Problem 1. Consider the following switched system
\[ \dot{x} = A_\sigma x, \quad x(t) \in \mathbb{R}^n, \quad \sigma(t) \in \{1, 2\} \quad (1) \]
where the matrix \( A_1 \) is asymptotically stable and the matrix \( A_2 \) is unstable. Show that there exist constants \( \tau_D, \alpha > 0 \) such that (1) is uniformly asymptotically stable for any set of switching signals \( S \) for which
\[ N_\sigma(\tau, t) \leq N_0 + \frac{t - \tau}{\tau_D}, \quad \text{and} \quad T_\sigma(\tau, t) \leq T_0 + \alpha(t - \tau), \quad \forall t > \tau \geq 0, \]
for every \((\sigma, x) \in S\). In the above equation, \( N_\sigma(\tau, t) \) denotes the number of discontinuities of \( \sigma \) in the open interval \((\tau, t)\) and \( T_\sigma(\tau, t) \) denotes the amount of time in the interval \((\tau, t)\) that \( \sigma \) is equal to 2, i.e., \( T_\sigma(\tau, t) := \int_\tau^t (\sigma(s) - 1) ds \).

Problem 2. Consider the following switched system
\[ \dot{x} = A_\sigma x \quad x = x^- \quad (\sigma, x) \in S[\chi] \]
where \( S[\chi] \) is a current-state dependent set of switching signals for which \( x(t) \in \chi_{\sigma(t)} \), \( \forall t \geq 0 \), with
\[ \chi_q := \{ z \in \mathbb{R}^n : E_q [\hat{z}] \geq 0 \} \quad \chi_q \cap \chi_p \subset \{ z \in \mathbb{R}^n : f'_{pq} [\hat{z}] = 0 \}, \quad \forall p, q \in Q. \]
The following was proved in class:

**Theorem 1.** Suppose that there exist symmetric matrices \( P_q \in \mathbb{R}^{(n+1) \times (n+1)}, \forall q \in Q \); vectors \( k_{pq} \in \mathbb{R}^{n+1}, \forall p, q \in Q \); constants \( \epsilon, \delta > 0 \); and symmetric matrices with nonnegative entries \( U_q, W_q, \forall q \in Q \) (not necessarily positive definite) such that (1) for every \( p, q \in Q \) for which \( \chi_q \) and \( \chi_p \) have a common boundary we have that
\[ P_q - P_p = k_{pq} f'_{pq} + f_{pq} k_{pq} \]
and (2) for every \( q \in Q \)
\[ \Pi^T A_q^T \Pi P_q + P_q \Pi^T A_q \Pi + E_q^T U_q E_q \leq -\epsilon \Pi^T \Pi \quad \quad P_q - E_q^T W_q E_q \geq \delta \Pi^T \Pi \]
where \( \Pi := [I_{n \times n}, 0_{1 \times n}] \), then the origin is uniformly exponentially stable. \( \square \)

Generalize this result for a switched system of the form
\[ \dot{x} = A_\sigma x + b_\sigma \quad x = x^- \quad (\sigma, x) \in S[\chi]. \]
where each \( b_q, q \in Q \) is a constant vector.