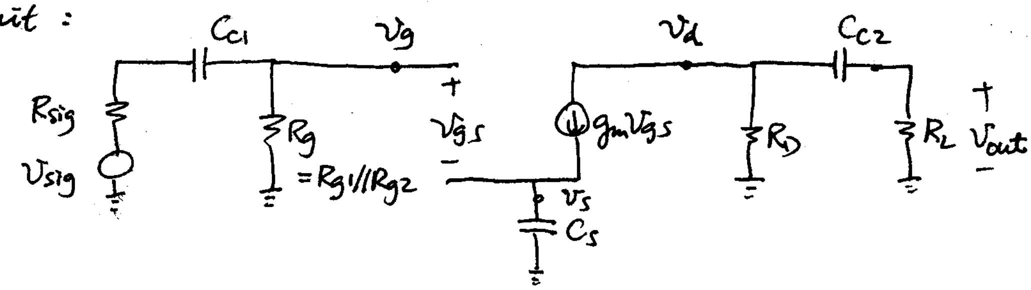
Short-Circuit Time Constant Method for Estimating $\omega_{-3dB\ LF}$

Low-freq response of a CS amplifier. (Fig. 4.51 in text)

Eq. Circuit:



Eq. 4.139:

$$A_{v\_LF} = \frac{v_{out}}{v_{sig}} = \underbrace{-g_m(R_D//R_L)\frac{R_g}{R_g+R_{sig}}}_{A_{v\_midband}} \cdot \left(\frac{s}{s+\omega_{p1}}\right)\left(\frac{s}{s+\omega_{p2}}\right)\left(\frac{s}{s+\omega_{p3}}\right)$$

where $\omega_{p1} = \dfrac{1}{(R_g+R_{sig})C_{C1}}$, $\omega_{p2} = \dfrac{g_m}{C_s}$, $\omega_{p3} = \dfrac{1}{(R_D+R_L)C_{C2}}$

Rewrite 4.139 as

$$A_{v\text{-}LF} = A_{v\_midband}\left(\frac{1}{1+\frac{\omega_{p1}}{s}}\right)\left(\frac{1}{1+\frac{\omega_{p2}}{s}}\right)\left(\frac{1}{1+\frac{\omega_{p3}}{s}}\right)$$

$$= A_{v\_midband}\left(\frac{1}{1+\frac{\omega_{p1}}{s}+\frac{\omega_{p2}}{s}+\frac{\omega_{p3}}{s}+\frac{\omega_{p1}\omega_{p2}}{s^2}+\frac{\omega_{p2}\omega_{p3}}{s^2}+\frac{\omega_{p1}\omega_{p3}}{s^2}+\frac{\omega_{p1}\omega_{p2}\omega_{p3}}{s^3}}\right)$$

$$= A_{v\_midband}\left(\frac{1}{1+\frac{\omega_{p1}+\omega_{p2}+\omega_{p3}}{s}+\frac{\omega_{p1}\omega_{p2}+\omega_{p2}\omega_{p3}+\omega_{p1}\omega_{p3}}{s^2}+\frac{\omega_{p1}\omega_{p2}\omega_{p3}}{s^3}}\right)$$

sub $s = j\omega$, and for $\omega > \omega_{p1}, \omega_{p2}, \omega_{p3}$, we can ignore the s^{-2} and s^{-3} terms:

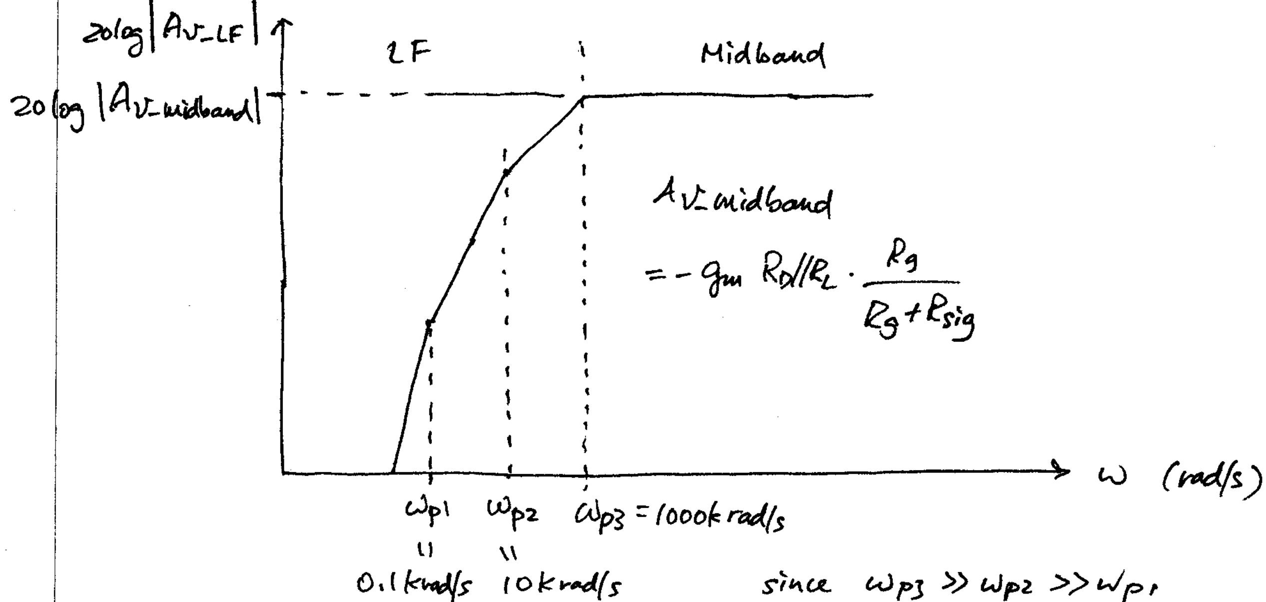
$$\therefore A_{v\text{-}LF} \approx A_{v\_midband}\frac{1}{1+\frac{\omega_{p1}+\omega_{p2}+\omega_{p3}}{s}} = A_{v\_midband}\frac{s}{s+(\omega_{p1}+\omega_{p2}+\omega_{p3})}$$

therefore, the lower corner frequency is $\boxed{\omega_{LF} \approx \omega_{p1}+\omega_{p2}+\omega_{p3}}$

e.g. $\omega_{p3} = 1000$ krad/s, $\omega_{p2} = 10$ krad/s, $\omega_{p1} = 0.1$ krad/s

i.e. $\omega_{p3} \gg \omega_{p2} \gg \omega_{p1}$, $\omega_{LF} \approx \omega_{p3}$ the dominate corner frequency.

sketch of the LF Bode plot :



$A_{V\_midband}$

$= -g_m R_D // R_L \cdot \dfrac{R_g}{R_g + R_{sig}}$

since $\omega_{p3} \gg \omega_{p2} \gg \omega_{p1}$

$\omega_{LF} \approx \omega_{p3}$.

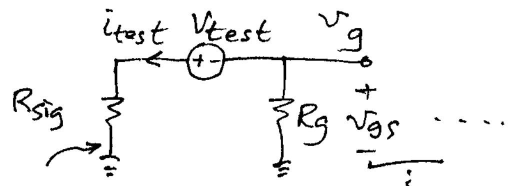
From the above derivation, we can generalize a quick estimation technique for computing ω_{LF}

$$\omega_{LF} = \sum_{i=1}^{n} \left(\frac{1}{\tau_i}\right)$$

where τ_i is the time constant associated with the ith ac-coupling or bypass capacitor.

To find τ_i, we need to find the effective resistance (R_i) seen by C_i. $\tau_i = R_i C_i$. When solving for R_i, all other capacitors are <u>short circuited</u>. This technique is sometimes referred to the $\boxed{\text{Short Circuit Time Constant (SCTC) method}}$ for solving the low-frequency corner freq (ω_{LF}).

(A) For the above example, $\tau_1 = (R_g + R_{sig}) C_{c1}$, b/c the effective R seen by C_{c1} is $R_g + R_{sig}$:

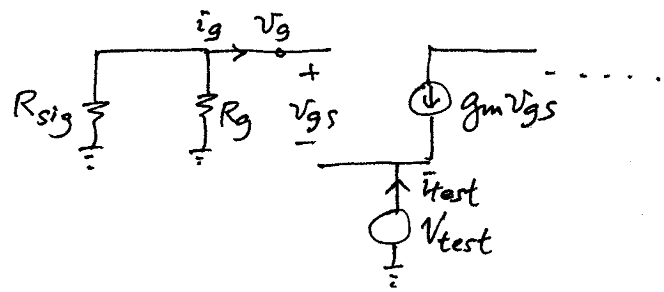


$$R_{C_{c1}} = \frac{V_{test}}{i_{test}} = R_{sig} + R_g .$$

$$\therefore \boxed{\tau_1 = R_{C_{c1}} C_{c1} = (R_g + R_{sig}) C_{c1}}$$

v_{sig}, the independent source is

(b) τ_2 is associated with C_s, to find R_{C_s}, again we remove v_{sig} and short circuit all other capacitors in the eq. circuit (i.e. C_{C1} and C_{C2}), then apply a test voltage source to replace C_s and solve for V_{test}/i_{test}



Perform KCL at the source node. $i_{test} = -g_m v_{gs}$

Also, since $i_g = 0$, $v_g = 0$.

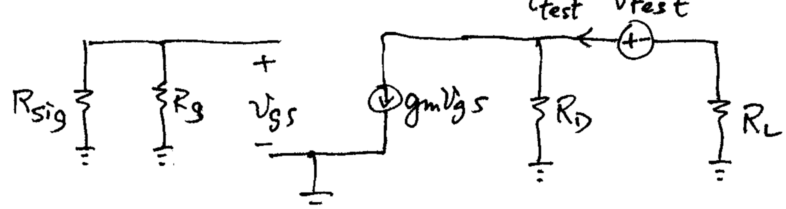
Therefore $v_{gs} = v_g - v_s = -v_s$, which is $= -v_{test}$.

$\Rightarrow \quad v_{gs} = -v_{test}$

Now we can find $\dfrac{v_{test}}{i_{test}} = \dfrac{-v_{gs}}{-g_m v_{gs}} = \dfrac{1}{g_m}$ (as expected)

$\therefore \quad \boxed{\tau_2 = \frac{1}{g_m} \cdot C_s}$

(c) The 3rd time constant, τ_3, is associated with C_{C2}: The eq. circuit is



since both v_g and v_s are both 0V, $g_m v_{gs} = 0$,

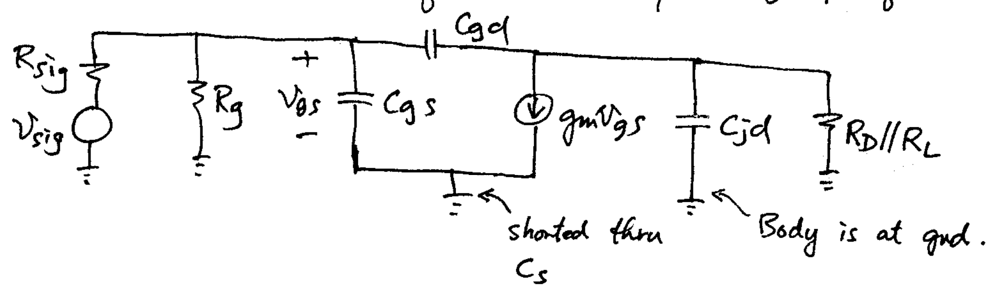
so $R_{C_{C2}} = R_D + R_L = V_{test}/i_{test}$.

$\therefore \quad \boxed{\tau_3 = R_{CC2} \cdot C_{C2} = (R_D + R_L)\, C_{C2}}$.

To recap, once we know the time constants, we estimate

$$\omega_{LF} = \sum_{i=1}^{n} \frac{1}{\tau_i} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} = \omega_{p1} + \omega_{p2} + \omega_{p3}.$$

Now, we can solve for the HF response of the CS amp. Let's include the effects of C_{gs}, C_{gd} and C_{jd} in the small-signal model. Therefore, the eq. circuit for high-freq analysis is



C_{c1}, C_{c2} and C_s are all replaced by short circuit.

Because at high freq, $\frac{1}{sC_{c1}} \rightarrow \frac{1}{j\omega C_{c1}} \rightarrow 0$ for large ω.

$\frac{1}{j\omega C_{c2}} \rightarrow 0$ "

$\frac{1}{j\omega C_s} \rightarrow 0$ "

Just like we did in the midband analysis.

The technique to use is called open-circuit time constant (OCTC) method.