Noncooperative Eigencoding for MIMO Ad hoc Networks

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***Abstract*—A new noncooperative eigencoding algorithm is introduced for multiple-input multiple-output (MIMO) ad hoc networks. The algorithm performs generalized waterfilling with respect to its transmit covariance matrix and the receive node covariance matrix, both of which can be estimated locally. A cooperative algorithm is also developed as a benchmark in simulations. Simulation results show that the noncooperative algorithm performance is close to that of the cooperative method and is much better than greedy optimization. A game theoretic interpretation of the algorithm is also provided.**

***Index Terms*—Ad hoc networks, game theory, generalized eigencoding, multiple-input multiple-output (MIMO), waterfilling.**

I. Introduction

The problem of minimizing total transmit power in an ad hoc network under capacity constraints is considered. Each node employs multiple antennas so as to increase the channel capacity by exploiting multipath scattering [1], [2]. The joint transmitter–receiver optimization involves multiple-input multiple-output (MIMO) precoding and decoding at the transmitter and the receiver. The problem of precoding/decoding under minimum mean square error (MMSE) optimization criteria for a single point-to-point link has been extensively studied in [3]–[8]. The application of MMSE techniques to multiuser optimization for a single-cell MIMO network has also been addressed in [9]–[11]. In the context of ad hoc networks, the precoder optimization was considered in [12]. In [12], the authors formulate the problem as maximizing the total capacity subject to power constraints at each node and also introduce a centralized optimization procedure based on gradient projection. Though a global optimum is not guaranteed, it was shown by simulation in [12] that the centralized algorithm performs best.

The idea of using local information to enable distributed optimization in a MIMO network is presented in [13]. A time-division duplexing (TDD) network is considered in [13], where the objective is to maximize the minimum capacity of each substream in the network. For TDD, the transmit vector proportional to the complex conjugate of the receive vector is shown to be optimal in [13]. Another approach to finding the transmit vector without explicitly estimating the channel matrix is given in [14]. The technique used in [14] is based on the power algorithm to find the dominant eigenvector and is then extended to full eigencoding.

We previously considered distributed beamforming for ad hoc networks in [15]. This paper extends the results of [15] to full eigencoding. The main results of this paper can be summarized as follows: a) the concept of a total interference function (TIF) previously developed for spatial beamforming is extended here to full eigencoding; b) a new algorithm, Noncooperative Generalized Eigencoding with Taxation (NGET), is developed to minimize the TIF while maintaining the quality of service (QoS) for every pair of nodes; c) the NGET algorithm is interpreted using a game theoretic approach; d) the existence of Nash equilibrium points for NGET is proven; and (e) a centralized optimization algorithm is developed as a benchmark for NGET.

Section II presents the narrowband MIMO ad hoc network model, the optimization problem, and the TIF function. The NGET algorithm is presented in Section III. Section IV presents numerical simulation results for the NGET, greedy, and centralized optimization algorithms.

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II. Narrowband Network Model

Consider an ad hoc network of N nodes with each unicasting node i communicating with node $l(i), i = 1 \ldots N$. Each node uses M transmit and M receive antennas. The channels are assumed to be flat fading. Let $\mathbf{H}_{i,j} \in \mathbb{C}^{M\times M}$ be the channel response from node j to node i. Channel reciprocity is assumed [13] such that for every i, j we have $\mathbf{H}_{i,j} = \mathbf{H}_{j,i}^T$.

Let $\mathbf{G}_i \in \mathbb{C}^{M\times M}$ and $\mathbf{P}_i \in \mathbb{R}^{M\times M}$ be the normalized transmit precoder and power matrix of node i. The unnormalized transmit precoder of node i is then $\tilde{\mathbf{G}}_i = \mathbf{G}_i\mathbf{P}_i^{1/2}$, where $\mathbf{P}_i$ is a diagonal matrix with nonnegative entries and $\mathbf{G}_i$ has unit norm columns. Let $\mathbf{b}_i \in \mathbb{C}^M$ be the symbol vector sent from node i to node $l(i)$. The signal received at node $l(i)$ is

$$\mathbf{r}_{l(i)} = \mathbf{H}_{l(i),i}\tilde{\mathbf{G}}_i\mathbf{b}_i + \sum_{k\neq i,l(i)} \mathbf{H}_{l(i),k}\tilde{\mathbf{G}}_k\mathbf{b}_k + \mathbf{n}_{l(i)}. \tag{1}$$

In (1), the first term represents the desired signal from node i, while the second term is multiuser interference, and the third term is circular white Gaussian noise. It is assumed w.l.o.g. when all receivers have the same noise figure that $\mathrm{E}\{\mathbf{n}_{l(i)}\mathbf{n}_{l(i)}^H\} = \mathbf{I}$. The decoupled capacity of the channel from node i and node $l(i)$ is then [12]

$$c_{l(i)} = \log\left|\mathbf{I} + \tilde{\mathbf{G}}_i^H\mathbf{H}_{l(i),i}^H\mathbf{R}_{l(i)}^{-1}\mathbf{H}_{l(i),i}\tilde{\mathbf{G}}_i\right| \tag{2}$$

where $\mathbf{R}_{l(i)}$ is the interference plus noise covariance at node $l(i)$ and can be computed as follows:

$$\mathbf{R}_{l(i)} = \mathbf{I} + \sum_{k\neq i,l(i)} \mathbf{H}_{l(i),k}\tilde{\mathbf{G}}_k\tilde{\mathbf{G}}_k^H\mathbf{H}_{l(i),k}^H. \tag{3}$$

The objective is to minimize the total transmit power in the network, while maintaining channel capacity $c_{l(i)} \geq r$ at all $l(i)$.

The network optimization problem is then

$$\begin{aligned}&\text{minimize} && \sum_{i=1}^{N}\mathbf{Tr}(\tilde{\mathbf{G}}_i\tilde{\mathbf{G}}_i^H),\\ &\text{subject to} && \log\left|\mathbf{I} + \tilde{\mathbf{G}}_i^H\mathbf{H}_{l(i),i}^H\mathbf{R}_{l(i)}^{-1}\mathbf{H}_{l(i),i}\tilde{\mathbf{G}}_i\right| \geq r, \quad i = 1,\ldots N.\end{aligned} \tag{4}$$

Optimizing the total network transmit power in (4) directly is a nonconvex problem and does not have a closed-form solution. The greedy algorithm in [12] minimizes $\mathrm{Tr}(\tilde{\mathbf{G}}_i\tilde{\mathbf{G}}_i^H)$ independently at each node i while considering the interference from other users as colored Gaussian noise. This approach results in a waterfilling procedure for each pair of nodes as in [1].

In the following, we will present an alternative objective function called the total interference function (TIF). For each transmission link, instead of minimizing the transmit power directly, each node minimizes the TIF. The TIF is the total interference power of the substreams of users in the network.

$$\mathrm{TIF} = \sum_i \sum_{k \neq i, l(i)} \mathbf{Tr}\left(\tilde{\mathbf{G}}_i^H \mathbf{H}_{i,k}^* \tilde{\mathbf{G}}_k^* \tilde{\mathbf{G}}_k^T \mathbf{H}_{i,k}^T \tilde{\mathbf{G}}_i\right) + 2\sum_i \mathbf{Tr}(\tilde{\mathbf{G}}_i \tilde{\mathbf{G}}_i^H). \tag{5}$$

The TIF function defined here is a generalized version of that defined in [15] and is similar to total squared correlation in [16]. The interference interpretation follows from the matrices $\tilde{\mathbf{G}}_i^H \mathbf{H}_{i,k}^* \tilde{\mathbf{G}}_k^* \tilde{\mathbf{G}}_k^T \mathbf{H}_{i,k}^T \tilde{\mathbf{G}}_i$. These can be interpreted as the covariance of the interference vector at node k as the result of transmission at node i when we set the decoder at node k, $\tilde{\mathbf{W}}_k$, equal to the complex conjugate of the precoder at that node: $\tilde{\mathbf{W}}_k = \tilde{\mathbf{G}}_k^*$. It is emphasized that when channel reciprocity holds, setting the decoder proportional to the complex conjugate of the precoder at each node is a standard technique that has been widely applied in wireless communications for cellular networks [17], [18] and for ad hoc networks [19], [13], [15]. The term $\mathbf{Tr}(\tilde{\mathbf{G}}_i \tilde{\mathbf{G}}_i^H)$ in (5) represents the thermal noise contribution at node i when the decoder is $\tilde{\mathbf{W}}_i = \tilde{\mathbf{G}}_i^*$. The TIF function can be decomposed as

$$\mathrm{TIF} = 2\mathrm{Tr}\left(\tilde{\mathbf{G}}_i^H \mathbf{R}_i^* \tilde{\mathbf{G}}_i\right) + f(\tilde{\mathbf{G}}_{-i}) \tag{6}$$

where here the game theoretic notation $f(\tilde{\mathbf{G}}_{-i})$ implies functional dependence on all $\tilde{\mathbf{G}}_l, l \neq i$. Equation (6) is very important in the sense that it is decomposable. To minimize TIF with respect to $\tilde{\mathbf{G}}_i$, we only need to minimize the first term in (6), and the only knowledge required is the interference covariance matrix, which can be estimated locally.

III. Noncooperative Generalized Eigencoding for MIMO Networks

At each node, the proposed noncooperative optimization algorithm solves the local problem

$$\begin{aligned} &\text{Minimize} \quad \mathbf{Tr}\left(\tilde{\mathbf{G}}_i^H \mathbf{R}_i^* \tilde{\mathbf{G}}_i\right), \\ &\text{subject to} \quad \log\left|\mathbf{I} + \tilde{\mathbf{G}}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1} \mathbf{H}_{l(i),i} \tilde{\mathbf{G}}_i\right| \geq r, \quad i = 1 \dots N \end{aligned} \tag{7}$$

where r is a predefined transmission rate and $\mathbf{R}_{l(i)}$ is held fixed. Note that (7) corresponds to minimization of TIF with regard to $\tilde{\mathbf{G}}_i$

Proposition 1: At the optimal point, the constraints in (7) are active, that is the inequalities in (7) become equalities.

Proof: We generalize [18]. Suppose that at the optimum point there exists a user i such that the inequality in (7) is strict, then we can reduce the transmit power of that user by replacing its transmit matrix by $(1-\epsilon)\tilde{\mathbf{G}}_i$, where $\epsilon > 0$ is arbitrarily small. The transmit powers of other users do not change, but the transmit power of user i decreases; therefore, both the total transmit power and the objective function in (7) decrease. This contradicts the optimality of the current point. ■

Theorem 1: A (nonunique) solution $\tilde{\mathbf{G}}_i$ to the local optimization problem (7) is the generalized eigenmatrix of $\mathbf{A}_i = \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1} \mathbf{H}_{l(i),i}$ and $\mathbf{B}_i = \mathbf{R}_i^*$, with $\mathbf{A}_i \tilde{\mathbf{G}}_i = \mathbf{B}_i \tilde{\mathbf{G}}_i \mathbf{\Lambda}_i^{AB}$. The elements of the diagonal matrix $\mathbf{\Lambda}_i^{AB} \in \mathbb{R}^{M \times M}$ are the generalized eigenvalues of $\mathbf{A}_i$ and $\mathbf{B}_i$.

Proof: Define Cholesky decomposition $\mathbf{b}_i = \mathbf{L}_i \mathbf{L}_i^H$ and transformed precoder $\bar{\mathbf{G}}_i = \mathbf{L}_i^H \tilde{\mathbf{G}}_i$. The optimization problem (7) is rewritten as

$$\begin{aligned} &\text{minimize} \quad \mathrm{Tr}(\bar{\mathbf{G}}_i^H \bar{\mathbf{G}}_i) \\ &\text{subject to} \quad \log\left|\mathbf{I} + \bar{\mathbf{G}}_i^H \mathbf{L}_i^{-1} \mathbf{A}_i (\mathbf{L}_i^{-1})^H \bar{\mathbf{G}}_i\right| \geq r. \end{aligned} \tag{8}$$

Let $\mathbf{Q}_i = \bar{\mathbf{G}}_i \bar{\mathbf{G}}_i^H$, and $\mathbf{K}_i = \mathbf{L}_i^{-1} \mathbf{A}_i (\mathbf{L}_i^{-1})^H$. Let $\mathbf{U}_i$ be the eigenmatrix of $\mathbf{K}_i$. The spectral decomposition of $\mathbf{K}_i$ is

$$\mathbf{K}_i = \mathbf{U}_i \mathbf{\Lambda}_i^K \mathbf{U}_i^H$$

where $\mathbf{\Lambda}_i^K$ is diagonal. Applying matrix properties: $|\mathbf{I} + \mathbf{AB}| = |\mathbf{I} + \mathbf{BA}|$ and $\mathrm{Tr}(\mathbf{AA}^H) = \mathrm{Tr}(\mathbf{A}^H \mathbf{A})$, the optimization in (8) becomes

$$\begin{aligned} &\text{minimize} \quad \mathrm{Tr}(\mathbf{Q}_i) \\ &\text{subject to} \quad \log\left|\mathbf{I} + \mathbf{U}_i^H \mathbf{Q}_i \mathbf{U}_i \mathbf{\Lambda}_i^K\right| \geq r. \end{aligned} \tag{9}$$

Denote $\tilde{\mathbf{Q}}_i = \mathbf{U}_i^H \mathbf{Q}_i \mathbf{U}_i$. Because $\mathbf{U}_i$ is unitary $\mathrm{Tr}(\tilde{\mathbf{Q}}_i) = \mathrm{Tr}(\mathbf{Q}_i)$. For fixed $\mathrm{Tr}(\tilde{\mathbf{Q}}_i)$, by the Hadamard inequality [20]

$$\begin{aligned} \log\left|\mathbf{I} + \mathbf{U}_i^H \mathbf{Q}_i \mathbf{U}_i \mathbf{\Lambda}_i^K\right| &= \log\left|\mathbf{I} + \tilde{\mathbf{Q}}_i \mathbf{\Lambda}_i^K\right| \\ &\leq \sum_{k=1}^{M} \log(1 + \tilde{\mathbf{Q}}_i(k,k) \mathbf{\Lambda}_i^K(k,k)) \end{aligned} \tag{10}$$

where $\tilde{\mathbf{Q}}_i(k,k)$ and $\mathbf{\Lambda}_i^K(k,k)$ are the kth diagonal elements of $\tilde{\mathbf{Q}}_i$ and $\mathbf{\Lambda}_i^K$. The equality occurs iff $\tilde{\mathbf{Q}}_i$ is diagonal (because $\mathbf{\Lambda}_i^K$ is diagonal). Note that the condition that $\mathrm{Tr}(\tilde{\mathbf{Q}}_i)$ is fixed involves only diagonal elements of $\tilde{\mathbf{Q}}_i$ and the right-hand side (RHS) of (10) also involves only diagonal elements of $\tilde{\mathbf{Q}}_i$. This means that if the optimum $\tilde{\mathbf{Q}}_i$ is not diagonal, we can set all the nondiagonal elements of $\tilde{\mathbf{Q}}_i$ to zero and thereby make the constraint in (8) inactive without violating the condition that $\mathrm{Tr}(\tilde{\mathbf{Q}}_i)$ is fixed. But an inactive constraint contradicts Proposition 1, and therefore $\tilde{\mathbf{Q}}_i$ must be diagonal.

Therefore

$$\tilde{\mathbf{Q}}_i^{(-1)/(2)} \mathbf{U}_i^H \mathbf{Q}_i \mathbf{U}_i \tilde{\mathbf{Q}}_i^{(-1)/(2)} = \tilde{\mathbf{Q}}_i^{-\frac{1}{2}} \mathbf{U}_i^H \bar{\mathbf{G}}_i \bar{\mathbf{G}}_i^H \mathbf{U}_i \tilde{\mathbf{Q}}_i^{-\frac{1}{2}} = \mathbf{I}.$$

This means that $\mathbf{T} = \bar{\mathbf{G}}_i^H \mathbf{U}_i \tilde{\mathbf{Q}}_i^{-\frac{1}{2}}$ is a unitary matrix or

$$\bar{\mathbf{G}}_i = \mathbf{U}_i \tilde{\mathbf{Q}}_i^{\frac{1}{2}} \mathbf{T}^H.$$

The optimal solution only depends on $\mathbf{Q}_i$. Thus, if $\bar{\mathbf{G}}_i$ is an optimal solution, then the product of $\bar{\mathbf{G}}_i$ with an arbitrary unitary matrix is also optimal. Therefore, $\bar{\mathbf{G}}_i$ can be replaced by $\bar{\mathbf{G}}_i \mathbf{T}$ yielding

$$\bar{\mathbf{G}}_i = \mathbf{U}_i \tilde{\mathbf{Q}}_i^{\frac{1}{2}}.$$

Because $\tilde{\mathbf{Q}}_i$ is diagonal and $\mathbf{U}_i$ is an eigenmatrix of $\mathbf{K}_i$, $\bar{\mathbf{G}}_i$ is an eigenmatrix of $\mathbf{K}_i$ with unnormalized columns. Thus

$$\mathbf{L}_i^{-1} \mathbf{A}_i (\mathbf{L}_i^{-1})^H \bar{\mathbf{G}}_i = \bar{\mathbf{G}}_i \mathbf{\Lambda}_i^{AB}$$

where $\mathbf{\Lambda}_i^{AB}$ is diagonal and $\bar{\mathbf{G}}_i$ is orthogonal. Substituting back $\bar{\mathbf{G}}_i = \mathbf{L}_i^H \tilde{\mathbf{G}}_i$, we have

$$\mathbf{A}_i \tilde{\mathbf{G}}_i = \mathbf{L}_i \bar{\mathbf{G}}_i \mathbf{\Lambda}_i^{AB} = \mathbf{L}_i \mathbf{L}_i^H \tilde{\mathbf{G}}_i \mathbf{\Lambda}_i^{AB} = \mathbf{B}_i \tilde{\mathbf{G}}_i \mathbf{\Lambda}_i^{AB}. \tag{11}$$

Thus, $\tilde{\mathbf{G}}_i$ is the generalized eigenmatrix of $\mathbf{A}_i$ and $\mathbf{B}_i$. ■

Because $\tilde{\mathbf{G}}_i$ is the generalized eigenmatrix of $\mathbf{A}_i$ and $\mathbf{B}_i$, $\mathbf{G}_i$ is also the generalized eigenmatrix of $\mathbf{A}_i$ and $\mathbf{B}_i$. Note that $\mathbf{G}_i$ is not an orthonormal matrix unless $\mathbf{B}_i = \alpha\mathbf{I}$ for some $\alpha \in \mathbb{R}$. However, the generalized eigenmatrix $\mathbf{G}_i$ has the following diagonalization properties [21]:

$$\mathbf{G}_i^H\mathbf{A}_i\mathbf{G}_i = \mathbf{\Lambda}_i^A, \quad \mathbf{G}_i^H\mathbf{b}_i\mathbf{G}_i = \mathbf{\Lambda}_i^B \tag{12}$$

where $\mathbf{\Lambda}_i^A$ and $\mathbf{\Lambda}_i^B$ are diagonal. Substitute $\tilde{\mathbf{G}}_i = \mathbf{G}_i\mathbf{P}_i^{(1)/(2)}$ into the objective and constraint functions to obtain a new local optimization problem, as follows:

$$\begin{aligned}\text{minimize} \quad & \mathrm{Tr}(\mathbf{P}_i^{1/2}\mathbf{G}_i^H\mathbf{B}_i\mathbf{G}_i\mathbf{P}^{1/2}) \\ & = \sum_{k=1}^{M}\mathbf{P}_i(k,k)\mathbf{\Lambda}_i^B(k,k), \\ \text{subject to} \quad & \log|\mathbf{I} + \mathbf{P}^{1/2}\mathbf{\Lambda}_i^A\mathbf{P}^{1/2}| \\ & = \sum_{k=1}^{M}\log(1+\mathbf{P}_i(k,k)\mathbf{\Lambda}_i^A(k,k)) \geq r \end{aligned} \tag{13}$$

where $\mathbf{P}_i(k,k)$, $\mathbf{\Lambda}_i^A(k,k)$ and $\mathbf{\Lambda}_i^B(k,k)$ are the kth elements of the respective diagonal matrices. The objective function (13) is linear in $\mathbf{P}_i(k,k)$ while the constraint sets in (13) are convex. This combined with the constraints that $\mathbf{P}_i(k,k) \geq 0$ which is also convex makes the optimization problem convex. Any local solution is thus global. The problem can be solved by the generalized waterfilling algorithm. The Lagrangian is

$$\begin{aligned} L(\mathbf{P}_i, \boldsymbol{\mu_i}) = & \sum_{k=1}^{M}\mathbf{P}_i(k,k)\mathbf{\Lambda}_i^B(k,k) \\ & + \mu_i\left(r - \sum_{k=1}^{M}\log(1+\mathbf{P}_i(k,k)\mathbf{\Lambda}_i^A(k,k))\right). \end{aligned} \tag{14}$$

Taking the derivative of the Lagrangian with respect to $\mathbf{P}_i(k,k)$, we have

$$\frac{\partial L(\mathbf{P},\mu_i)}{\partial \mathbf{P}_i(k,k)} = \mathbf{\Lambda}_i^B(k,k) - \frac{\mu_i\mathbf{\Lambda}_i^A(k,k)}{1+\mathbf{\Lambda}_i^A(k,k)\mathbf{P}_i(k,k)}. \tag{15}$$

At a stationary point, (15) equals zero; therefore

$$\mathbf{P}_i(k,k) = \left[\frac{\mu_i\mathbf{\Lambda}_i^{AB}(k,k) - 1}{\mathbf{\Lambda}_i^A(k,k)}\right]^+ \tag{16}$$

where $\mathbf{\Lambda}_i^{AB}(k,k)$ is the kth generalized eigenvalue of $(\mathbf{A}_i, \mathbf{B}_i)$ and equals $\mathbf{\Lambda}_i^{AB}(k,k) = \mathbf{\Lambda}_i^A(k,k)/\mathbf{\Lambda}_i^B(k,k)$.

The waterfilling level is

$$\mu_i = \exp\left(\frac{r - \sum_{k:\mathbf{P}_i(k,k)>0}\log\left(\mathbf{\Lambda}_i^{AB}(k,k)\right)}{N_m}\right). \tag{17}$$

N_m is the number of modes for which $\mathbf{P}_i(k,k) > 0$. The NGET algorithm for a MIMO ad hoc network is summarized in Table I.

Theorem 2: The NGET algorithm presented in Table I is a noncooperative game in which the utility function of each user is concave. Therefore, there exists at least one fixed point.

Proof: Let us define the strategy of each user i as a pair $\{\mathbf{G}_i, \mathbf{r}_i\}$, where $\mathbf{G}_i \in \mathbb{C}^{M\times M}$ determines directions of transmission of the precoder and $\mathbf{r}_i \in \mathbb{R}^{+M}$ is the rate allocation vector of user i. The kth element of $\mathbf{r}_i$ is

$$\mathbf{r}_i(k) = \log\left(1+\mathbf{P}_i(k,k)\mathbf{\Lambda}_i^A(k,k)\right). \tag{18}$$

TABLE I
NGET ALGORITHM FOR MIMO AD HOC NETWORKS

```
For t = 1 ...
  For i = 1 to N
    Estimate the interference covariance matrix R_i at node i
    Receive the interference covariance matrix sent from node l(i) R_{l(i)}
    A_i = (H_{l(i),i})^H (R_{l(i)})^{-1} H_{l(i),i}
    B_i = R_i^*
    [G_i, Λ_i^{AB}] = eig(A_i, B_i)
    Λ_i^A = (G_i^k)^H A_i^i G_i^k
    P_i(k,k) = [ (μ_i Λ_i^{AB}(k,k) - 1) / Λ_i^A(k,k) ]^+ ,   k = 1, ... M
    G̃_i = G_i (P_i)^{1/2}
  End
End
```

For clarity, define $\hat{\mathbf{G}}_i$ as the generalized eigenmatrix of $\mathbf{A}_i$ and $\mathbf{B}_i$ with unit norm columns in (12). At node i, the optimization minimizing (13) can be reformulated using game theoretic notations as

$$\{\mathbf{r}_i, \mathbf{G}_i\} = \arg\max_{\mathbf{r}_i,\mathbf{G}_i} u_i(\mathbf{r}_i, \mathbf{G}_i, \mathbf{r}_{-i}, \mathbf{G}_{-i})$$

where

$$\begin{aligned} u_i(\mathbf{r}_i, \mathbf{G}_i, \mathbf{r}_{-i}, \mathbf{G}_{-i}) = & -\sum_{k=1}^{M}\frac{\mathbf{\Lambda}_i^B(k,k)(e^{\mathbf{r}_i(k)}-1)}{\mathbf{\Lambda}_i^A(k,k)} \\ & -\mathrm{Tr}\left((\mathbf{G}_i - \hat{\mathbf{G}}_i)(\mathbf{G}_i - \hat{\mathbf{G}}_i)^H\right) \end{aligned} \tag{19}$$

is the utility function of that node. The strategy set is $\{\mathbf{r}_i \in \mathbb{R}^{+M}, \mathbf{G}_i \in S|\mathbf{1}^T\mathbf{r}_i = r\}$, where S is any convex set in $\mathbb{C}^{M\times M}$ containing the set of matrices with unit norm columns. The strategy set is convex and compact. The first term in the utility function in (19) forces (13) to be minimized while the second term in (19) forces $\mathbf{G}_i = \hat{\mathbf{G}}_i$ as the optimal strategy for user i. Note that the first term is concave in $\mathbf{r}_i$. The second term is concave in $\mathbf{G}_i$. Because the first term depends only on $\mathbf{r}_i$ and the second term depends only on $\mathbf{G}_i$, the utility function (19) is a concave function in $\{\mathbf{G}_i, \mathbf{r}_i\}$. Therefore, the noncooperative algorithm has at least one fixed point [22], [23].

Simulation results in Section IV show that NGET always outperforms the greedy algorithm and gives the same result as a centralized algorithm based on the augmented Lagrangian technique. A game theoretic interpretation of the algorithm is presented next to suggest why NGET works better than the greedy counterpart. At each step, node i's utility function can be expressed as follows:

$$\tilde{\mathbf{G}}_i = \arg\max_{\tilde{\mathbf{G}}_i\in\{\tilde{\mathbf{G}}_i\in\mathbb{C}^{M\times M}|r_{l(i)}(\tilde{\mathbf{G}}_i)\geq r\}} u_i'(\tilde{\mathbf{G}}_i, \tilde{\mathbf{G}}_{-i}) \tag{20}$$

where

$$\begin{aligned} u_i'(\tilde{\mathbf{G}}_i, \tilde{\mathbf{G}}_{-i}) &= -\mathrm{Tr}\left(\tilde{\mathbf{G}}_i^H\mathbf{R}_i^*\tilde{\mathbf{G}}_i\right) \\ &= -\mathrm{Tr}(\tilde{\mathbf{G}}_i^H\tilde{\mathbf{G}}_i^H) - \mathrm{Tr}(\tilde{\mathbf{G}}_i^H\tilde{\mathbf{R}}_i'^*\tilde{\mathbf{G}}_i) \end{aligned} \tag{21}$$

and

$$\tilde{\mathbf{R}}_i' = \sum_{k\neq i,l(i)}\mathbf{H}_{i,k}\tilde{\mathbf{G}}_k\tilde{\mathbf{G}}_k^H\mathbf{H}_{i,k}^H$$

is the covariance of the interference from other nodes in the network. At each iteration at node i the greedy algorithm minimizes the power $\mathrm{Tr}(\tilde{\mathbf{G}}_i^H\tilde{\mathbf{G}}_i)$ only. Thus, the greedy utility function is the first term on the RHS of (21). Due to the channel reciprocity in the network, the second term in (21) represents the total interference power from node

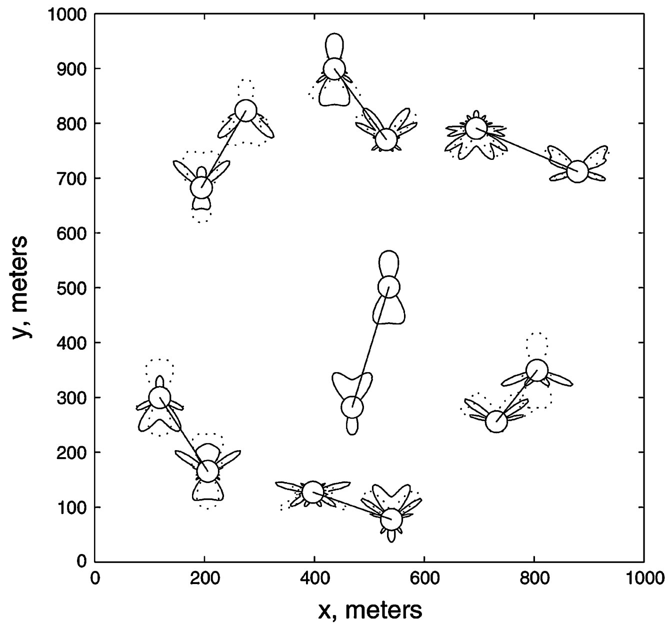

Fig. 1. Node locations and beampatterns corresponding to the largest eigenmode of the network.

i to other nodes in the network when the decoder is set equal to the complex conjugate of the precoder $\tilde{\mathbf{W}}_i = \tilde{\mathbf{G}}_i^*$. This term can also be interpreted as a tax to the utility function imposed on node i. This makes node i while maintaining its QoS, attempt to minimize its interference to other nodes. In the greedy algorithm, each node acts selfishly by minimizing power alone, and thus the equilibrium point is not necessarily the optimum from the social viewpoint. By appropriately pricing (i.e., via taxation) the system resource, the NGET algorithm thus guides user behavior toward a more socially efficient point [24].

IV. Numerical Results and Conclusion

Simulations were carried out to find the total transmit power of the nodes in the network. The first network considered has 14 nodes in a 1000 m × 1000 m area as in Fig. 1. Each node has $M = 8$ transmit and receive antennas. The channels are assumed to be full rank with angle of arrival uniformly distributed over $[-\pi/4, \pi/4]$. Each multipath has a Rayleigh fading magnitude. Transmit power is normalized to the noise power such that in the absence of fading, a transmit power of unity on an omnidirectional antenna creates a signal-to-noise ratio (SNR) of 10 dB at the receive antenna. The pathloss exponent is 4. The required channel capacity for each link is $r = 2$ b/s/Hz. Fig. 2 shows the total transmit power of the following algorithms: NGET, the greedy algorithm, and the centralized algorithm using an augmented Lagrangian algorithm. The figure shows that NGET outperforms the greedy algorithm and its performance is comparable to that of the cooperative algorithm. To further evaluate the performance of NGET, for each value of the number of nodes in the network $N \in \{10, 14, 18, 22, 26, 30\}$, 200 random networks were generated on a square of 1000 m × 1000 m. The distance between the transmitter and the receiver is uniformly distributed between 0 and 200 m. Other parameters are the same as in Figs. 1 and 2. The average total transmit power of nodes over all the network instances of NGET, the greedy algorithm, and the centralized algorithm are shown in Table II. The results show that performance of NGET is better than that of the greedy algorithm and comparable to that of the centralized algorithm.

To conclude, a noncooperative eigencoding algorithm with a rate constraint was developed incorporating taxation. Simulation results for an ad hoc network demonstrated that the resulting NGET algorithm outperforms greedy optimization from the standpoint of minimizing total transmit power, and performs almost identically to a cooperative augmented Lagrangian approach.

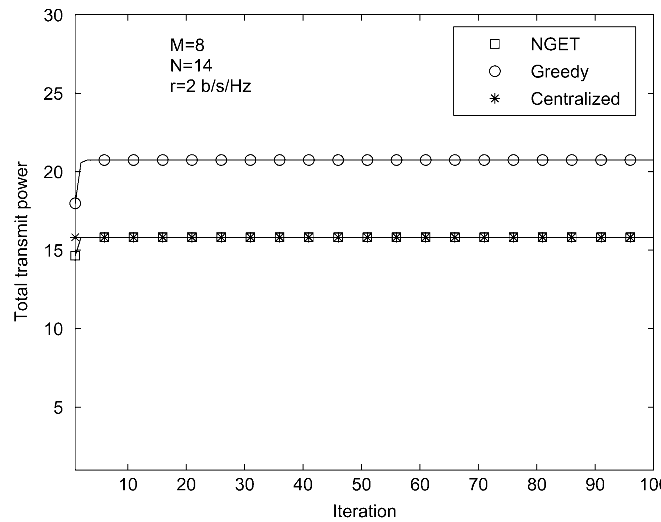

Fig. 2. Total transmit power of nodes in the network.

TABLE II
AVERAGE TOTAL TRANSMIT POWER OF NGET, GREEDY, AND CENTRALIZED ALGORITHMS

Algorithm	N=10	N=14	N=18	N=22	N=26	N=30
NGET	8.432	10.470	16.668	22.244	27.357	36.666
Greedy	8.778	11.666	18.100	26.039	34.288	49.263
Centralized	8.432	10.471	16.667	22.244	27.259	36.710

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