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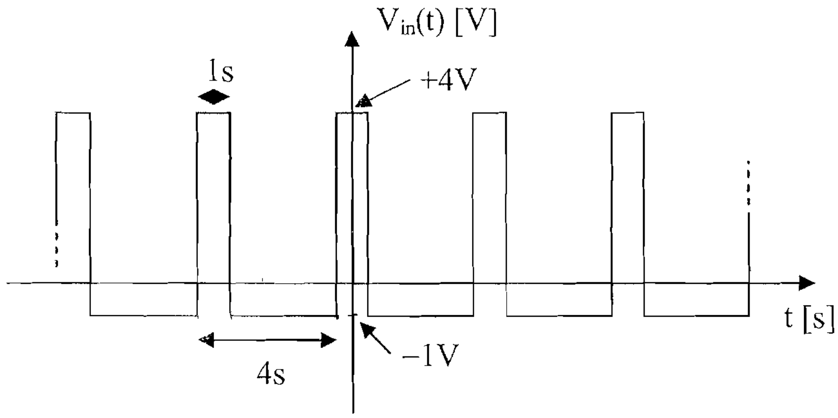
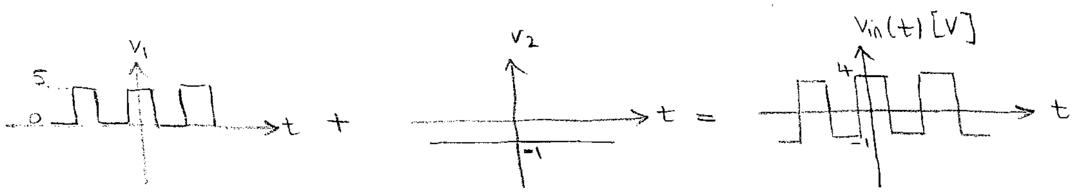
Problem 1 [Total 30 pt.]

(a) [15 pt.] Consider the periodic signal below, find the trigonometric Fourier series representation of this waveform. (A table of Fourier coeeficients for commonly usedwaveforms is provided with the exam.)



$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)\right]$$



$T_0 = 4S$, $T = 1S$

$A = 5$, $\frac{T}{T_0} = \frac{1}{4}$

$a_0 = \frac{AT}{T_0} = \frac{5}{4}$

$b_n = 0$ for all n

$a_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi T}{T_0}\right)$

$a_n = \frac{10}{n\pi} \sin\left(\frac{n\pi}{4}\right)$

+

$a_0 = -1$

$a_n = 0$ for all n

$b_n = 0$ for all n.

=

$a_0 = \frac{5}{4} - 1 = \frac{1}{4} \rightarrow \boxed{a_0 = \frac{1}{4}}$

$\boxed{b_n = 0 \text{ for all } n}$

$\boxed{a_n = \frac{10}{n\pi} \sin\left(\frac{n\pi}{4}\right)}$ for all n

$T_0 = 4S \rightarrow f_0 = \frac{1}{4}$ HZ, $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\rightarrow V_{in}(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \left[\frac{10}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cos\left(2\pi n \frac{1}{4} t\right)\right] \Rightarrow$$

$$\boxed{V_{in}(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \left[\frac{10}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cos\left(\frac{n\pi t}{2}\right)\right]} \text{ [V]}$$

$a_1 = \frac{10}{\pi} \sin\left(\frac{\pi}{4}\right) = \frac{10}{\pi} \cdot (0.71) = 2.25$ V

$a_2 = \frac{10}{2\pi} \sin\left(\frac{2\pi}{4}\right) = \frac{5}{\pi} \overbrace{\sin\left(\frac{\pi}{2}\right)}^{1} = 1.59$V

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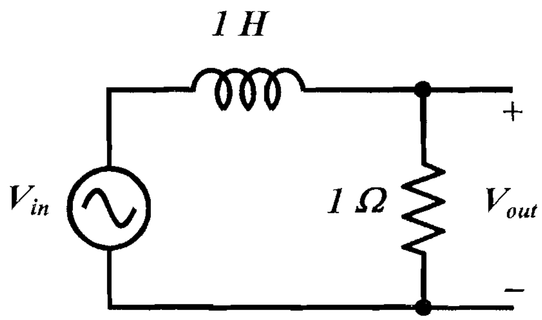
Problem 1 (continued)
(b) [15 pt.] Apply the above waveform as Vin to the single-time-constant circuit below. Write down the amplitude and phase of the <u>first 2 non-zero components</u> of Vout.



$$T(S) = \frac{1}{1+SL} = \frac{1}{1+S} \rightarrow T(j\omega) = \frac{1}{1+j\omega}$$

$T(0) = 1 \rightarrow a_0 = (1)(\frac{1}{4}) = \frac{1}{4}$ amplitude $\measuredangle = \phi$: dc term

$\left|T(j\frac{\pi}{2})\right| = \frac{1}{\sqrt{1^2 + (\frac{\pi}{2})^2}} = 0.54 \rightarrow a_1 = (0.54)(2.25) = 1.21$ amplitude

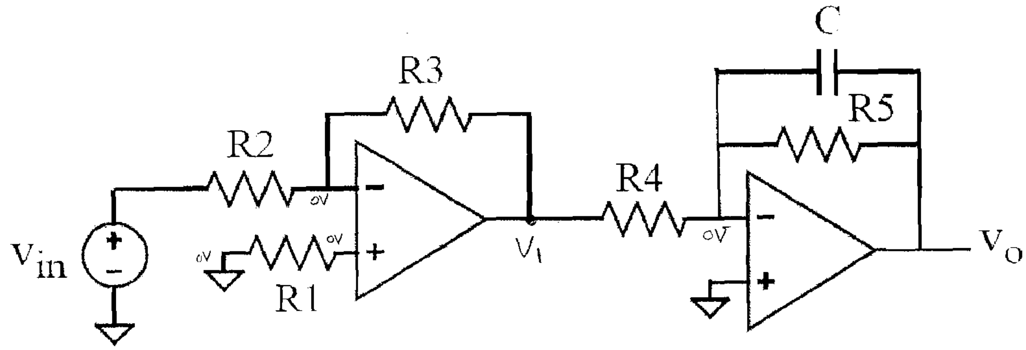
$\measuredangle = \phi - \tan^{-1}(\frac{\pi}{2}) = -57.5°$ phase

$\left|T(j\pi)\right| = \frac{1}{\sqrt{1+\pi^2}} = 0.30 \rightarrow a_2 = (0.30)(1.59) = 0.48$ amplitude

$\measuredangle = \phi - \tan^{-1}(\pi) = -72.34°$ phase

$\rightarrow V_{out} = \frac{1}{4} + 1.21 \cos(\frac{\pi t}{2} - 57.5°) + 0.48 \cos(\pi t - 72.34°) + \ldots$

Problem 2 [Total 30 pt.]
Calculate the transfer function, $H(s)= v_o/v_{in}$ assuming ideal op amps (the open loop gains are infinite as well as everything else being ideal).



$$V_o = \frac{-\frac{\frac{1}{SC}\cdot R_5}{\frac{1}{SC}+R_5}}{R_4} V_1 = \frac{\frac{-R_5}{R_5SC+1}}{R_4} V_1 = \frac{-R_5}{R_4(R_5SC+1)} V_1 = \frac{-R_5}{R_4R_5CS+R_4} V_1$$ (10)

$$V_1 = \frac{-R_3}{R_2} V_{in}$$ (10)

$$\rightarrow V_o = \frac{+R_5}{R_4R_5CS+R_4}\left(\frac{+R_3}{R_2}\right) V_{in} \rightarrow$$

$$\boxed{H(S) = \frac{V_o}{V_{in}} = \left(\frac{R_5}{R_4R_5CS+R_4}\right)\left(\frac{R_3}{R_2}\right)} = \frac{R_3R_5}{R_2R_4R_5CS+R_2R_4}$$ (10)

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Problem 3 [Total 40 pt.]

(a) [20 pt.] Find I_3 and V_4, in the circuit shown below, for $\mu_n C_{ox}$=50μA/V^2, $\mu_p C_{ox}$=20μA/V^2, V_{tn}=1V, V_{tp} =–1V, $\lambda_n=\lambda_p=0$, $L_n=L_p$=10μm, W_n=30μm, and W_p=75μm.

$M_n C_{ox} = 50 \times 10^{-6} \ \frac{A}{V^2}$

$M_p C_{ox} = 20 \times 10^{-6} \ \frac{A}{V^2}$

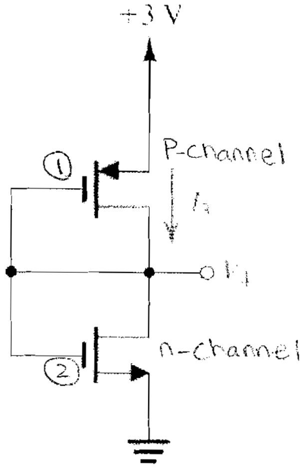
$V_{tn} = 1V \ , V_{tp} = -1V$

$\lambda_n = \lambda_p = 0$

$L_n = L_p = 10 \ \mu m$

$W_n = 30 \ \mu m$

$W_p = 75 \ \mu m$



(b)

1:

$i_D = \frac{1}{2}(M_p C_{ox})(\frac{W_p}{L_p})(V_{SG} - |V_t|)^2$, $V_{S_1} = 3V$

$i_3 = \frac{1}{2}(20 \times 10^{-6})(\frac{75}{10})(3 - V_4 - 1)^2$

For saturation we want

$V_{sg} > 1 \rightarrow 3 - V_4 > 1 \rightarrow$

$-V_4 > -2 \rightarrow V_4 < 2$

2:

$i_D = \frac{1}{2}(M_n C_{ox})(\frac{W_n}{L_n})(V_{GS} - V_t)^2$

$V_{GS} = V_G - V_S = V_4 - 0 = V_4$, $V_{S_2} = 0$

condition for saturation.

$i_3 = \frac{1}{2}(50 \times 10^{-6})(\frac{30}{10})(V_4 - 1)^2$

$V_{gs} > 1 \rightarrow V_4 > 1$

Dividing 1 by 2 : $1 = \frac{20 \times 75 \ (2 - V_4)^2}{50 \times 30 \ (V_4 - 1)^2} \rightarrow 50 \times 30 \times (V_4 - 1)^2 = 20 \times 75 \ (2 - V_4)^2 \rightarrow$

$(V_4 - 1)^2 = (2 - V_4)^2 \rightarrow \begin{cases} V_4 - 1 = 2 - V_4 \rightarrow 2V_4 = 3 \rightarrow \boxed{V_4 = 1.5V} \\ V_4 - 1 = V_4 - 2 \quad X \end{cases}$

which satisfies both conditions for saturation.

$i_3 = \frac{1}{2}(50 \times 10^{-6})(3)(1.5 - 1)^2 = 18.75 \ \mu A \rightarrow \boxed{i_3 = 18.75 \ \mu A}$

nice!

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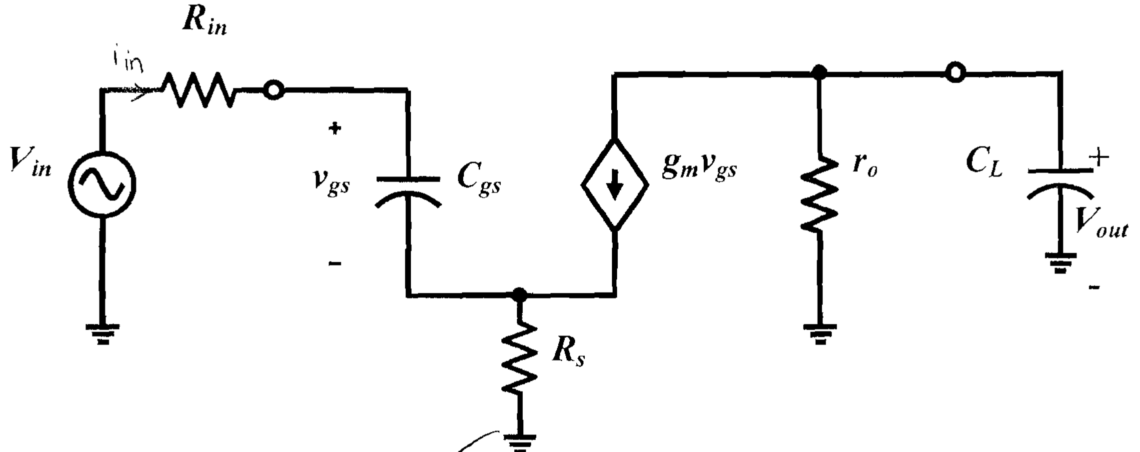
Problem 3 (continued)

(b) [20 pt.] Shown below is the simplified small-singal equaivalent circuit of a common-source amplifier with source degeneration resistor. Find the transfer function, $V_{out}/V_{in}(s)$.

Hints: (1) the impedance of a capacitance, C, is 1/(sC). (2) find Vout in terms of Vgs and Vin in terms of Vgs, then take the ratio.



$$V_{out} = -g_m V_{gs} \left(r_o \parallel \frac{1}{sC_L} \right)$$

$$V_{in} - i_{in} R_{in} - V_{gs} - R_s (g_m V_{gs} + i_{in}) = 0 \text{, and } V_{gs} = i_{in} \left(\frac{1}{sC_{gs}} \right) \text{, } i_{in} = V_{gs} \cdot sC_{gs}$$

$$\rightarrow V_{in} = R_{in} sC_{gs} V_{gs} + V_{gs} + R_s g_m V_{gs} + R_s sC_{gs} V_{gs} = V_{gs} \left(1 + g_m R_s + sC_{gs} (R_s + R_{in}) \right).$$

$$\rightarrow \frac{V_{out}}{V_{in}} = \frac{-g_m \left(r_o \parallel \frac{1}{sC_L} \right)}{1 + g_m R_s + sC_{gs} (R_s + R_{in})} = \frac{-g_m \left(\frac{r_o \cdot \frac{1}{sC_L}}{r_o + \frac{1}{sC_L}} \right)}{1 + g_m R_s + sC_{gs} (R_s + R_{in})}$$

$$\rightarrow \frac{V_{out}}{V_{in}} = \frac{-g_m \left(\frac{r_o}{r_o s C_L + 1} \right)}{1 + g_m R_s + sC_{gs} (R_s + R_{in})}$$