Predictive Fusion Coding of Spatio-temporally Correlated Sources

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Coding of Correlated Sources …

- Early interest in source coding with side-info (Slepian-Wolf (1973), Wyner-Ziv (1976))
- Other flavors: multi-terminal source coding, distributed source coding
- Applications: distributed compression in sensor networks (DISCUS (1999), Network VQ (2001))

\[ \mathcal{X}, \mathcal{Y}, \mathcal{Z} \]
Coding Correlated Sources for Storage

- New setting: Storage Media
- **Joint encoding/compression/storage of sources**
- **Selective retrieval of sources**

\[
X(Q) = X_j X_m X_l X_k
\]

Storage Rate \( R_s \) vs. Retrieval Rate \( R_r \) vs. Distortion \( D \)
Minimizing Storage Rate

- Compress all sources together
  - minimizes storage
  Lossless Coding $\Rightarrow R_s = H(X_1, \ldots, X_M)$

- Retrieves all stored data for *all queries*
  - high retrieval time!
  $R_r = R_s = H(X_1, \ldots, X_M)$
Minimizing Retrieval Rate

- Compress each subset separately
  - minimizes retrieval rate/time

Lossless Coding \( \Rightarrow R_r = \sum q \ P(q) H(X_{(q)}) \)

- Req'd storage grows with size of query set
  - (combinatorially) high storage rate!

\[ R_s = \sum q \ H(X_{(q)}) \gg H(X_1, \ldots, X_M) \]
Impact/Applications

- Storage, search and retrieval of correlated streams of data e.g. from sensor networks, stocks

A 2D Sensor Field: boxes are regions of interest
Prior Work on Fusion Storage Coding

• Achievable rates (lossless coding) characterized by Nayak et al (2005)
• “Lossy” fusion coders by Ramaswamy et. al. (2007)
• Storage devices have fixed (limited) storage capacity ($R_s$)
• Allowed $R_s$, trade-off between distortion ($D$) and retrieval rate ($R_r$) optimized:
  \[ \min D + \lambda R_r \]
The Fusion Coder (FC)

Encoder: \( \mathcal{E} \)

Bit-Selector: \( \mathcal{S} \)

Decoder: \( \mathcal{D} \)

Correlated Sources: \( X_1 \ldots X_M \)

Query-dependent bit-(subset) selection

Query: \( Q \sim P(q) \)

Design by Iterative Optimization
Exploiting time-correlations

• Sensor data exhibit time-correlations
  ⇒ fusion code over large blocks (?)
• Coding over large blocks impractical
  - encoding complexities $O(2^{NR_s})$
• Predictive coding – a low complexity alternative
Optimal Predictive Fusion Coding

\[ X_{(Q)} \rightarrow PZ^{-1} \rightarrow D \rightarrow S \rightarrow \mathcal{E} \rightarrow R_s \]

Query \( Q \sim P(q) \)
Complexity of Optimal Predictive Fusion Coding

• $Q$ - set of queries
• $|Q|$ prediction loops necessary
• $|Q|$ prediction error residuals
  - grows (combinatorially) with sources
• Dimensionality of input to encoder = $M|Q|
  - $M|Q| \gg M \Rightarrow$ high-complexity!!
Constrained Predictive Fusion Coding

- Constraints imposed for practical designs
- Allow only K prediction loops
- K chosen according to complexity possible
- Queries “share” the K predictors
- Zero “drift” between encoder and decoder
- Prediction bit-selector $S_P$ vs. Query bit-selector $S(q)$
Constrained Predictive Fusion Coding: Encoder

Only One Predictor i.e. K=1

\[ X \xrightarrow{(-)} e \xrightarrow{\epsilon} R_s \xrightarrow{S_P} D_P \xrightarrow{\hat{e}} PZ^{-1} \xrightarrow{\sim X} X \]

\[ \begin{bmatrix} 0 & 1 & \ldots \\ 1 & 0 & \ldots \\ 0 & 1 & \ldots \\ 0 & 0 & \ldots \end{bmatrix} \]
Constrained Predictive Fusion Coding: Decoder

STORED BITS

0 1 ...
1 0 ...
0 1 ...
0 0 ...

$S_p$ $\rightarrow$ $D_p$ $\rightarrow$ $^\wedge e$ $\rightarrow$ $PZ^{-1}$

$S$ $\rightarrow$ $D$ $\rightarrow$ $^\wedge e_{(Q)}$ $\rightarrow$ $^\wedge x_{(Q)}$

Query $Q \sim P(q)$
Issues in Predictive Coder Design

• Open loop design
  – generate prediction residuals separately
  – design quantizer for residuals
  – codebooks & predictor mismatched

• Closed loop design
  – close prediction loop; iteratively design encoder & decoder
  – residuals (training set) change unpredictably during design
  – unstable (feedback loop) at low rates
Asymptotic Closed Loop Design

• Always design in open loop
  ⇒ stable design
• Gradually change training set in between design iterations
• Asymptotically loop is closed
  ⇒ no mismatch of codebooks and predictor
• ACL design necessary for PFC design
  - since $R_s < M$ (low compression rate)
ACL Update Rules

1. Reconstruct source sequence $\hat{X}(n)=\tilde{X}(n)+\hat{e}(n)$
   - for next iteration, new $\tilde{X}(n)=P\hat{X}(n-1)$

2. Create new prediction residuals $e(n)=X(n)-\tilde{X}(n)$
   (in one go, avoiding the prediction loop)

3. Update all encoder and decoder mappings

4. Evaluate cost.
   If converged STOP,
   ELSE go to step 1

FOR DETAILS, REFER PAPER
Experiments

- M Correlated 1st order Gauss-Markov sources
- $E(W_i W_j) = \rho_{ij} = \rho^{|i-j|}$ \equiv linear sensor array
- $X_m(n) = \beta_m X_m(n-1) + W_m(n), \quad \forall \ 1 \leq m \leq M$
- "Neighborhoods" of $n$ sources queried

- $M=100$ sources, $\rho=0.95$, $\beta_m=0.8 \quad \forall m$
- $n=10$, Uniform query distribution, $|Q|=91$
Results

$M=100$ sources, STORAGE CAPACITY $R_s=6$ bits/sample

![Graph showing distortion vs. retrieval rate for different compression techniques.](image)
Conclusions

• Fusion coding of correlated sources - an important *storage* problem
• Exploit time-correlations by prediction
• Optimal predictive fusion coder (PFC) has high encoder complexity
• Constrained PFC designed by ACL principle
• Significant gains over memoryless FC and joint compression (VQ)