Energy-Efficient Joint Source-Channel Coding for Optical Wireless Underwater Networks

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Abstract—We describe a novel joint source-channel coding scheme for short-range optical wireless links in underwater sensor networks. We present a procedure that maps source symbol sequences to ON-OFF waveforms in the modulation space such that the energy consumption of modulation per source symbol is minimized. We apply this procedure to two settings: the single-dimensional and vector i.i.d. source processes. We show that the energy consumption per source symbol decreases as the length of the modulation waveform increases; however, for a fixed length modulation waveform, this decrease is not monotonic in the length of the source symbol sequence used. We present simulation results for two Gaussian processes that are correlated with each other, and quantitatively demonstrate the effects of the correlation coefficient between the two processes on the minimum energy of modulation that can be achieved per vector source symbol.

Index Terms—optical, underwater, energy, source-channel, network

I. INTRODUCTION

This paper describes a novel joint source-channel coding scheme for short-range optical wireless links in underwater sensor networks. The sensor nodes in underwater networks typically face severe battery limitations, as the sensors must remain underwater for long periods of time. The sensor data can be collected via acoustic or optical communications, and recent results [1] have demonstrated the viability of autonomous underwater vehicles (AUVs) that can collect data from the underwater sensors on optical wireless links. These optical wireless links are able to operate at high energy efficiency (in bits per Joule) over short links, and are hence an attractive option, over acoustic communications, for collecting data from sensors, if AUVs are available.

The main problem addressed in this paper is the development of energy-efficient joint source-channel codes that can achieve high energy efficiency by mapping directly from the source symbols (collected by the sensors) to energy-efficient waveforms. This scenario is found in two applications: (1) When a sensor is communicating with an AUV that probes it via an optical wireless link for sensor data, the sensor sends the data it has collected so far, on an optical wireless link to the AUV, and (2) A sensor node may communicate with other sensor nodes in clear water conditions [2], in a dense sensor network deployment. The first scenario is currently feasible, and has been discussed in [1], and the second scenario might become feasible in the future if the sensor costs can be lowered so that dense, ad hoc networks of sensors can become a reality. We focus mostly on the first scenario in this paper in which the sensor sends its data to an AUV that probes it, via an optical wireless link. Because the sensor node is severely constrained in its energy supply, the transmission from the sensor node to the AUV must be optimized for minimum energy.

Almost all of the results in optical communications in the literature have been developed for the bandwidth-limited regime, where bandwidth rather than energy, has been assumed to be the main limitation to performance. Even for the underwater optical wireless links, past papers [4] have used phase shift keying (PSK) systems that transmit at a constant power, and have analyzed the performance of these pulses under realistic optical channel conditions. However, in underwater sensor networks, because the sensor nodes are severely limited in their battery energy, much more energy-efficient modulation schemes must be utilized than PSK. In this paper, we use ON-OFF waveforms over \( n \) subsequent intervals, each of length \( T \) seconds. Each slot in this waveform can be OFF, where the sensor node spends no energy, or it can be ON, in which case it sends a signal of energy \( E \) at constant amplitude. There are \( 2^n \) such waveform patterns for length \( n \). The pulse length, \( T \), is chosen such that it is much longer than the delay spread of the optical channel. (For short-range optical wireless links, the delay spread occurs mainly due to scattering through the medium, rather than multipath reflections.) Since our focus will be on energy efficiency, we focus on Joules per source symbol as our main cost metric in this paper.

The main idea in this paper is to map from the alphabet of source symbols of the collected sensor measurements directly to the alphabet of these energy-efficient modulation waveforms. As a category, this falls under joint source-channel coding in information theory; however, the basic technique we develop here is very different from classical treatments that aim to map source symbols to the channel codes for bandwidth-limited transmission. In contrast, we aim to derive joint source-channel codes in the energy-limited regime where we want to minimize the average energy used per source symbol, rather than the average codeword length subject to a fixed amount of distortion.

Further, the transmission of few sensor data bits in these networks, rather than long multimedia streams, calls for different techniques that have not been explored in the literature. Here, a small, finite number of bits of data, must be transmitted in an energy-efficient fashion to the receiver. The compression techniques, such as Lempel-Ziv codes, that are suitable for very
long streams of data cannot be used in this setting. Hence, the

 technique that we advocate for tiny, energy-limited sensors is
to map the source (namely the sensor data) directly to energy-

efficient modulation waveforms, bypassing any compression. This
may require an efficient look-up table in hardware, and
we aim to quantify in our future work the hardware energy
costs and the complexity of implementation associated with
this direct mapping.

In this paper, we derive the optimal joint source-channel
coding scheme from a \( k \)-symbol vector i.i.d. channel source
to an \( n \)-symbol modulation scheme, using only ON-OFF
waveforms in each of the \( n \) intervals. We model the correlation
between different measurements (such as temperature and
conductivity) at the same point in space, as correlated Gaussian
random variables.

The rest of this paper is organized as follows: In Section II,
we present our model and problem formulation. In Section III,
we describe how we map the length-\( k \) sequences of source
symbols to length-\( n \) sequences of modulation waveforms such
that either an ON or OFF symbol is used at each slot. In
Section IV, we present our simulation results for both a scalar
and a vector discrete source process, and quantify the
effect of correlation between the two processes on the energy
consumption. In Section V, we present our conclusions and
directions for future work.

II. MODEL AND PROBLEM FORMULATION

Our first problem formulation is as follows: Given a source
(that captures the data collected by the sensor, such as tempera-
ture, or conductivity) with probability distribution \( \{ p(i) \} \), find
the transmission scheme that minimizes the average energy
that it takes to send a finite number \( k \) of i.i.d. source symbols
across the optical wireless link. The problem is important be-
cause it can lead to very energy-efficient designs for operating
short-range optical wireless links underwater.

In this paper, we show the optimal assignment scheme from
a discrete source to energy-efficient waveforms. The mapping
is performed as follows: The sequences of source symbols, each of which is of finite length \( k \), are first ordered according
to their probabilities ranking from the highest probability to
the lowest probability. The highest-probability source symbol
sequence is assigned the length-\( n \) zero-energy waveform, the
next source sequence is assigned a singleton waveform which
consumes the unit energy \( E \) on only one of the \( n \) slots, and
has zero energy on the remaining \( n - 1 \) slots. This is repeated
until all of the source symbol sequences have been assigned.
(The process bears a similarity to filling the energy levels
with electrons in quantum physics, starting with the lowest
energy level, and proceeding toward higher energy levels. The
similarity arises from the objective of minimization of energy.)
It is required that \( |X|^{k} \leq 2^{n} \) (where \( X \) is the discrete source
alphabet) so that all of the source symbol sequences can be
mapped to the waveform set.

The channel waveform length \( n \), is limited potentially by
two possible delay constraints: First, the time that the AUV
spends at each sensor node for data extraction. Second, the
time for which all of the processor circuitry needs to remain
ON, at the sensor node, during the transmission process. We
have found that the second one is the limiting factor for sensor
networks. The AUV can spend on the order of seconds to
minutes at each sensor node (or be passing in its vicinity) whereas having the processor circuitry ON, on the order of
minutes would substantially deplete the energy supply of the
sensor. Hence, the second constraint places a limitation on how
large \( n \) can be. In this paper, we use \( n \) up to 400 for realistic
implementations.

In order to make further progress, we must have a model
of the source process itself, so that we can work with realistic
\( \{ p(i) \} \) distributions. For example, when a thermistor string
collects temperature measurements, the distribution of these
temperature measurements may be modelled by a Gaussian
distribution around an average value (that is modelled as
constant for that time of the day and that region). Assume
that we take the average temperature of that region of the
sea as the base temperature, which is known to both the
sensor node and the AUV that probes the sensor for the set of
collected measurements. The sensor node then needs to send
only the difference between the observed measurement and
this base temperature. Since sensing itself requires the node’s
circuits to be turned on, we usually operate these sensor nodes
to minimize the sensing intervals. If the sensing interval is
chosen such that the subsequent temperature measurements are
roughly statistically decorrelated, unnecessary measurements
in between will be avoided, and the resulting collected values
of differences from the base temperature will be statistically
independent. We first work under this assumption, and assume
that the sensor has collected \( k \) such identically distributed,
independent random variables, where \( k \) is a small number.
When the AUV arrives and probes the sensor node, it must
send these \( k \) source symbols with the minimum amount of
energy possible.

We map from this Gaussian source directly to energy-
efficient channel waveforms. In order to establish a connection
with our discrete symbol alphabet results above, we quantize
the analog Gaussian source to obtain a discrete distribution,
and we apply the techniques that we developed in the first part
to obtain the optimal assignment to energy-efficient channel
waveforms.

In this work, the sensing interval has been chosen large
enough such that the energy for waking up the node for sensing
can be minimized. The sensing intervals can thus be made
roughly on the order of the coherence time of the underlying
(temperature, etc.) process. However, oversampling this is also
of value in real systems, since the coherence time might
change over time. Further, the average of the sensed data might
drift. Such sampling leads to correlated measurements. We
plan to explore the optimal mapping from a correlated source
process to energy-efficient waveforms in our future work.

III. OPTIMAL MAPPING AND AVERAGE ENERGY
CONSUMPTION

We shall focus first on the single-dimensional source
process, which we denote by \( X[m] \). Our source process is
discrete, with an underlying probability mass function \( p_{X}(x) \).
$k$ subsequent samples are taken from this discrete process, and the samples are i.i.d. The modulation waveforms used are ON-OFF waveforms, of length $n$, with a single energy value $E$ for each time slot that uses ON. Note that in this paper, we take the ON-OFF modulation waveforms as a fixed family of waveforms to be used, and we map in a minimum energy fashion to this family. That is, we do not design the optimal waveforms themselves in this paper.

The following example illustrates the mapping from the source process to the modulation waveform: Take $k = 3$, and $n = 4$, and let $p_X(x)$ be Bernoulli with parameter 0.3; that is, it is two-valued with probability mass function ("pmf") $[0.3 \ 0.7]$. Then, list all of the possible source sequences of length $k = 3$, in order of decreasing probability. In this case, the most probable sequence has probability $0.7 \times 0.7 \times 0.7 = 0.343$, and the least probable sequence has probability $0.3 \times 0.3 \times 0.3 = 0.027$. The pmf of the length-3 sequences is $[0.343 \ 0.147 \ 0.147 \ 0.063 \ 0.063 \ 0.063 \ 0.027]$ in order of decreasing probability. Now, our assignment scheme assigns the highest probability sequence (that has probability 0.343) to the all-zero waveform; that is, to the modulation waveform with $[0 \ 0 \ 0 \ 0]$ in all of the time slots. After this, the second most probable sequence with probability 0.147 is assigned to the first “singleton” waveform, that has a single energy $E$ in one of the slots and none elsewhere; namely $[E \ 0 \ 0 \ 0]$. (Note that we denote the energy in each of the time intervals, rather than amplitude.) The third source sequence, with probability 0.147, is mapped to $[0 \ E \ 0 \ 0]$; the fourth one, with probability 0.147, is mapped to $[0 \ 0 \ E \ 0]$; the fifth one, with probability 0.063, is mapped to $[0 \ 0 \ 0 \ E]$; the sixth one, with probability 0.063, is mapped to the first “doubleton” $[E \ E \ 0 \ 0]$; the seventh one, with probability 0.063 is mapped to $[E \ 0 \ E \ 0]$; and the final one, with probability 0.027, is mapped to $[E \ 0 \ 0 \ E]$. We see that, here, we have run out of source sequences at the doubletons, and the remaining doubletons, as well as the higher energy waveforms are unused.

Now, the average energy per source symbol can be calculated as

$$ E = (0.343 \times 0 + 0.147 \times E + 0.147 \times E + 0.147 \times E + 0.063 \times E + 0.063 \times 2E + 0.063 \times 2E + 0.027 \times 2E)/3 = 0.27E $$

In general, let $p_X(i)$ denote the probability mass function of the discrete source process $X[n]$, where $m$ stands for discrete time. Then, the average energy of modulation per source symbol can be written as

$$ \bar{E} = \frac{1}{k} \sum_{s \in S} p(S)e(S) $$

where $S$ is the set of all source sequences of length $k$, $p(S)$ is the probability of source sequence $S$, and $e(S)$ is the energy of the modulation waveform to which the above mapping assigns the sequence $S$.

The average energy per source symbol is the main figure of merit that we care about in these mappings directly from the source space to the modulation space. It can be proved that for the problem of mapping from a discrete source space of $k$ subsequences to the modulation space consisting of only ON-OFF waveforms of energy $E$ for each ON interval, the above procedure that we described is optimal; that is, it is the one that minimizes the average energy per source symbol.

Next, we turn our attention to the vector measurement process, where, for example, the sensor node may collect simultaneous temperature and conductivity measurements. As described in the previous section, we assume that the subsequent samples are taken wide apart to minimize the energy consumption in sensing such that each process is i.i.d., but simultaneous measurements of these two different processes are correlated. For simplicity, assume that there are only two such processes; that is, the vector is of length 2. Then, a joint probability mass function $p_{X,Y}(x, y)$ describes this vector process.

We pose the same question, namely: What is the optimal mapping from the source space of sequences of length $k$ to the modulation space of ON-OFF waveforms, of length $n$? The optimal procedure is the generalization of the one for the 1-D case; however, we also need to include the correlation between the simultaneous measurements of the different processes. We outline the procedure here, again via an example: Assume that $k = 2$, and $n = 4$, and there are two measurement processes with alphabets $X = \{a, b\}$ and $Y = \{0, 1\}$. Their joint pmf is $p_{X,Y}(a, 0) = 0.3$, $p_{X,Y}(a, 1) = 0.1$, $p_{X,Y}(b, 0) = 0.4$, $p_{X,Y}(b, 1) = 0.2$. We again list all the 2-vector sequences of length 2 (since $k = 2$), in terms of decreasing probability. The pmf for such sequences is:

$$ [0.16 \ 0.12 \ 0.12 \ 0.09 \ 0.08 \ 0.08 \ 0.06 \ 0.06 \ 0.04 \ 0.04 \ 0.04 \ 0.03 \ 0.03 \ 0.03 \ 0.02 \ 0.02 \ 0.01] $$

After this, the same procedure of mapping the highest probability sequence to the all-zero waveform, and mapping the next one to the first singleton, etc. is used to fill the waveforms. In this particular case, since $n = 4$, and the pmf has 16 possible source sequences, all of the waveforms will be used; that is, the energy levels will be filled completely. The average energy per source symbol can be calculated via the same technique as before.

Note that energy savings are obtained as the number of modulation intervals, $n$, becomes large. In the limit as $n$ becomes very large, the most probable source sequence is mapped to the zero waveform, and all of the others are mapped to the singleton waveforms. Hence, in the limit as $n$ becomes large (and $k$ remains fixed), the average energy per source symbol will be close to $E/k$ Joules, since all source sequences except one are mapped to singletons in this case.

### IV. Simulations

The purpose of this section is to apply the framework and the procedure developed in the previous section.

#### A. Single Dimensional Source Process

First, we again focus on a single-dimensional process, such as the collected temperature measurements around a mean temperature. Without loss of generality, we shall take the mean to be zero, and assume that the measurements have an
underlying Gaussian distribution around this mean. We obtain a discrete pmf from this by uniformly quantizing this Gaussian distribution.

In our simulations, the variance of the underlying Gaussian distribution is \( \sigma^2 = 3 \), and 5 levels of quantization are used, with the centers at \([-2, -1, 0, 1, +2] \). and the boundaries half-way between these centers. The entire probability mass of the analog Gaussian distribution is assigned to the center of that bin, in constructing the pmf. (Note that the edges +2 and -2 are still called “centers”; they will accumulate all of the probability mass from the tails of the Gaussian distribution.)

We apply the procedure developed in the previous section, and investigate the average energy consumption per source symbol as a function of the waveform length \( n \), and parameterize the results with the number of subsequent source symbols \( k \). In our graphs, we present the “normalized” energy per source symbol, where the normalization has been performed by \( E \). Fig. 1 displays the results for \( k \in \{3, 5, 7, 9\} \) and \( 22 \leq n \leq 400 \). First, we see that the average energy per source symbol decreases as a function of \( n \), for fixed \( k \); however, the behavior is not monotonic in \( k \) for fixed \( n \).

In order to interpret the results in more detail, examine the curve for \( k = 3 \) in Fig. 1. Note the cusp that occurs at \( n = 125 \), from which point onwards, the normalized energy per source symbol is flat. Because the source has 5 possible symbols (the 5 quantized levels) in this simulation set-up, beginning with \( n = 5^3 \), all of the sequences except one are mapped to the singleton waveform, and one of them is mapped to the zero waveform. Hence, the energy per source symbol is approximately \( 1/k = 1/3 = 0.333 \).

A similar result would be observed for \( k = 5 \) at \( n = 5^5 \), and similarly for higher \( k \)’s at \( n = 5^k \); however, we have not graphed beyond \( n = 400 \) in this figure.

We also see that, in general, even before this regime is reached, the curves for \( k \in \{5, 7, 9\} \) have cusps with distinct regions between the cusps. These cusps occur at points where \( n \) reaches the critical point where a marginal increase in \( n \) will allow us to use only the lower energy levels; e.g. when it allows us to drop from using \( 3E \) as an energy level to only doubletons, singletons, and the zero waveform.

Figures such as Fig. 1 are useful because based on such plots, we can determine the \( n \) required to achieve a certain energy consumption target.

**B. Vector Source Process**

We now turn to a vector, i.i.d. source process, of two types of measurements (such as temperature and conductivity) that are correlated for simultaneous measurements, but are assumed to be taken at wide apart intervals such that there is no temporal correlation between samples.

We apply the procedure that we described in Section III. In our simulation set-up, we have two processes \( X[n] \) and \( Y[n] \), which are discretized versions of an underlying jointly Gaussian 2-vector random process with correlation coefficient \( \eta \) between two simultaneous measurements of two different types.

More precisely, the underlying analog processes \( X(t) \) and \( Y(t) \) are jointly Gaussian with covariance matrix \( K_{XY} = [\sigma_X^2 \ \eta \sigma_X \sigma_Y; \ \eta \sigma_X \sigma_Y \ \sigma_Y^2] \). In our simulations, we take \( \sigma_X^2 = 3 \) and \( \sigma_Y^2 = 5 \). We vary the value of \( \eta \) in order to investigate the effects of correlation between the two measurement processes, on the normalized energy consumption per source symbol. The discretization of this joint process is performed as follows: Again, 5 levels are used along each axis. The centers of the bins along the \( x \) axis are \([-2.0, -1.0, 0.0, +1.0, +2.0] \) as was the case for the single-dimensional process. The centers along the \( y \) axis are \( \sqrt{\frac{5}{2}}[-2.0, -1.0, 0.0, +1.0, +2.0] \); that is, it is scaled with respect to the difference in the standard deviations of the two processes. Hence, each two-dimensional bin is a rectangle of size 1.0 by \( \sqrt{\frac{5}{2}} \). All of the probability
mass that falls into that bin is assigned to the center for that bin. (Note that the edge values are still called “center”’s.)

Fig. 2 shows the normalized energy consumption per source symbol as a function of the waveform length $n$, for $k \in \{3, 5\}$, and $\eta = 0.1$. Hence, the two processes are only slightly correlated. We see that, for fixed $k$, the energy consumption is monotonically decreasing in $n$. Just as in the single-dimensional case, this decrease is not a concave function of $n$, and displays cusps, as shown, at the critical points where an energy level becomes unfilled, and the highest energy level ceases to be used. We also see that the curve for $k = 3$ is above that for $k = 5$; however, this is in general, not always true, as the curves for different $k$’s may criss-cross, as was the case for the single-dimensional source in the previous subsection.

Fig. 3 shows the normalized energy consumption per source symbol as a function of $n$, for $\eta = 0.8$; that is, the simultaneous measurements of the two processes are highly correlated in this case. The same trend of a monotonic, though non-concave, decrease in $n$ holds. When we compare this with Fig. 2, we see that the energy consumption is lower for $\eta = 0.8$ than for $\eta = 0.1$. The reason is that the higher the correlation between the two processes, the more this correlation can be exploited to reduce the energy consumption per source symbol. This result bears some resemblance to the fact that, in the bandwidth-limited regime, the higher the correlation, the more we can “compress” the bitstream to a few bits. In that case, we would be aiming to maximize the usage of each time slot that goes by. In our case, time slots themselves are not the main constraint, but energy is. Hence, we cannot speak of “compression”, but we see that the energy consumption per source symbol can be reduced by exploiting the correlation between two different measurements that are taken simultaneously. (Note that our processes here do not show temporal correlation, and are i.i.d. in time. But we expect similar results to hold when temporal correlation is introduced.)