “signature family of a cell”. A node entering a new cell must first listen to what user signatures are active in that cell and pick one that is not active and use it for its transmission.

In order to make the transmissions of users in different cells within the same block distinguishable, each signature within a block must be distinct, i.e. the signature families within a block must be disjoint. The set of all the signatures in a block is called the “signature family of a block”, which is the union of the signature families of its N cells.

Because we concatenate blocks to cover the deployment region, the signature families must be placed far apart enough to control the co-channel interference. The *code reuse parameter* M is defined to be the number of distinct signature families of blocks used on the deployment region. Then, the total number of distinct user signatures needed on the deployment region is LM .

The designer must first determine the desired range of transmission and choose the block size to exceed this range. The cell size can be chosen to have one user per cell on average to leverage the inter-cell communication capability of this design.

In mobile networks, as a node moves from one cell to the next, it must switch from using the duplex schedule of the old cell to that of the new cell. It must also find a user signature in the new cell. Assume that Node A falls in cell 1 and uses user signature S_1 as shown in Figure 2. The velocity vector and the current position of Node A obtained from its GPS module can be used to predict that Node A will enter cell 2. Before entering, Node A can use this information to start its search for a new user signature in cell 2 among the family of user signatures for that cell. Once a new signature S_2 has been found, Node A can release its old signature S_1 .

We now address the intra-cell communication. Since two nodes that fall in the same cell would have the same duplex schedule, they would not be able to communicate with

each other under this scheme. For this reason, a separate range of channels for intra-cell communication is required. One strategy for intra-cell communication is to use the traditional CSMA scheme to handle this "residual" communication. Therefore, when there are multiple users in adjacent cells, they will be competing to access the same medium. As a result, under this strategy, the intra-cell communication would have a performance similar to that of CSMA.

IV. EXAMPLE OF SYSTEM DESIGN WITH P-CDMA/MUD

In this section, we give an example of the design of a system that uses P-CDMA/MUD. This example illustrates the choice of the block and cell sizes, the design parameters N , L and M and the spatial allocation for the signature families of blocks.

In this example, the transmission range for nodes is chosen to be 200 meters with omnidirectional antennas. The node density is modeled as uniform over the deployment region with an average of 1 node per 10000 square meters.

In order to attain an average of 1 node per cell, we pick the size of each square cell to be 100 meters on each side. In order to meet the transmission range objective, we choose the side of each block to be 400 meters with 16 cells in one block.

Assume that the required minimum signal to co-channel interference ratio (SCCIR) is 15 dB in this system and that the transmit power roll-off on this terrain is $1/d^4$. Since the roll-off is strong, it suffices to consider the co-channel interference from only the nearest interferers.

Figure 3 displays a design with the code reuse parameter $M = 4$. The number in each cell of the lower left-hand block in the figure identifies the signature family of that cell. This particular spatial allocation of signature families for $M = 4$ minimizes the CCI. The SCCIR must be calculated at the receive node assuming the worst-case positions for the transmit node in its cell and the receive node at the maximum transmission range of the transmit node. It can be shown that the positions of the transmit and receive nodes and the nearest interferers displayed in Figure 3 give the highest (i.e. worst-case) CCI. The CCI power is calculated as the sum of the powers of the nearest interferers as shown in the figure. This design attains a SCCIR of 16.4 dB which meets the specification.

In this particular design, we will support at most $L = 2$ users per cell. As a result, the total number of distinct codes needed on the deployment region is $LNM = 2 * 16 * 4 = 128$. For this design, we pick the family of Gold codes with constraint length $m = 7$ to serve as user signatures. There are 129 such Gold codes. We dispense with one of the Gold codes and use the remaining 128 Gold codes as the required 128 user signatures.

V. THROUGHPUT ANALYSIS OF P-CDMA/MUD

In this section, we develop an analytical model and derive the throughput performance of P-CDMA/MUD. The throughput is defined as the number of useful bits per second received over the total number of useful bits sent to a node.

We use the following model to analyze throughput: From the perspective of any one of the nodes (called the receive node) in the ad hoc network, there are K users transmitting to it, where K is a variable number. The whole discussion in this section is from the perspective of this receive node. The data rate of each transmitting node is D bits per second.

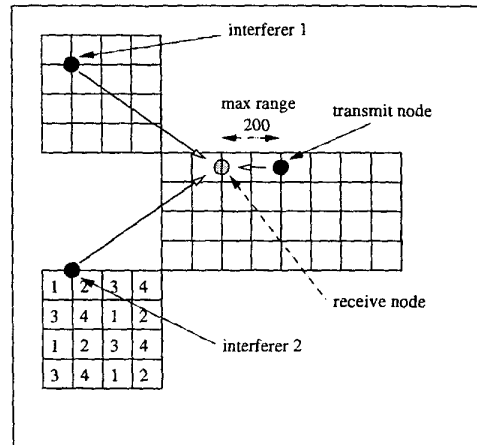


Fig. 3: The computation of co-channel interference from the nearest two interferers

A. CHANNEL AND RECEIVER MODELS FOR P-CDMA/MUD

We model the narrowband interference by a Rayleigh fading channel. For P-CDMA/MUD, we assume that the receiver uses a decorrelating detector, which does not require knowledge of the amplitudes of the K active users and achieves the optimal near-far resistance [7].

B. PROBABILITY OF ERROR AND THROUGHPUT ANALYSIS

The k th user's average probability of bit error under BPSK modulation for a decorrelating detector in Rayleigh fading is given [7] by

$$P_b^{DD}(k) = \frac{1}{2} \left(1 - \frac{1}{\sqrt{1 + \frac{1}{\bar{\eta}_k SNR_k}}} \right) \quad (1)$$

where $\bar{\eta}_k$ denotes the near-far resistance of the decorrelating detector [7] for the k th user and the SNR_k is the average received SNR per bit for the k th user.

Since nothing is known a priori about the K users, we will assume that their SNR's at the receiver are identically distributed with the same Rayleigh parameter. Comparing this expression with the single-user average probability of bit error in Rayleigh fading given [3] by

$$P_b^{single} = \frac{1}{2} \left(1 - \frac{1}{\sqrt{1 + \frac{1}{SNR}}} \right) \quad (2)$$

we see that the loss in the average SNR for the decorrelating detector is by a factor of $\bar{\eta}_k$.

Our strategy is to compensate for this loss in SNR by channel coding for each user in the multiuser system. Because the decorrelating detector completely decouples the transmissions of different users at the cost of a reduction in the SNR of each user by $\bar{\eta}_k$, each user's bitstream can be channel-coded without affecting the performance of other received users.

Necessary Coding Gain		
K	$\frac{1}{\bar{\eta}_k^B}$ (dB)	$\frac{1}{\bar{\eta}_k}$ (dB)
1	0	0
2	0.23	0.17
3	0.49-0.83	0.33-0.68
4	0.91-1.56	0.83-1.45
5	1.25-2.01	0.96-1.74
6	1.91-3.19	1.62-2.65
7	2.24-4.53	2.09-3.72
8	2.99-6.02	2.55-4.67
9	3.13-6.47	2.90-4.97

Tab. 2: Necessary coding gains for P-CDMA/MUD for $P_{out} = 0.01$

$S_{P-CDMA/MUD}$ (kbps)							
K	$\nu = 2$	$\nu = 3$	$\nu = 4$	$\nu = 5$	$\nu = 6$	$\nu = 7$	$\nu = 8$
1	5.25	5.25	5.25	5.25	5.25	5.25	5.25
2	9.65	9.65	9.65	9.65	9.65	9.65	9.65
3	14.5	14.5	14.5	14.5	14.5	14.5	14.5
4	19.3	19.3	19.3	19.3	19.3	19.3	19.3
5	23.6	24.1	24.1	24.1	24.1	24.1	24.1
6	25.6	27.9	28.5	28.9	29.0	29.0	29.0
7	-	27.5	30.5	31.9	32.4	33.0	33.4
8	-	-	-	31.9	34.6	36.0	37.1
9	-	-	-	32.9	36.8	38.1	40.0

Tab. 3: Throughput of P-CDMA/MUD for a single user data rate of $D = 10$ kbps

In this analysis, we will assume that an inner channel coding block brings $P_b^{DD}(k)$ down to P_b^{single} and that an outer channel coding block brings P_b^{single} down to a target probability of error denoted by P_b^{target} . We use this separation into inner and outer coding to make the comparison with the CSMA system easier where the CSMA system uses only the outer block for channel coding. Further, since the rate loss due to the outer encoder is common to both systems, we will not include it in our throughput expression for either system. The code rate of the inner encoder for the k th received user in the P-CDMA/MUD system is denoted by $R_{coding,k}$.

In practical systems, we specify an outage probability, denoted by P_{out} , that specifies the percentage of the time our design is allowed not to meet the P_b^{target} specification. In this analysis, we assume that all time shifts of the K users are equally likely and compute the histogram of $\bar{\eta}_k$ for each user. From the histogram and the P_{out} specification, we determine the minimum coding gain necessary to meet P_b^{target} . The necessary coding gain is thus

$$\gamma_k = \max \left\{ \gamma : P \left\{ \frac{1}{\bar{\eta}_k} < \gamma \right\} \leq P_{out} \right\} \quad (3)$$

In P-CDMA/MUD, a bandwidth of W Hz is shared by all the K transmitting users. If the processing gain is denoted by n , the data rate under BPSK modulation is $D = W/n$ bits per second. The receiver can receive K bitstreams at the same time. If these K nodes were allowed to transmit continuously to the receive node, the throughput of P-CDMA/MUD would be bounded below by

$$S_{P-CDMA/MUD}^{cont} \geq D(1 - P_{out}) \sum_{k=1}^K R_{coding,k} \quad (4)$$

bits per second.

However, according to the duplex scheduling scheme we proposed in Section II, these K nodes can in fact transmit only during their transmit slots. The throughput under a set of duplex schedules assigned to $K + 1$ nodes (including the receive node) over the throughput under continuous transmission from K nodes to the receive node will be called the *duplex schedule efficiency*.

We model the receive node's duplex schedule to be picked randomly from the set of N duplex schedules. The duplex schedules of the K nodes must be different from the receive

node's duplex schedule for the communication to be possible. In our model, the duplex schedule of each of the transmit nodes is picked randomly from the remaining $N - 1$ duplex schedules and independently of each other. For simplicity, we further assume that N is chosen so that $\log_2 N$ is an integer. Under this stochastic model, the duplex schedule efficiency is a random variable whose expected value is computed to be

$$R_{duplex} = \frac{N \log_2 N - 1}{2(N - 1) \log_2 N} \quad (5)$$

which is approximately $1/2$ for large N . Therefore, the average throughput of the P-CDMA/MUD design is given by

$$S_{P-CDMA/MUD} \geq D(1 - P_{out}) R_{duplex} \sum_{k=1}^K R_{coding,k} \quad (6)$$

in bits per second. Since the average duplex schedule efficiency is fixed for a given N , in order to maximize the throughput, we need a channel code for each user to achieve the highest code rate possible while fulfilling a coding gain of γ_k and typically meeting a complexity constraint such as the constraint length of a convolutional code.

C. THROUGHPUT PERFORMANCE OF P-CDMA/MUD

In this section, we examine the design of a particular P-CDMA/MUD system that uses Gold codes with constraint length $m = 5$ and employs a decorrelating detector at the receiver of each node. Since there are 33 Gold codes in this design for the whole deployment region, it can be used in a simple system such as one with design parameters $(L, N, M) = (1, 16, 2)$.

We assume a data rate of $D = 10$ kbps with BPSK modulation and use a chip rate of $Dn = 10 * 31 = 310$ kHz. As a result, the average chip duration is 3.22 microseconds.

We assume that the maximum transmission range is 200 meters and choose the cell size to be 100 meters by 100 meters which implies a block size of 400 meters by 400 meters. For this transmission range, we assume a multipath delay profile where 99% of the energy is concentrated in the range 0 - 6 microseconds. With GPS synchronization, this results in a channel that is asynchronous by at most 2 chips.

For our computations, we assume that the number of active users K is at most 9. The average duplex schedule efficiency

of this design with $N = 16$ is calculated to be 63/120 from Equation 5.

In order to compute $\bar{\eta}_k$ for this design, we first fix K and choose a subset of the Gold codes ($m = 5$) that has K elements. We report the range of values for which the near-far resistance satisfies the outage probability specification of $P_{out} = 0.01$ for each K .

Table 2 displays K and the range of values for $1/\bar{\eta}_k$ in dB which expresses the necessary coding gains as well as $1/\bar{\eta}_k^{LB}$ where $\bar{\eta}_k^{LB}$ is the Lupas-Verdu lower bound [1] for $\bar{\eta}_k$. We will use $1/\bar{\eta}_k$ as the required coding gain in the rest of the paper.

Yasuda et al. have derived high-rate punctured convolutional codes for soft-decision Viterbi decoding [9]. We assume that for a fixed K , all of the received users use the same constraint length ν for the convolutional encoder. Then, for a given K and ν , we first determine the necessary code rate for each of the K users. Then, we use Equation 6 to compute the throughput of P-CDMA/MUD for a given K and ν . The results are displayed in Table 3.

The results show that for a sufficiently high constraint length, the maximum code rate achievable by Yasuda's punctured codes (namely 13/14) can be attained.

VI. PERFORMANCE COMPARISON WITH 1-PERSISTENT CSMA

We use a Rayleigh fading channel model for the CSMA system. The probability of bit error under Rayleigh fading for BPSK modulation is given by Equation 2.

The GPS synchronization makes this a slotted CSMA system. Takagi derived the throughput of slotted 1-persistent CSMA [5] under the following assumptions: There are K users transmitting to a single receive node. The maximum propagation delay is α seconds, and each time slot is chosen to last α seconds. Each node senses the medium at the beginning of every time slot. If it senses a transmission, it does not transmit for the next $1/\alpha$ slots. The probability that a transmission is generated in a time slot is denoted by g . The parameter g is used to optimize S_{CSMA} .

In Table 4, we give a comparison between the maximum throughput achieved by CSMA and the throughput of P-CDMA/MUD. Since the CSMA system uses a bandwidth of W/n and the P-CDMA/MUD system uses a bandwidth of W , we have divided the throughput of P-CDMA/MUD by n for a fair comparison. We refer to $S_{P-CDMA/MUD}/n$ as the normalized throughput. For the CSMA system, we assume a link data rate of $D = 10$ kbps and a maximum propagation delay of 6 microseconds. For the P-CDMA/MUD system, we assumed convolutional encoders with constraint length $\nu = 8$.

The results show that beyond $K = 4$, P-CDMA/MUD has a throughput performance advantage over 1-persistent CSMA that grows quickly with increasing K . The throughput of the CSMA system degrades with increasing number of users whereas the normalized throughput of the P-CDMA/MUD system increases with K . This performance advantage becomes very significant at a high number of users as shown in the fourth column of Table 4.

The normalized throughput of P-CDMA/MUD is always bounded above by $DKR_{duplex}R_{coding}^{max}/n$ where R_{coding}^{max} is the maximum coding gain possible with the employed channel encoders. (With Yasuda's codes, $R_{coding}^{max} = 13/14$). The normalized throughput increases less than linearly with K since the $R_{coding,k}$ necessary to compensate for the loss in SNR_k is higher for larger K . For a given constraint length, since the

K	$S_{CSMA}(\text{bps})$	$S_{P-CDMA}/n(\text{bps})$	Ratio
1	7311	170	0.0233
2	5624	311	0.0553
3	3014	467	0.155
4	1435	623	0.434
5	645	778	1.21
6	280	934	3.34
7	119	1077	9.05
8	49.5	1184	23.9
9	20.3	1291	63.6

Tab. 4: Performance comparison of P-CDMA/MUD and 1-persistent CSMA

necessary coding gain may be achievable only at code rates less than $R_{coding}^{max} = 13/14$, the above expression serves as an upper bound. As the constraint length is increased, the throughput of P-CDMA/MUD will always improve for any K until R_{coding}^{max} is reached for each user.

VII. CONCLUSION

We have demonstrated that P-CDMA/MUD, a novel position-based MAC protocol that uses multiuser detection at the physical layer, has the potential to achieve a throughput much higher than that of 1-persistent CSMA. Whereas the throughput of CSMA degrades quickly as the number of users increases, P-CDMA/MUD achieves data rates one to two orders of magnitude above CSMA by virtue of its capability to detect the bitstreams of multiple users simultaneously.

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