



phase-modulator. Based on the model depicted in Fig. 1, a set of nonlinear differential equations describing total phase error  $\varphi_e(t) = \varphi_s(t) - \varphi_{LO}(t)$  and the output RF signal  $V_{out}(t)$  are derived. A detailed description of the baseband model is reported in [6]. For the sampling loop, the continuous-wave (CW) optical source is exchanged with the pulsed optical source and the rest of the equation system remains the same. We furthermore include the effect of timing and amplitude jitter associated with the optical pulse source. Timing jitter is modeled as Brownian motion phase error [2]. Amplitude jitter is modeled as colored noise where the power spectrum density (PSD) is phenomenologically described as  $G(f) = \eta / (1 + (f/f_R)^2)$  [3],  $\eta$  is low-frequency jitter PSD, and  $f_R$  is the roll-off frequency. Loop gain is defined as  $K = \pi P_{ls}^{av} A R_{pd} R_L / (V_\pi \tau_{LF})$ , where  $P_{ls}^{av}$  is the average power of the pulsed optical source,  $A$  is the gain of the amplifier,  $R_{pd}$  is the responsivity of the photodiodes,  $R_L$  is the load resistance, and  $\tau_{LF}$  is the inversely proportional to the bandwidth (BW) of loop filter. ( $V_\pi$  assumed equal for both phase modulators.) In order to characterize the SFDR of the system, the input RF signal is assumed to consist of two closely spaced frequencies:  $\omega_1$  and  $\omega_2$ . The nonlinear response of the phase detector, pulsed optical source, etc., will result in intermodulation distortion of the demodulated signal. Third-order intermodulation products  $\{2(\omega_1 - \omega_{ls}) - (\omega_2 - \omega_{ls}), 2(\omega_2 - \omega_{ls}) - (\omega_1 - \omega_{ls})\}$  fall at a frequency close to the demodulated signal and they cannot be filtered away, limiting the SFDR of the system [3]. The demodulated signal obtained by the sampling phase-locked optical demodulator is then characterized by the SIR which is the ratio between the power of the demodulated signal and the third-order intermodulation products. When the tracking LO phase-modulator nonlinearities are set to zero and assuming that the phase detection process is linear,  $\sin[\varphi_e(t)] \approx \varphi_e(t)$ , the total phase error for the (locked) baseband loop (CW optical source) including the loop filter becomes [6]

$$\varphi_e(t) = \frac{1}{K^2 + \omega_1^2} (M_{in} \omega_1^2 \cos[\omega_1 t] - K \sin[\omega_1 t]) \quad (1)$$

where  $M_{in} = \pi V_1 / V_\pi$ . In (1), it is assumed for simplicity that the input RF signal consists of only one tone at frequency  $\omega_1$  with the amplitude  $V_1$ . Since there are no nonlinearities in the loop,  $\varphi_e(t)$  contains only a frequency component at  $\omega_1$ , as expected. Equation (1) also shows that as  $K$  approaches infinity, the total phase error  $\varphi_e(t)$  approaches zero, i.e., the demodulated signal  $V_{out}(t)$  is a replica of a input RF signal. Under the same assumption as for the baseband loop, the total phase error for the sampling loop is governed by the following equation:

$$\varphi_e(t) = M_{in} \cdot \left( \cos[\omega_1 t] - \frac{K \sin^3[\omega_1 t]}{6\omega_1} - \frac{A_0 K \sin[\omega_1 t] - \omega_1 t \cos[\omega_1 t]}{2\omega_1} \right) - \frac{1}{2} \frac{A_0 K (\cos[\omega_1 t] \sin[\omega_1 t] - \omega_1 t)}{\omega_1} \quad (2)$$

where we have assumed raised cosine pulse shape.  $A_0$  is the amplitude of the pulses. It is observed in (2) that  $\varphi_e(t)$  not only contains the frequency components at  $\omega_1$ , but also the multiples of  $\omega_1$  (harmonics). The amplitude of harmonics increases as the loop gain is increased. This is in contrast with the baseband loop. Furthermore, by inspecting (2), we can qualitatively

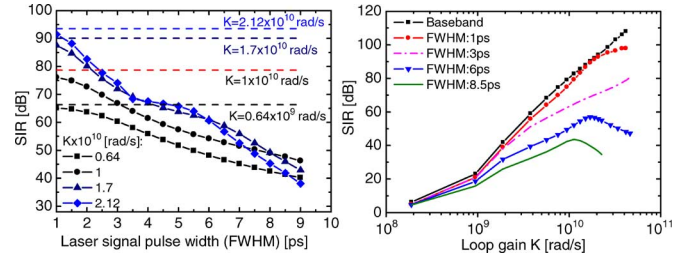


Fig. 2. (a) SIR as a function of pulsewidth (FWHM) of the laser signal for selected values of the loop gain. (b) SIR as a function of input signal frequency for selected values of the loop gain. Pulsewidth of the laser signal: 1 ps.

conclude that the amplitude of the harmonics can be reduced by increasing the frequency  $\omega_1$ . In Section III, it is confirmed that the sampling induces extra nonlinearities in the overall loop response by using the time domain large signal numerical model based on nonlinear differential equations describing  $\phi_e(t)$  and  $V_{out}(t)$ .

### III. SAMPLING VERSUS BASEBAND LOOP

In this section, the linearity of the optical phase demodulator, based on sampling PLL, is investigated by computing the SIR of the demodulated signal. An input RF signal modulation depth of  $\pi/2$  is assumed in all simulation results and the pulse shape of the optical pulse source is Gaussian. In Fig. 2(a), the SIR is computed as a function of the full-width at half-maximum (FWHM) of the pulsed optical source signal for selected values of  $K$ . The input RF signal frequencies are  $\omega_1 = 20 \text{ GHz} + 0.9 \text{ GHz}$ ,  $\omega_2 = 20 \text{ GHz} + 1.6 \text{ GHz}$ , and  $\omega_{ls} = 20 \text{ GHz}$ . The down-converted IF components, at which the loop will operate, are  $\omega_{IF,1} = \omega_1 - \omega_{ls}$  and  $\omega_{IF,2} = \omega_2 - \omega_{ls}$ . The constant lines correspond to the SIR obtained by the baseband loop for the corresponding values of the loop gain. For the baseband case,  $\omega_1 = 0.9 \text{ GHz}$  and  $\omega_2 = 1.6 \text{ GHz}$ . In general, Fig. 2(a) shows that the SIR decreases as the pulsewidth is increased and thereby indicates that the sampling induces a penalty in the SIR. This is in accordance with (2). However, it should be noted that for the open-loop sampling system, there is no degradation in the SIR as the pulsewidth is varied. Furthermore, we observe from Fig. 2(a) that as the pulsewidth is decreased, the SIR of the sampling loop approaches the value obtained by the baseband loop. However, very short pulses ( $\sim 1$  ps) are required in order to preserve the SIR. Fig. 2(a) furthermore indicates that for relatively high values of the pulsewidth, increasing  $K$  does not improve the SIR. We, therefore, need to investigate how the SIR of the demodulated signal is affected by increasing the loop gain. In Fig. 2(b), the SIR is computed as a function of the loop gain for different values of the pulsewidth. As a reference, we also plot the SIR obtained by the baseband loop in the same figure. Fig. 2(b) shows that for increasing loop gain and pulsewidth, the SIR of the demodulated signal, obtained by the sampling loop deviates from the SIR obtained by the baseband loop. The penalty in the SIR is increased as the loop gain is increased. This is in accordance with (2), as discussed earlier. However, the penalty decreases for decreasing pulsewidth. Fig. 2(b) also shows that the SIR is less sensitive to the pulsewidth for low values of the loop gain. Furthermore, the SIR curve starts to decrease, for the pulsewidth of 6 and 8.5 ps, as the loop gain is sufficiently increased. The value of the loop gain for which the

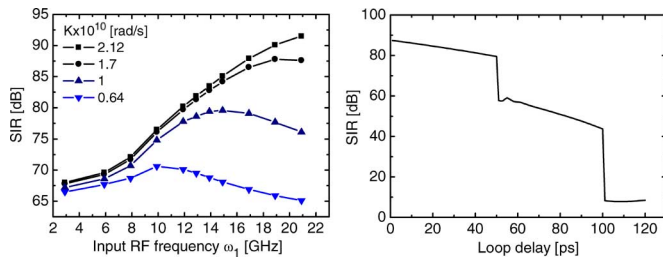


Fig. 3. (a) SIR as a function of loop gain  $K$  for selected values of the FWHM of the pulsed laser source. (b) SIR as a function of loop delay in a sampled second-order feedback loop with 20-GHz pulse rate and 1-GHz IF.

SIR starts to decrease, decreases with increasing pulsewidth. For the pulsewidth of 8.5-ps pulsewidth, the loop loses its lock if the loop gain is increased beyond  $2.5 \times 10^{10}$  rad/s. Next, it is investigated how SIR is affected by input RF signal frequency [Fig. 3(a)]. The pulsewidth is set to 1 ps. The frequency difference  $\omega_1 - \omega_{1s}$  and  $\omega_2 - \omega_{1s}$  is held constant and the sampling loop will, therefore, operate at a constant intermediate frequency. Fig. 3(a) indicates that for relatively high values of loop gain,  $2.12 \times 10^{10}$  and  $1.17 \times 10^{10}$  rad/s, the SIR of the demodulated signal increases as the input signal frequency is increased. For relatively low values of the input RF signal frequency, the dependence of the SIR on the loop gain is negligible. In that case, the SIR is limited by the nonlinearities of the low-frequency optical pulsed source. For the relatively low value of the loop gain ( $K = 0.64 \times 10^{10}$  rad/s), the SIR does not seem to be much affected by increasing  $\omega_1$ . Furthermore, for  $K = 0.64 \times 10^{10}$  and  $1 \times 10^{10}$  rad/s, there is an optimal value of the input signal frequency for which the SIR is increased. So far, the effect of the finite delay in the loop has been considered to be small. In a baseband loop, the effect of loop delay is well known to limit the available loop gain, while maintaining stability [7]. The delay gets more significant the higher the frequency. In a sampled loop, the effective feedback delay is given by that of the pulse rate. No information can in this case be forwarded when no pulse is present in the loop, assuming the pulsewidth is narrower than the physical delay of the loop. In other words, the sampled loop will operate more efficiently at higher input frequencies. Fig. 3(b) shows the effect of a physical loop delay in a sampled loop. A 20-GHz pulse rate and 1-GHz IF has been assumed. Further, idealized delta pulses have been assumed to clearly isolate the impact of the loop delay. The performance degrades slowly with increasing delay up to the point where the delay exceeds the pulse repetition ratio, after which step degradation in performance occurs corresponding to a delayed feedback to the second proceeding pulse. It can be observed that the effect of loop delay is relatively weak compared to the pulse rate.

In Fig. 4(a), signal-to-noise ratio (SNR) of the demodulated signal is computed as a function of timing jitter of the pulsed source. As expected, the SNR decreases as the timing jitter in-

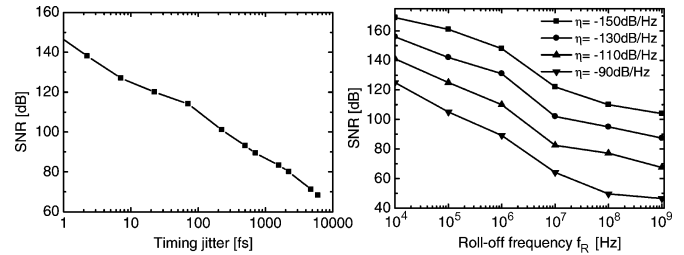


Fig. 4. SNR of the demodulated signal in 500-MHz BW. (a) SNR as a function of timing jitter of the pulsed source. Integration range: 100 Hz–10 GHz. (b) SNR as a function of roll-off frequency  $f_R$  when  $\eta$  is varied.

creases. In Fig. 4(b), the effect of amplitude jitter on SNR is investigated. Even though  $\eta$  is relatively large, high values of SNR can be obtained by having low  $f_R$ .

#### IV. CONCLUSION

A novel approach of using optical sampling, in order to increase operation frequency of the optical phase demodulator, has been theoretically investigated. The optical sampling inherently induces a penalty in the SIR compared to the baseband loop. However, for very short pulsewidths ( $< 2$  ps) and high input signal frequencies, the penalty is very small making this technique promising for high-frequency analog optical links. From the practical point of view, one needs to deal with close path matching of very narrow pulses, close polarization matching, bandwidth limitations due to loop group delay, etc.

#### ACKNOWLEDGMENT

The authors would like to thank L. Coldren, L. Lembo, S. Pappert, and R. Smith for useful conversations and input.

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