This homework requires the material covered in Lectures #11 and #12.

Exercise 1 (Triangular switched systems). Given two asymptotically stable upper triangular $2 \times 2$ matrices $A_1$ and $A_2$, show that it is always possible to find a common quadratic Lyapunov function for the switched system

$$\dot{x} = A_\sigma x$$

of the form $V(x) = x'Px$ with $P$ diagonal.

Exercise 2 (Switching between controllers). Consider the switching controller for the single-link flexible manipulator that you designed for the first Exercise in Homework #3.

Construct realizations for the two controllers that guarantee stability of the switched feedback system regardless of when the position of the tip is available. This would actually be quite important because vision systems are prone to often not providing measurements (even when the tip would be in the camera’s field of view) because of difficulties in image processing.

Hint: This setup differs slightly from the one considered in the lecture #12 because the $C$ matrix of the process changes. However, you can use the controller for which the tip position is not available as a “nominal” controller, because it always stabilizes the process.