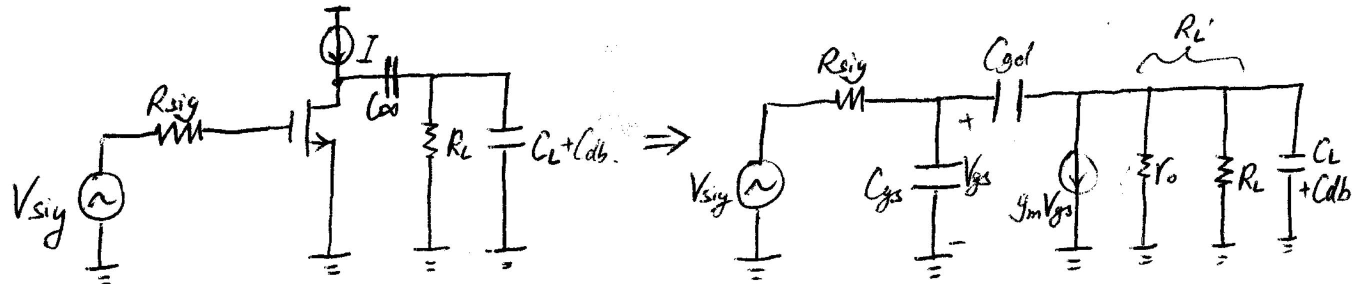
ECE 2C HW#6 Solution

By Li Sun.

Problem 1:

(a) The circuit skematic and equivalent circuit is as:



where $C_L + C_{db} = 1pF$, $R_{sig} = R_L = 20k = r_o$, $g_m = 5mA/V$, $C_{gs} = 2pF = 10 \cdot C_{gd}$.

obviously $A_m = -g_m (R_L // r_o) = -5 \times 10 = -50$

As for the f_H, we need to calculate τ_H first.

$$\tau_H = C_{gs} \cdot R_{gs}^o + C_{gd} \cdot R_{gd}^o + (C_L + C_{db}) \cdot R_L^{10}$$

where $R_{gs}^o = R_{sig}$, $R_{L'}^o = R_L // r_o$. $R_{gd}^o = (1 + g_m R_L') R_{sig} + R_L'$

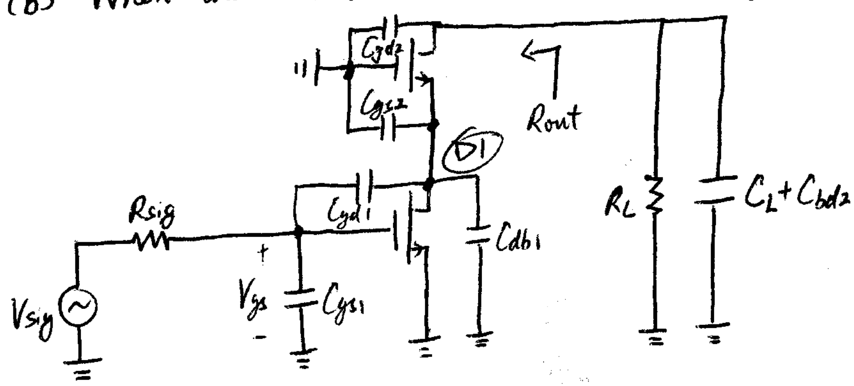
$$\therefore \tau_H = 2pF \times 20k\Omega + 1pF \times 10k\Omega + 0.2pF \times (51 \times 20k\Omega + 10k\Omega)$$

$$= 40 + 10 + 206 \ ns = 256 \ ns$$

therefore, $f_H = \frac{1}{2\pi \tau_H} = 621.7 \ kHz$.

$\therefore$ BGP $= f_t = 50 \times 621.7 \ kHz = 31.1 \ MHz$.

(b) When there is a CG transistor, the equivalent circuit will be:



Here, we can find 4 terms in time constant, corresponding to

C_{gs1}, C_{gd1}, $C_{db1} + C_{gs2}$, and $C_{gd2} + C_L + C_{bd2}$, respectively.

First, we need to calculate R_{D1}, which is the resistance looking into note D1.

This R_{D1} is also the corresponding open-time constant resistance of $C_{bd1} + C_{gs2}$.

$$R_{D1}^{o} = R_{D1} = r_{o1} \,//\, \left[\frac{1}{g_{m2} + g_{mb2}} + \frac{R_L}{A_{vo2}} \right]$$

where $g_{mb2} = \chi g_{m2}$, we can get $g_m' = g_m + g_{mb} = 1.2 \cdot g_m = 6\ mS$

Also. $A_{vo2} = 1 + g_m' \cdot r_{o2} = 1 + 1.2 \times 5\,mS \times 20\,k\Omega = 121$

$\therefore R_{D1}^{o} = R_{D1} = 20\,k\Omega \,//\, 0.332\,k\Omega \approx 0.32654\,k\Omega$

Now we can write $\tau_{H\_cascade} = C_{gs1} \cdot R_{gs1}^{o} + C_{gd1} \cdot R_{gd1}^{o} + (C_{db1} + C_{gs2}) \cdot R_{D1}^{o} + (C_{gd2} + C_L + C_{bd2}) \times (R_L // R_{out})$

And
$$\begin{cases} R_{gs1}^{o} = R_{sig} \\ R_{gd1}^{o} = R_{D1} + (1 + g_m' \cdot R_{D1}) \cdot R_{sig} \\ R_{out} = r_{o2} + A_{vo2} \cdot r_{o1} \approx A_o \cdot r_o \\ A_o = g_m' \cdot r_o \approx A_{vo2} = 121 \end{cases} \Rightarrow \begin{cases} R_{gs1}^{o} = 20\,k\Omega \\ R_{gd1}^{o} = 59.511\,k\Omega \\ R_{D1}^{o} = 0.32654\,k\Omega \\ R_L // R_{out} \cong 121 \times 20\,k\Omega \end{cases}$$

Therefore, $$\tau_{H\_cascode} = 2pF \times 20k\Omega + 0.2pF \times 53k\Omega + 2.2pF \times 0.32654k\Omega + 1.2pF \times 20k\Omega$$

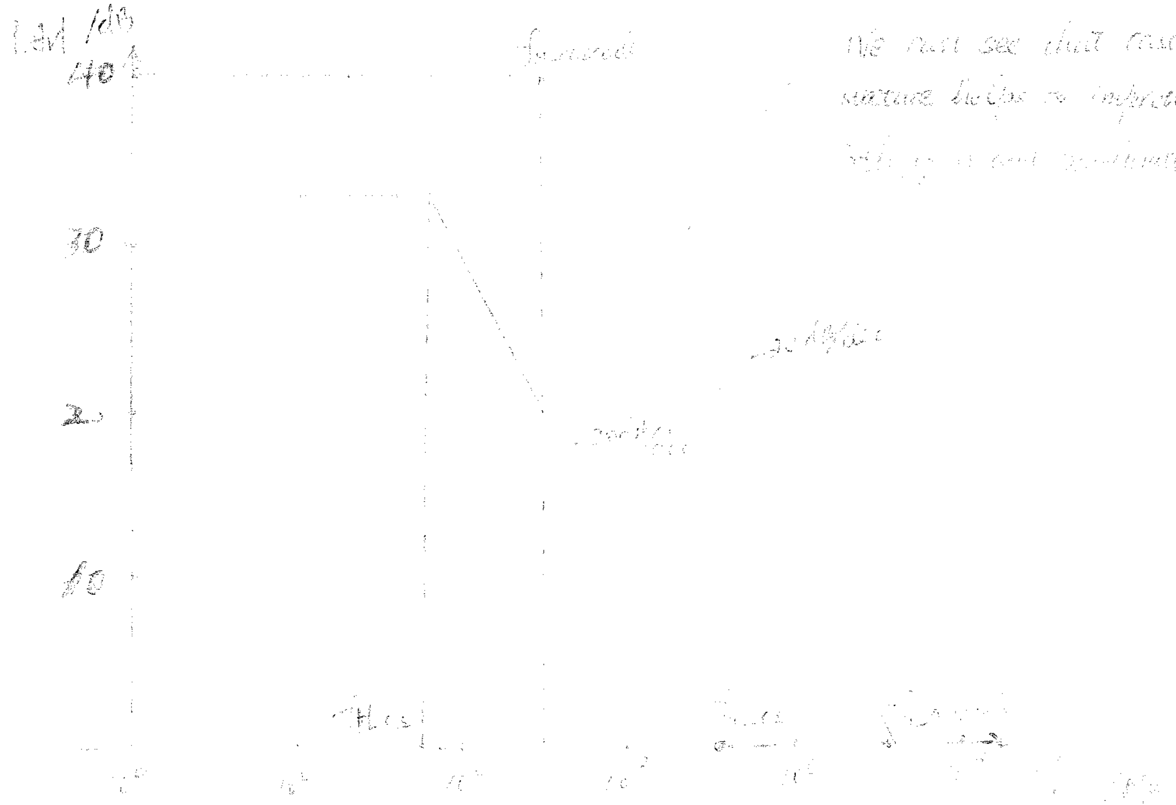
$$= 40ns + 10.6ns + 0.7184ns + 24ns$$

$$= 75.32ns$$

$$\therefore f_{H\_cascode} = \frac{1}{2\pi \tau_{H\_cascode}} = 2.113 MHz$$

Because $A_{v\_cascode} = A_{v1} A_{v2} \frac{R_L}{R_L + A_{v2} r_o} = A_{v1} A_{v2} \frac{r_o}{r_o + A_{v2} r_o} = -98.4 V/V$

we have $f_T = |A_{v\_cascode}| \times f_{H\_cascode} = 207.924 MHz$



We can see that cascode structure helps to improve [illegible]

As for the 2I current source, because the equivalent resistance seen by this source is $R_{eq} = r_{o1} // \left[\frac{1}{g_{m2}+g_{mb2}} + \frac{R_L}{A_{vo2}}\right]$

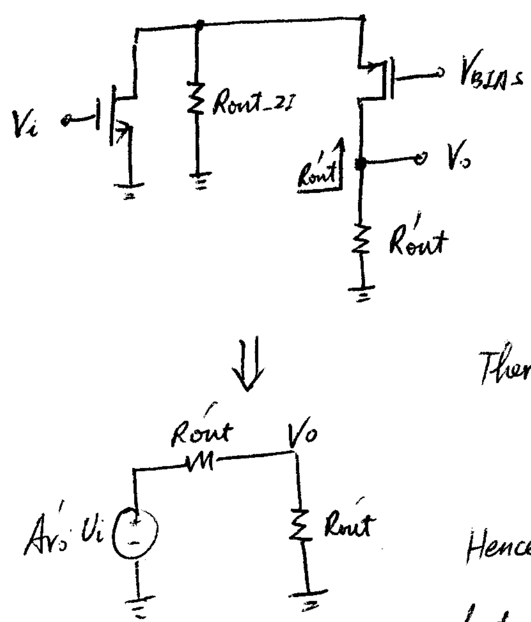
where R_L is the output resistance of I current source

Therefore, $R_{eq} \approx r_{o1} // r_{o2} \approx r_{o1}/2$

As long as the output resistance of 2I current source $R_{out\_2I} \gg R_{eq}$.

we can assume this source is ideal.

When taking into account of the output resistance of 2I source, we have:



Now, we still can calculate A_{vo}' first.

$$A_{vo}' = A_{vo1}' \cdot A_{vo2}'$$

$$= -g_{m1}(r_{o1} // R_{out\_2I}) \cdot [1 + g_{m2} \cdot r_{o2}]$$

Therefore. $\frac{V_o}{V_i} = \frac{A_{vo}'}{2} = -\frac{g_{m1}}{2}(r_{o1} // R_{out\_2I})(1 + g_{m2} r_{o2})$

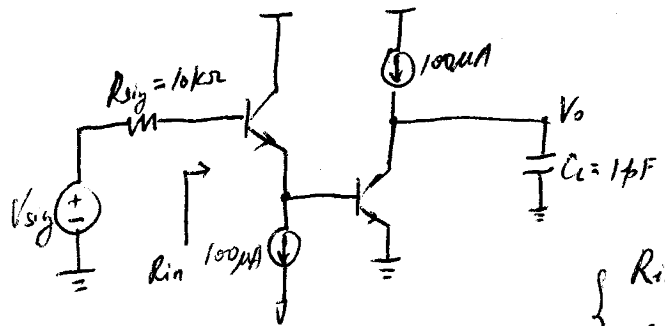
Hence, if the total gain reduced by 1%, means that $r_{o1} // R_{out\_2I} = 0.99 \cdot r_{o1}$

$$\Rightarrow R_{out\_2I} = 99 \cdot r_{o1} = 4.95\,M\Omega.$$

Problem 3:

$\beta_0 = 100$, $V_A = 100V$, $C_\mu = 0.2\ pF$

$C_{je} = 0.8\ pF$, $f_T = 400\ MHz$



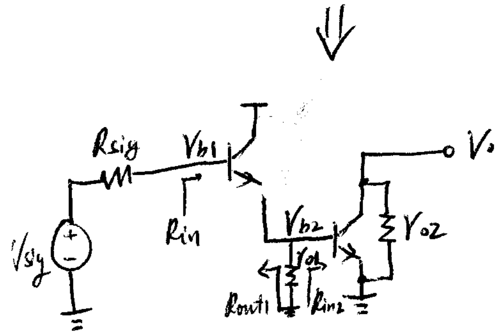
(a) we can get that. $g_m = \frac{100\mu A}{25mV} = 4mS$

$R_{in2} = r_{\pi 2} = \beta / g_m = 25k\Omega$

$r_o = V_A / I = 1M\Omega$

$R_{out1} = \left(r_{e1} + \frac{R_{sig}}{\beta+1}\right) = \left(250 + \frac{10k}{101}\right) = 349\Omega$

$R_{in} = (\beta+1)(r_{e1} + R_{in2}) = 101 \times 25.25k = 2.55M\Omega$



$\therefore \frac{V_o}{V_{sig}} = \frac{V_o}{V_{b2}} \cdot \frac{V_{b2}}{V_{b1}} \cdot \frac{V_{b1}}{V_{sig}}$

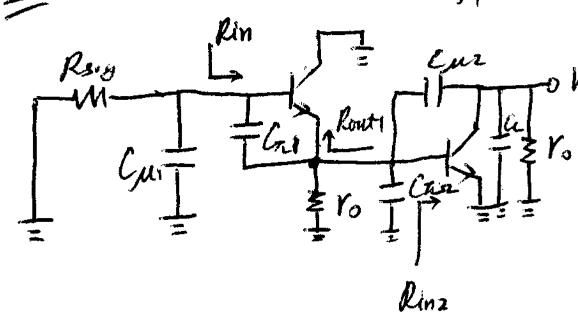
Here $r_{o1} = 1M\Omega >> R_{in2} = 25k\Omega$. we just neglect r_{o1}.

$\frac{V_o}{V_{b2}} = -g_m \cdot r_{o2}$, $\frac{V_{b2}}{V_{b1}} = \frac{R_{in2}}{R_{in2} + R_{out1}} = \frac{25000}{25000 + 349} \approx 0.98623$

$\frac{V_{b1}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{2550k\Omega}{2550k\Omega + 10k\Omega} \approx 0.9961$

Therefore, $\frac{V_o}{V_{sig}} = 0.9961 \times 0.98623 \times (-4ms \times 1M\Omega) = -3929.53$

(b) Because $C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = 1.59\ pF$ $\therefore C_\pi = 1.39\ pF$



We have 4 capacitors. therefore

$\tau_H = C_{\mu 1} \cdot R_{\mu 1}^{\circ} + C_{\pi 1} \cdot R_{\pi 1}^{\circ} + C_{\mu 2} \cdot R_{\mu 2}^{\circ} + C_{\pi 2} \cdot R_{\pi 2}^{\circ} + C_L \cdot r_o$

Obviously, $R_{\mu 1}^{0} = R_{sig} \,//\, R_{in} = 10k\Omega \,//\, 2.55M\Omega \approx 10k\Omega$

$$R_{\pi 1}^{0} = \frac{R_{sig} + R_L'}{1 + \frac{R_{sig}}{r_\pi} + \frac{R_L'}{r_e}} \quad \text{where } R_L' = r_o \,//\, R_{in2} \approx 25k\Omega$$

$$= \frac{10K + 25K}{1 + \frac{10K}{25K} + \frac{25K}{250}} \approx 345.16\,\Omega$$

$R_{\pi 2}^{0} = R_{out1} \,//\, r_o \,//\, R_{in2} \approx 349\Omega \,//\, 1M\Omega \,//\, 25k\Omega = 344.2\,\Omega$

$R_{\mu 2}^{0} = r_o + (1 + g_m r_o)\cdot(R_{out1} \,//\, r_o \,//\, R_{in2})$

$= 1M\Omega + (1 + 4mS \cdot 1M\Omega) \times 344.2\Omega \approx 2.377\,M\Omega$

Therefore, $\tau_H = 0.2pF \times 10k\Omega + 1.39\,pF \times 345.16\Omega + 1.39\,pF \times 344.2\Omega + 0.2pF \times 2.377M + 1pF \times 1M\Omega$

$\approx 1478.36\ ns$

$$\therefore f_H = \frac{1}{2\pi\tau_H} = 107.66\ kHz$$

Obviously, C_L is dominant because r_o is huge compared to others.

$C_{\mu 2}$ is second important capacitor.

(c) If current increas to 10 times, g_m will be $10 g_m$, r_o will be $r_o/10$

$$\therefore \frac{V_o}{V_{sig}} = -10 g_m \cdot \frac{r_o}{10} \times \frac{25K/10}{25K/10 + 25 + \frac{10K}{101}} \times \frac{101 \times (25 + 25K/10)}{101 \times (25 + 25K/10) + 10K} = -3667.2$$

which is not very different than before.

But as for frequency responce, because now $C_\pi' + C_\mu = \frac{10\, g_m}{2\pi f_T} = 15.9\,pF$

we have $C_\pi' = 15.7\,pF$

$$\begin{cases} R_{\mu_1}^{o} = R_{sig} \,//\, R_{in}' = 10k\Omega \,//\, 255.025\,k\Omega \approx 9.6\,k\Omega \\ R_{\pi_1}^{o} = \dfrac{R_{sig} + r_o' // R_{in2}'}{1 + \dfrac{R_{sig}}{r_\pi} + \dfrac{R_{in2}' // r_o'}{r_e}} = \dfrac{10k + 100k // 2500}{1 + \dfrac{10k}{2500} + \dfrac{100k//2500}{25}} = \dfrac{12439}{1 + 97.56 + 4} = 121.285\,\Omega \\ R_{\pi_2}^{o} = R_{out1}' \,//\, r_o' \,//\, R_{in2}' = 124\Omega \,//\, 100k \,//\, 2500 \approx 118.15\Omega \\ R_{\mu_2}^{o} = r_o' + (1 + g_m' r_o')(R_{\pi_2}^{o}) = 100k\Omega + 4001 \times 118.15\Omega = 572.72\,k\Omega \end{cases}$$

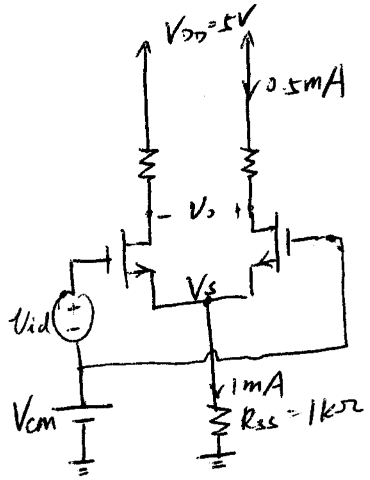
$$\therefore Z_H' = 0.2\,pF \times 9.6\,k\Omega + 15.7\,pF \times 121.285\Omega + 15.7\,pF \times 118.15\,\Omega + 0.2\,pF \times 572.72\,k\Omega$$
$$+ 1\,pF \times 100\,k\Omega$$

$$\approx 220.22\,ns$$

$$\therefore f_H' = \frac{1}{2\pi Z_{H}'} = 722.7\,kHz$$

Now we find that both C_L and $C_{\mu2}$ are dominant, and the f_H is increased by about 7 times.

Problem 4.

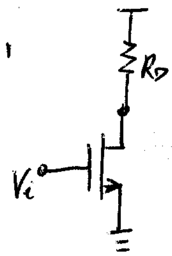


(a) Obviously, $0.5mA = \frac{1}{2} 2.5mA/V^2 \times (V_{cm} - V_S - |V_{th}|)^2$

where $V_S = 1mA \times 1k\Omega = 1V$

$\Rightarrow \quad V_{cm} = 2.332\,V$

(b)



we have that $A_d = g_m R_D = 8\ V/V$

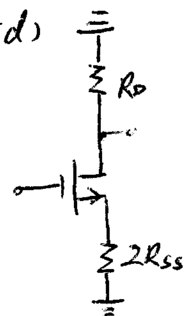
where $g_m = \dfrac{2I_D}{(V_{cm} - V_S - V_{th})} = 1.58\ mS.$

$\therefore R_D = 5.06\ k\Omega$

(c) Because the current of each transistor is $0.5mA$. Hence $V_D = 2.47V$

And because $V_D > V_{cm} - |V_{th}|$, Q_1 and Q_2 indeed are saturated.

(d)



Because we cannot neglect $1/g_m$.

We know $\dfrac{\Delta V_{D1}}{\Delta V_{cm}} = - \dfrac{g_m}{1 + g_m 2R_{SS}} \times R_D = \dfrac{-5.06k}{2k + 0.633k} = -1.92$

(e) If Q_1 and Q_2 get into triode region,

we know that $V_G - V_{th} = V_D$

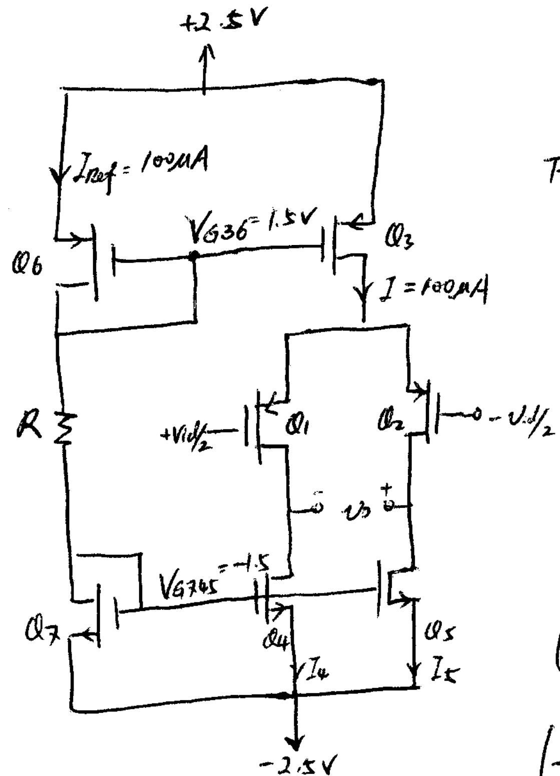
$2.332V + \Delta V_{cm} - 0.7V = 2.47V + \Delta V_D$

Based on (d), $\Delta V_D = -1.92\,\Delta V_{cm}$, hence,

$2.92\,\Delta V_{cm} = 0.838\ V$

$\Rightarrow \Delta V_{cm} = 0.287\ V$

Problem 3.



Obviously, $I_3 = I_6 = I = I_{ref} = 100\mu A$

$$I_1 = I_2 = I_4 = I_5 = \frac{I}{2} = 50\mu A.$$

Therefore, $\begin{cases} \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_6 \\ \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = \frac{1}{2}\left(\frac{W}{L}\right)_7. \end{cases}$

$$\begin{cases} I = 100\mu A = \frac{1}{2}\mu_p C_{ox} \left(\frac{W}{L}\right)_{3,6} (2.5 - 1.5 - 0.7)^2 \\ \frac{I}{2} = 50\mu A = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_{4,5} (-1.5 + 2.5 - 0.7)^2 \end{cases}$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_6 = 74.07, \quad \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = 12.346$$

$$\left(\frac{W}{L}\right)_7 = 2\left(\frac{W}{L}\right)_{4,5} = 24.69$$

If we want $A_d = 80\ V/V$. $g_{m1}(r_{on} // r_{op})$ must be 80.

where $g_{m1} = \sqrt{2 \cdot \frac{I}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_1}$ and $r_{on} = r_{op} = \frac{V_A}{I/2} = \frac{20V}{50\mu A} = 400k\Omega$

$$\therefore \sqrt{3000\ \mu A^2/V^2 \left(\frac{W}{L}\right)_1} \cdot 200k\Omega = 80$$

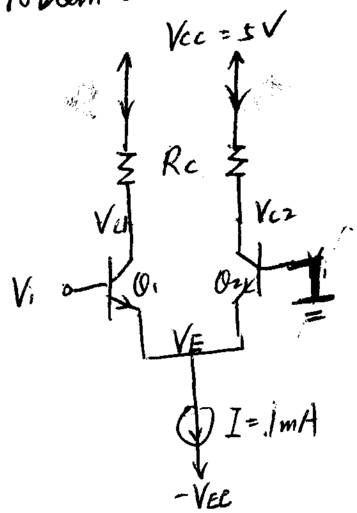
$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 53.33$$

As for R, we know $R = \frac{V}{I} = \frac{V_{G36} - V_{G745}}{I_{ref}} = \frac{3V}{100\mu A} = 30k\Omega$

For $V_{gs1,2}$, we know $50\mu A = \frac{1}{2}\mu_p C_{ox} \left(\frac{W}{L}\right)_{1,2} (V_{gs1,2} - 0.7)^2$

$$\Rightarrow V_{gs1,2} = 0.95V$$

Problem 6.



First we need to know what region the transistors are.

Because $V_i = 1V$. we can assume Q_1 works in saturation

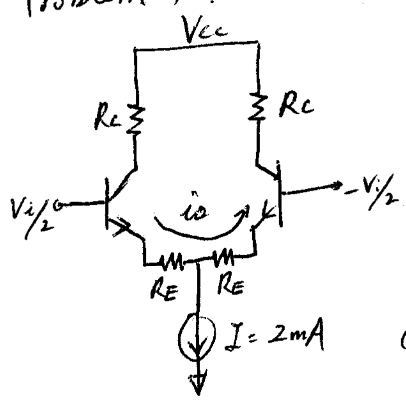
Hence, $V_E = 1 - 0.7 = 0.3V$. which means Q_2 cut off.

$\therefore$ $V_{C2} = V_{CC} = 5V$ $\quad I_{E1} = I = 1mA$.

$\therefore$ $I_{C1} = \alpha I_{E1} = \frac{\beta}{\beta+1} I_{E1} = 0.99\,mA$.

$\therefore$ $V_{C1} = V_{CC} - I_{C1} \cdot R_C = 2.03\,V$

Problem 7.



(a) $i_e = \frac{v_{id}}{2r_e + 2R_E} = \frac{0.1\,V}{50\Omega + 200\Omega} = 400\,\mu A$

$v_{be} = (v_{id} - 2R_E \cdot i_e)/2 = (0.1V - 200\Omega \times 400\mu A)/2$

$= 0.01\,V$

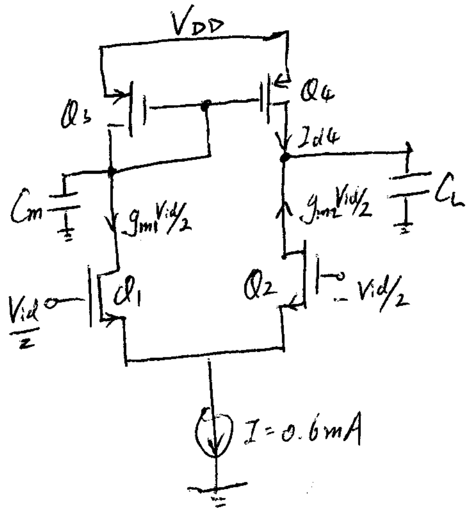
(b) $I_{E1} = \frac{I}{2} + i_e = 1.4\,mA$, $\quad I_{E2} = \frac{I}{2} - i_e = 0.6mA$.

(c)

$v_c = \pm \frac{R_C}{r_e + R_E} \times \frac{v_{id}}{2} = \pm \frac{5k\Omega}{25\Omega + 100\Omega} \times 0.05V = \pm 2V$

(d) $A_{vd} = \frac{R_C}{r_e + R_E} = 40\ V/V$

Problem 8:



we know $C_m = 0.1pF$, $C_L = 0.2pF$.

$$\begin{cases} g_{m1} = g_{m2} = \dfrac{2I/2}{V_{OV1,2}} = \dfrac{0.6mA}{0.3V} = 2mS \\ g_{m3} = g_{m4} = \dfrac{2I/2}{V_{OV3,4}} = \dfrac{0.6mA}{0.5V} = 1.2mS. \end{cases}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = r_o = \frac{V_A}{I/2} = \frac{9V}{0.3mA} = 30k\Omega$$

$$V_{g3} = V_{g4} = -g_{m1}V_{id}/2 \cdot \left[1/g_{m3} \,//\, r_o \,//\, r_o \,//\, \frac{1}{sC_m} \right]$$

$$= -g_{m1}V_{id}/2 \times \frac{1}{sC_m + 1.267mS}$$

$$I_{d4} = -g_{m4} \cdot V_{g3} = \frac{g_{m4}\, g_{m1}\, V_{id}/2}{sC_m + 1.267mS}$$

$$\Rightarrow I_o = I_{d4} + I_{d2} = \frac{g_{m4}\, g_{m1}\, V_{id}/2}{sC_m + 1.267mS} + g_{m2}V_{id}/2 \qquad \text{and} \qquad V_o = I_o \frac{1}{\frac{1}{R_o} + sC_L}$$

$$\Rightarrow V_o = \frac{V_{id}}{2} \times \left[g_{m2} + \frac{g_{m4}\, g_{m1}}{sC_m + 1.267mS} \right] \left[\frac{R_o}{1 + sC_L R_o} \right]$$

$$\Rightarrow A_V = \frac{V_o}{V_{id}} = \frac{g_{m1}}{2} \left[1 + \frac{g_{m4}}{sC_m + 1.267mS} \right] \left[\frac{15k\Omega}{1 + sC_L R_o} \right]$$

$$= g_{m1} \times 15k\Omega \times \frac{0.97356}{1 + s\, C_m/1.267mS} \times \frac{1 + s\, C_m/2.467mS}{1 + sC_L R_o}$$

Therefore $A_{Vo} = g_{m1} \times 15k\Omega \times 0.97356 = 29.2068$

$$f_{p1} = \frac{1}{2\pi C_L R_o} = \frac{1}{2\pi \times 0.2pF \times 15k\Omega} = 53.052MHz$$

$$f_{p2} = \frac{1}{2\pi\, C_m/1.267mS} = \frac{1}{2\pi \times 0.1pF/1.267mS} = 2.016GHz$$

$$f_z = \frac{1}{2\pi\, C_m/2.467mS} = 3.926GHz.$$