

Hybrid Control and Switched Systems

Lecture #1 Hybrid systems are everywhere: Examples

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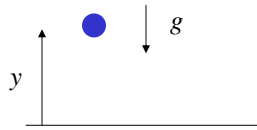


Summary

Examples of hybrid systems

1. Bouncing ball
2. Thermostat
3. Transmission
4. Inverted pendulum swing-up
5. Multiple-tank
6. Server
7. Supervisory control

Example #1: Bouncing ball



Free fall $\equiv \ddot{y} = -g$

Collision $\equiv y^+(t) = y^-(t) = 0$

$\dot{y}^+(t) = -c\dot{y}^-(t)$

$c \in [0,1] \equiv$ energy “reflected” at impact

Notation: given $x : [0, \infty) \rightarrow \mathbb{R}^n \equiv$ piecewise continuous signal

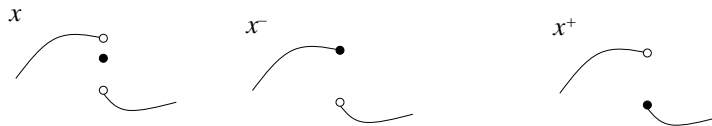
$x^- : (0, \infty) \rightarrow \mathbb{R}^n \quad x^-(t) := \lim_{\tau \uparrow t} x(t), \quad \forall t > 0$

$x^+ : [0, \infty) \rightarrow \mathbb{R}^n \quad x^+(t) := \lim_{\tau \downarrow t} x(t), \quad \forall t \geq 0$

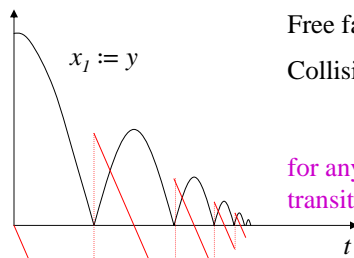
at points t where x is continuous $x(t) = x^-(t) = x^+(t)$

By convention we will generally assume right continuity, i.e.,

$x(t) = x^+(t) \quad \forall t \geq 0$



Example #1: Bouncing ball



Free fall $\equiv \ddot{y} = -g$

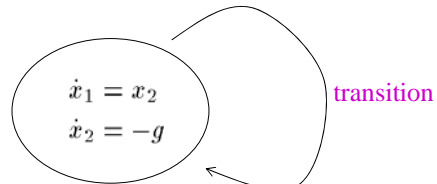
Collision $\equiv y^+(t) = y^-(t) = 0$

$\dot{y}^+(t) = -c\dot{y}^-(t)$

for any $c < 1$, there are infinitely many transitions in finite time (Zeno phenomena)

$x_2 := \dot{y}$ guard or jump condition

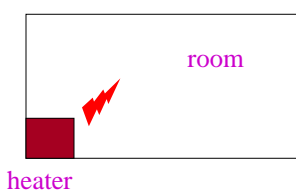
$x_1 = 0 \ \& \ x_2 < 0 ?$



state reset

$x_2 := -c x_2^-$

Example #2: Thermostat



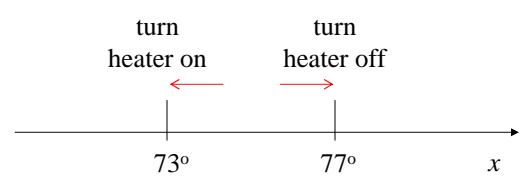
goal \equiv regulate temperature around 75°

$x \equiv$ mean temperature

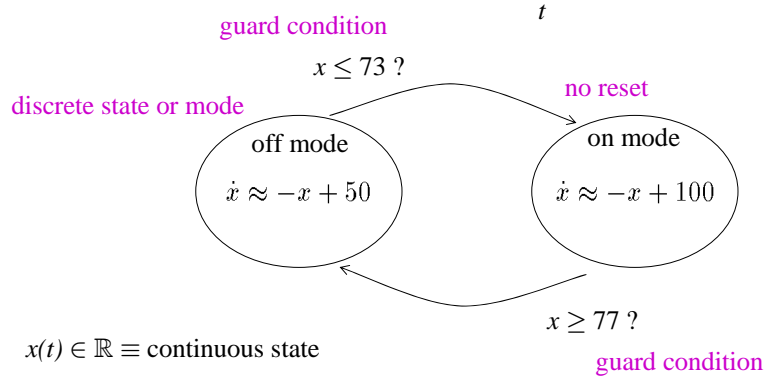
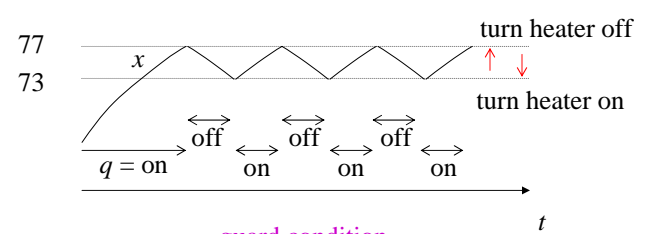
when heater is off: $\dot{x} \approx -x + 50 \quad (x \rightarrow 50^\circ)$

when heater is on: $\dot{x} \approx -x + 100 \quad (x \rightarrow 100^\circ)$

event-based control



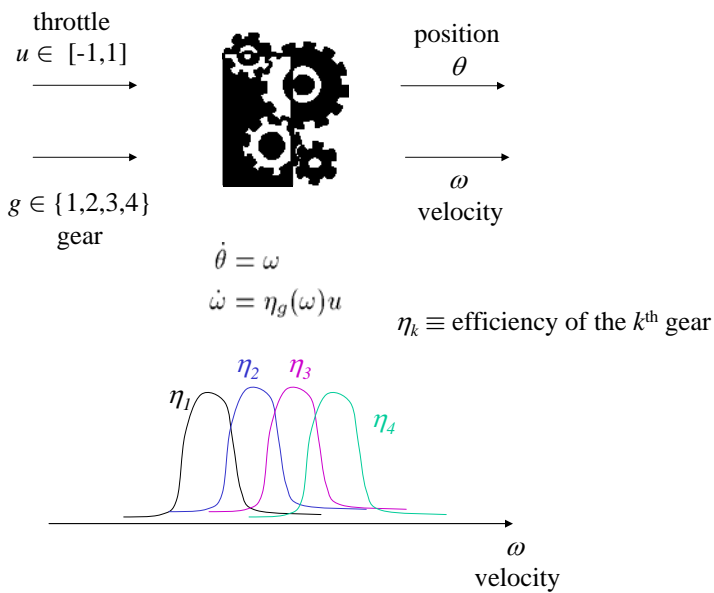
Example #2: Thermostat



$x(t) \in \mathbb{R} \equiv$ continuous state

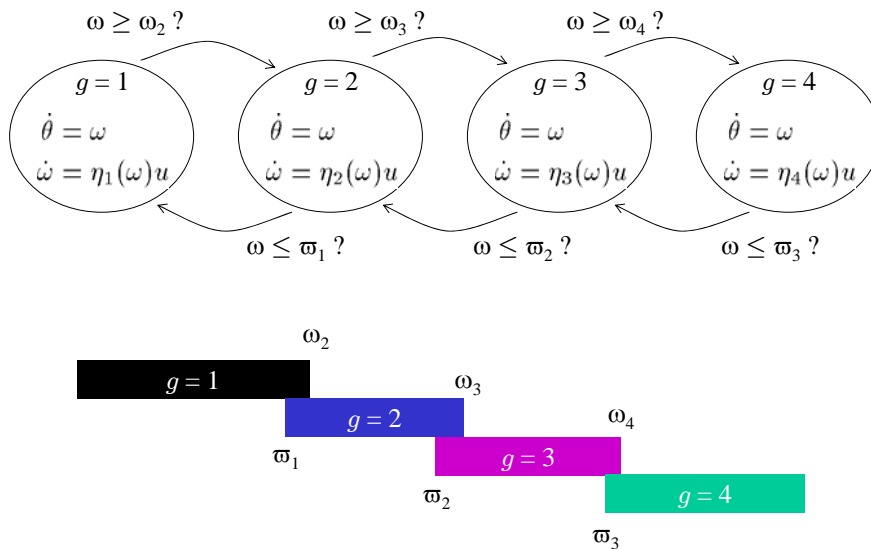
$q(t) \in \{ \text{off, on} \} \equiv$ discrete state

Example #3: Transmission



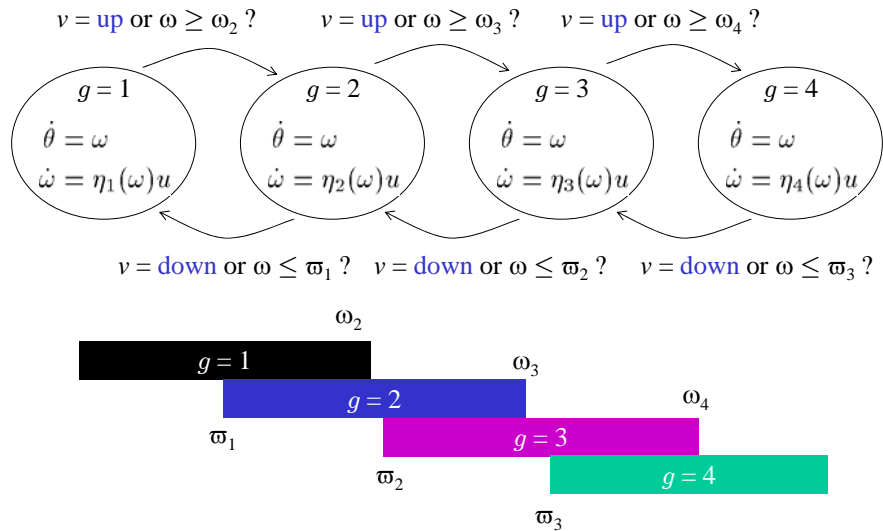
[Hedlund, Rantzer 1999]

Example #3: Automatic transmission

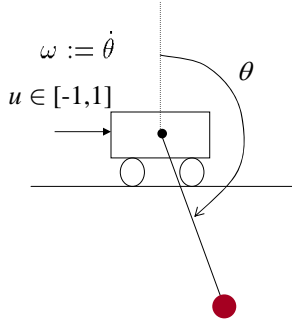


Example #3: Semi-automatic transmission

$v(t) \in \{ \text{up, down, keep} \} \equiv$ drivers input (discrete)



Example #4: Inverted pendulum swing-up



goal \equiv drive θ to 0 (upright position)

$$\ddot{\theta} = \sin \theta - u \cos \theta$$

$u \in [-1, 1] \equiv$ force applied to the cart

$$\text{Total system energy} \equiv E := \frac{1}{2} \omega^2 + (\cos \theta - 1)$$

kinetic potential

normalized to be zero at stationary upright position

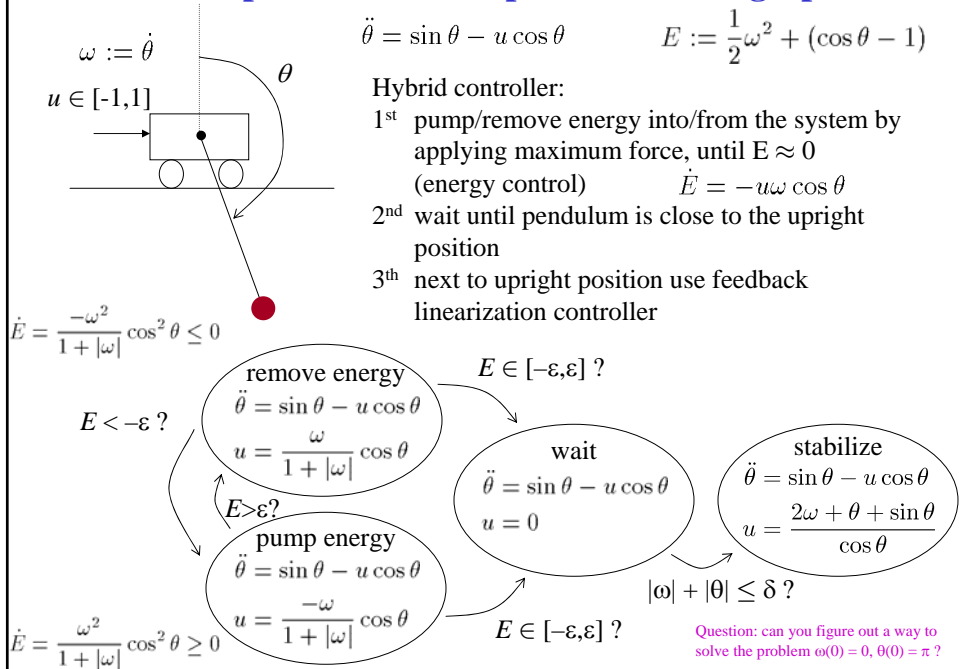
Feedback linearization controller: try to make

$$\ddot{\theta} + 2\dot{\theta} + \theta = 0 \quad \leftarrow \quad \theta(t) = e^{-t}(\theta(0) + \theta(0)t + \omega(0)t)$$

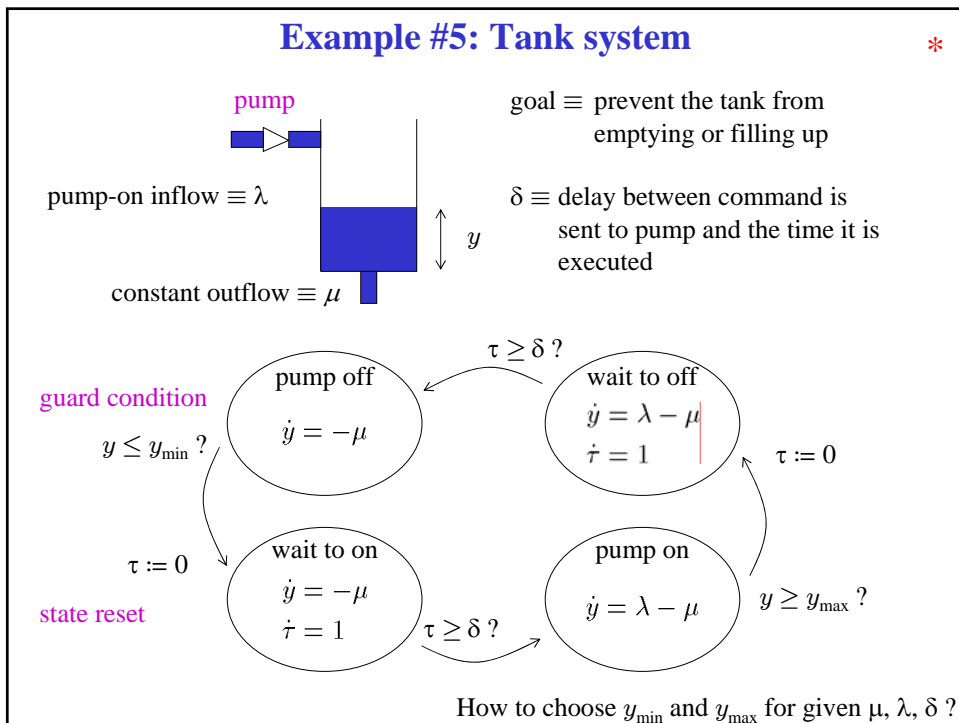
$$\begin{cases} \ddot{\theta} = \sin \theta - u \cos \theta \\ \ddot{\theta} + 2\dot{\theta} + \theta = 0 \end{cases} \Rightarrow u := \frac{2\dot{\theta} + \theta + \sin \theta}{\cos \theta} \quad \text{only in } [-1, 1] \text{ close to upright position } (\theta = \omega = 0)$$

[Astrom, Furuta 1999]

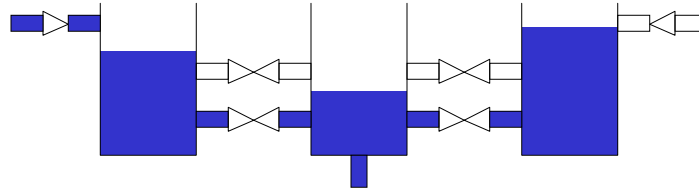
Example #4: Inverted pendulum swing-up



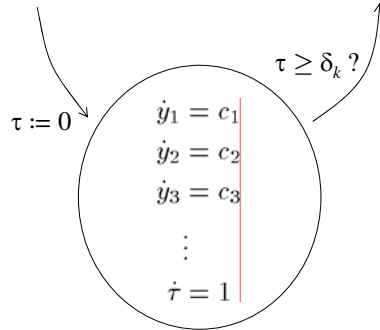
Example #5: Tank system



Example #5: Multiple-tank system



$$y_1 \leq y_{1,\min} \ \& \ y_2 \geq y_{2,\max} ?$$

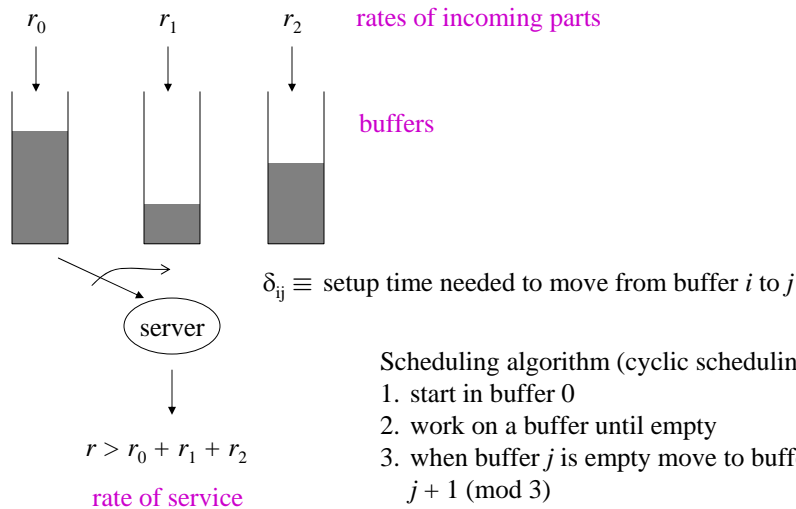


Initialized rectangular hybrid automata

- all differential equations have constant r.h.s.
- all jump cond. are of the form: state var. 1 \in fixed interval 1 & state var. 2 \in fixed interval 2 & etc.
- all resets have constant r.h.s.

Most general class of hybrid systems for which there exist completely automated procedures to compute the set of reachable states

Example #6: Switched server



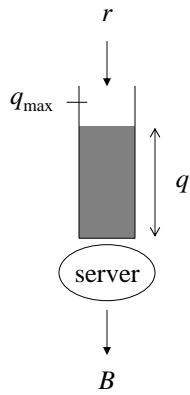
Scheduling algorithm (cyclic scheduling):

1. start in buffer 0
2. work on a buffer until empty
3. when buffer j is empty move to buffer $j + 1 \pmod{3}$

Often also an initialized rectangular hybrid automata ...

Example #7: Server system with congestion control

incoming rate

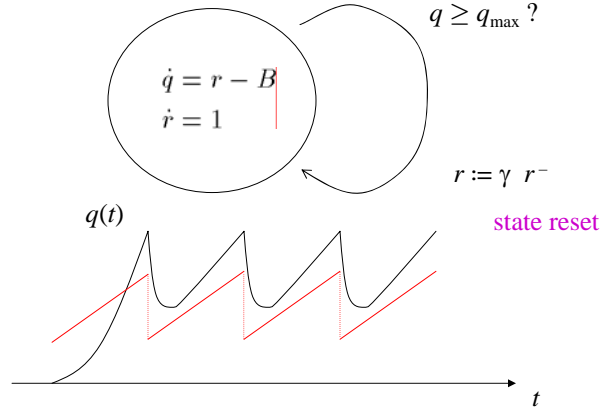


rate of service
(bandwidth)

Additive increase/multiplicative decrease congestion control (AIMD):

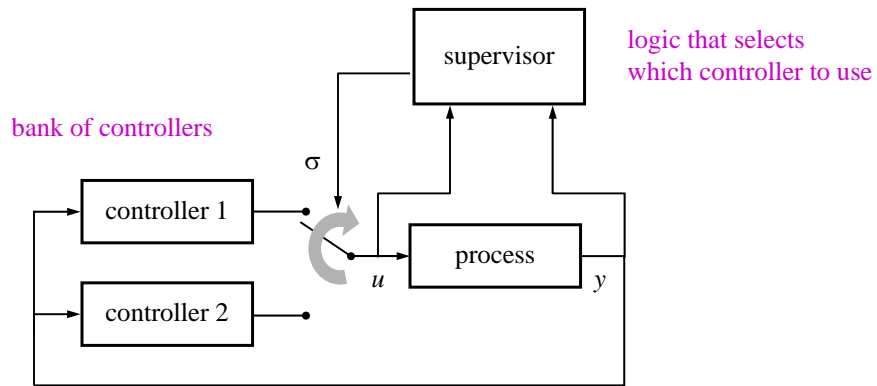
- while $q < q_{\max}$ increase r linearly
- when q reaches q_{\max} instantaneously multiply r by $\gamma \in (0,1)$

guard condition

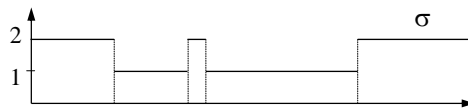


no longer an initialized rectangular hybrid automata ...

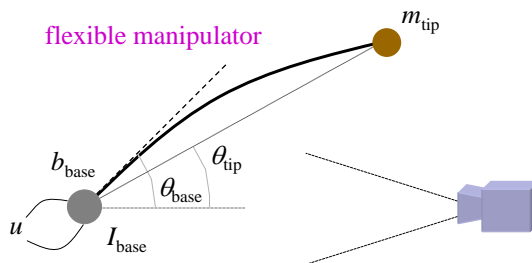
Example #8: Supervisory control



$\sigma \equiv$ switching signal taking values in the set $\{1,2\}$



E.g. #8 a): Vision-based control of a flexible manipulator



goal \equiv drive θ_{tip} to zero, using feedback from

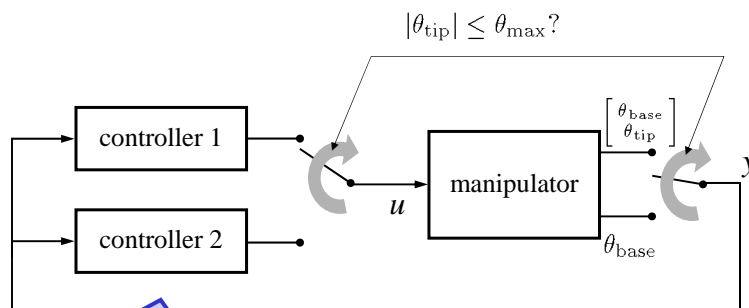
θ_{base} \rightarrow encoder at the base

θ_{tip} \rightarrow machine vision (needed to increase the damping of the flexible modes in the presence of noise)

To achieve high accuracy in the measurement of θ_{tip} the camera must have a **small field of view**

output feedback output:
$$y := \begin{cases} \begin{bmatrix} \theta_{base} \\ \theta_{tip} \end{bmatrix} & |\theta_{tip}| \leq \theta_{max} \\ \theta_{base} & |\theta_{tip}| > \theta_{max} \end{cases}$$

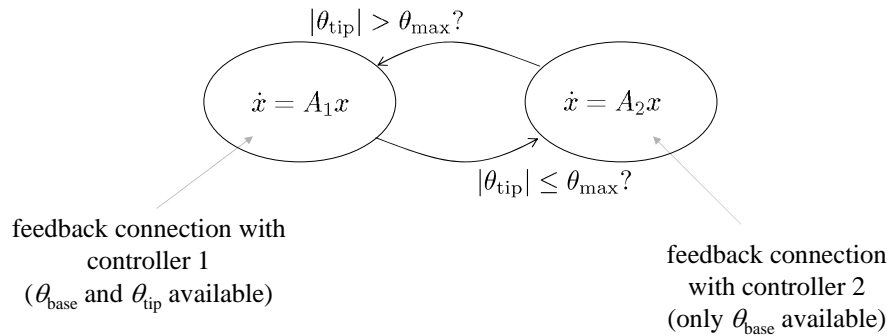
E.g. #8 a): Vision-based control of a flexible manipulator



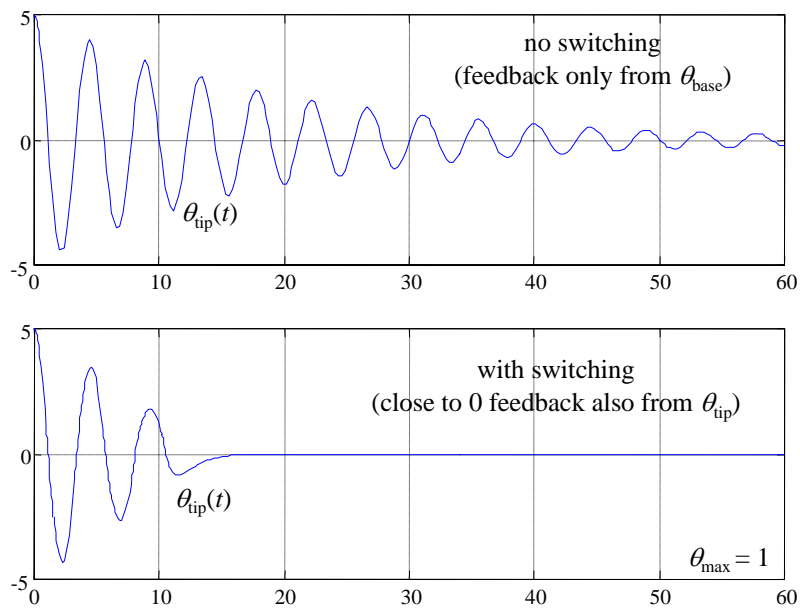
controller 1 optimized for feedback from θ_{base} and θ_{tip}
and
controller 2 optimized for feedback only from θ_{base}

E.g., LQG controllers that minimize $\lim_{T \rightarrow \infty} \frac{1}{T} E \left[\int_0^T \theta_{tip}^2 + \dot{\theta}_{tip}^2 + \rho u^2 dt \right]$

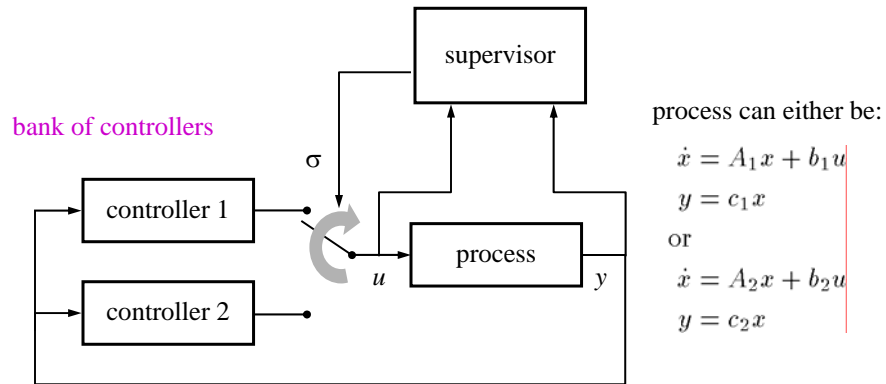
E.g. #8 a): Vision-based control of a flexible manipulator



E.g. #8 a): Vision-based control of a flexible manipulator



Example #8 b): Adaptive supervisory control



Goal: stabilize process, regardless of which is the actual process model

Supervisor must

- try to determine which is the correct process model by observing u and y
- select the appropriate controller

Next class...

1. Formal models for hybrid systems:

- Finite automata
- Differential equations
- Hybrid automata
- Open hybrid automaton

2. Nondeterministic vs. stochastic systems

- Non-deterministic automata and differential inclusions
- Markov chains and stochastic processes