Lecture #14
Computational methods to construct multiple Lyapunov functions & Applications

João P. Hespanha
University of California at Santa Barbara

Summary

• Computational methods to construct multiple Lyapunov functions—Linear Matrix Inequalities (LMIs)
• Applications (vision-based control)
**Current-state dependent switching**

\[ \chi := \{ \chi_q \in \mathbb{R}^n: q \in Q \} \equiv \text{(not necessarily disjoint) covering of } \mathbb{R}^n, \text{i.e., } \bigcup_{q \in Q} \chi_q = \mathbb{R}^n \]

**Current-state dependent switching**

\[ S[\chi] \equiv \text{set of all pairs } (\sigma, x) \text{ with } \sigma \text{ piecewise constant and } x \text{ piecewise continuous such that } \forall t, \sigma(t) = q \text{ is allowed only if } x(t) \in \chi_q \]

Thus \((\sigma, x) \in S[\chi] \text{ if and only if } x(t) \in \chi_{\sigma(t)} \forall t \]

**Multiple Lyapunov functions**

\[ \dot{x} = f_\sigma(x) \quad x = \rho(\sigma, \sigma^-, x^-) \quad (\sigma, x) \in S \subset S[\chi] \]

**Theorem:**
Suppose there exists \(\alpha_1 \in \mathcal{K}_\infty, \alpha_2 \in \mathcal{K}\) and a family of continuously differentiable, functions \(V_q: \mathbb{R}^n \to \mathbb{R}, q \in Q\) such that

\[ V_q(z) \geq \alpha_1(||z||), \quad \frac{\partial V_q}{\partial x}(z)f_q(z) \leq -\alpha_2(||z||) \quad \forall q \in Q, \ z \in \chi_q \]

and at any \(z \in \mathbb{R}^n\) where a switching signal in \(S\) can jump from \(p\) to \(q\)

\[ V_p(z) \geq V_q(\rho(q, p, z)) \]

Then the origin is (glob) uniformly asymptotically stable.
Multiple Lyapunov functions (linear)

\[ \dot{x} = A_\sigma x \quad x = x^- \quad (\sigma, x) \in \mathcal{S} \subset \mathcal{S}[\chi] \]

Theorem:
Suppose there exist constants \( \delta, \varepsilon \geq 0 \) and symmetric matrices \( P_q \in \mathbb{R}^{n \times n}, q \in Q \) such that

\[ z' P_q z \geq \delta \|z\|^2 \quad z' (A_q^T P_q + P_q A_q) z \leq -\varepsilon \|z\|^2 \quad \forall q \in Q, \quad z \in \chi_q \]

and at any \( z \in \mathbb{R}^n \) where a switching signal in \( \mathcal{S} \) can jump from \( p \) to \( q \)

\[ z P_q z \leq z' P_p z \]

Then the origin is uniformly asymptotically stable (actually exponentially).

Given the \( A_q, q \in Q \)
can we find \( P_q, q \in Q \) and \( \chi := \{ \chi_q : q \in Q \} \) so that we have stability?

stabilization through choice of switching regions

\[ \chi_q := \{ z \in \mathbb{R}^n : z' P_q z \geq z' P_p z \quad \forall p \in Q \} \]

\[ = \{ z \in \mathbb{R}^n : z' P_q z = \max_{p \in Q} z' P_p z \} \]

sigma must be equal to (one of) the largest

\[ V_q(z) := z' P_q z \Rightarrow \text{for switching to be possible at } z \in \mathbb{R}^n \text{ from } p \text{ to } q, \text{ we must have } V_q(z) = V_p(z) \Rightarrow \text{jump condition is always satisfied} \]
Multiple Lyapunov functions (linear)

\[ \dot{x} = A x \quad x = x^r \quad (\sigma, x) \in \mathcal{S} \subseteq \mathcal{S}[\chi] \]

**Theorem:**
Suppose there exist constants \( \delta, \varepsilon \geq 0 \) and symmetric matrices \( P_q \in \mathbb{R}^{n \times n}, q \in Q \) such that

\[ z' P_q z \geq \delta \|z\|^2 \quad z' (A_q' P_q + P_q A_q) z \leq -\varepsilon \|z\|^2 \quad \forall q \in Q, \; z \in \chi_q \]

and at any \( z \in \mathbb{R}^n \) where a switching signal in \( S \) can jump from \( p \) to \( q \)

\[ z P_q z \leq z' P_p z \]

Then the origin is uniformly asymptotically stable (actually exponentially).

It is sufficient to find \( P_q, q \in Q \) such that

\[ z' P_q z \geq \delta \|z\|^2 \quad z' (A_q' P_q + P_q A_q) z \leq -\varepsilon \|z\|^2 \quad \forall q \in Q, \; z \in \chi_q \]

or equivalently \( \forall q \in Q: \)

\[ z' P_q z \geq z' P_p z \quad \forall p \in Q \implies z' P_q z \geq \delta \|z\|^2 \quad z' (A_q' P_q + P_q A_q) z \leq -\varepsilon \|z\|^2 \]

**Finding multiple Lyapunov functions**

We need to find \( P_q, q \in Q \) such that \( \forall q \in Q \)

\[ z' P_q z \geq z' P_p z \quad \forall p \in Q \implies z' P_q z \geq \delta \|z\|^2 \quad z' (A_q' P_q + P_q A_q) z \leq -\varepsilon \|z\|^2 \]

Suppose we can find \( P_q, q \in Q \) and constants \( \gamma_{p,q}, \mu_{p,q} \geq 0, p,q \in Q \) such that

\[
\begin{align*}
    P_q &\geq \delta I + \sum_{p \in Q \setminus \{q\}} \mu_{p,q} (P_q - P_p) \\
    A_q' P_q + P_q A_q &\leq -\varepsilon I - \sum_{p \in Q \setminus \{q\}} \gamma_{p,q} (P_q - P_p)
\end{align*}
\]

Then

\[
\begin{align*}
    z' P_q z &\geq \delta \|z\|^2 + \sum_{p \in Q \setminus \{q\}} \mu_{p,q} z' (P_q - P_p) z \\
    z' (A_q' P_q + P_q A_q) z &\leq -\varepsilon \|z\|^2 - \sum_{p \in Q \setminus \{q\}} \gamma_{p,q} z' (P_q - P_p) z \\
    &\geq 0 \text{ when } z' P_q z \geq z' P_p z \forall p \in Q
\end{align*}
\]
Constructing multiple Lyapunov functions

\[ \dot{x} = A_\sigma x \quad x = x^- \quad (\sigma, x) \in \mathcal{S} \subset \mathcal{S}[x] \]

**Theorem:**
Suppose there exist constants \( \delta, \varepsilon, \gamma_p, \mu_p, q, \mu_q \geq 0, p, q \in \mathbb{Q} \) and symmetric matrices \( P_q \in \mathbb{R}^{n \times n}, q \in \mathbb{Q} \) such that

\[
P_q \geq \delta I + \sum_{p \in \mathcal{Q}\setminus\{q\}} \mu_{p,q} (P_q - P_p)
\]

\[
A_q^T P_q + P_q A_q \leq -\varepsilon I - \sum_{p \in \mathcal{Q}\setminus\{q\}} \gamma_{p,q} (P_q - P_p)
\]

Then, for

\[ \chi_q := \{ z \in \mathbb{R}^n : z^T P_q z \geq z^T P_p z \ \forall p \in \mathbb{Q} \} \]

the origin is uniformly asymptotically stable (actually exponentially).

---

**Linear Matrix Inequality (LMI)**

\[ F(x) \geq 0 \quad \text{where } F : \mathbb{R}^m \to \mathbb{R}^{n \times n} \equiv \text{affine function} \]

(i.e., \( F(x) - F(0) \) is linear)

System of linear matrix inequalities:

\[
\begin{align*}
F_1(x) & \geq 0 \\
F_2(x) & \geq 0 \\
& \vdots \\
F_k(x) & \geq 0
\end{align*}
\]

\[ \iff \quad \tilde{F}(x) \geq 0 \quad \tilde{F}(x) := \begin{bmatrix} F_1(x) & 0 & \cdots & 0 \\ 0 & F_2(x) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & F_k(x) \end{bmatrix} \]

System of linear matrix equalities and inequalities

\[
\begin{align*}
F(x) & \geq 0 \\
G(x) & = 0
\end{align*}
\]

\[ \iff \quad \begin{bmatrix} F(x) & 0 & \cdots & 0 \\ 0 & G(x) & \cdots & 0 \\ 0 & 0 & \cdots & -G(x) \end{bmatrix} \geq 0 
\]

(However most numerical solvers handle equalities separately more efficiently)

*There are very efficient methods to solve LMIs (type ‘help lmilab’ in MATLAB)*
Back to constructing multiple Lyapunov functions

\[ \dot{x} = A_\sigma x \quad x = x^- \quad (\sigma, x) \in \mathcal{S} \subset \mathcal{S}[\chi] \]

**Theorem:**
Suppose there exist constants \( \delta, \varepsilon, \gamma_{p,q}, \mu_{p,q} \geq 0, p,q \in Q \) and symmetric matrices \( P_q \in \mathbb{R}^{n \times n}, q \in Q \) such that

\[
\begin{align*}
P_q &\geq \delta I + \sum_{p \in \mathcal{E}(q)} \mu_{p,q} (P_q - P_p) \\
A_q^T P_q + P_q A_q &\leq -\varepsilon I - \sum_{p \in \mathcal{E}(q)} \gamma_{p,q} (P_q - P_p)
\end{align*}
\]

Then, for

\[ \chi_q := \{ z \in \mathbb{R}^n : z^T P_q z \geq z^T P_p z \forall p \in Q \} \]

the origin is uniformly asymptotically stable (actually exponentially).

* LMI on the unknowns \( \delta, \varepsilon > 0, \gamma_{p,q}, \mu_{p,q} \in \mathbb{R}, P_q \in \mathbb{R}^{n \times n}, p,q \in Q \).

Only useful if we have the freedom to choose the switching region \( \chi_q \)

Convex polyhedral regions

\[ \dot{x} = A_\sigma x \quad x = x^- \quad (\sigma, x) \in \mathcal{S} \subset \mathcal{S}[\chi] \]

\( \chi_q, q \in Q \equiv \) convex polyhedral with disjoint interiors

\[ \chi_q := \left\{ z \in \mathbb{R}^n : E_q \begin{bmatrix} z \\ 1 \end{bmatrix} \geq 0 \right\} \quad E_q \in \mathbb{R}^{m_q \times (n+1)} \]

boundary between regions \( p, q \in Q \):

\[ \chi_q \cap \chi_p \subset \left\{ z \in \mathbb{R}^n : f_{p,q}' \begin{bmatrix} z \\ 1 \end{bmatrix} = 0 \right\} \quad f_{p,q} \in \mathbb{R}^{n+1} \]
Theorem:
Suppose there exist
1. symmetric matrices $P_q \in \mathbb{R}^{(n+1) \times (n+1)}$, $q \in Q$
2. vectors $k_{pq} \in \mathbb{R}^{n+1}$, $p, q \in Q$
3. constants $\epsilon, \delta > 0$
4. symmetric matrices with nonnegative entries $U_q, W_q$, $q \in Q$ (not necessarily positive definite)
such that for every $p, q \in Q$ for which $\chi_q$ and $\chi_p$ have a common boundary
the origin is uniformly asymptotically stable (actually exponentially).

LMI on the unknowns $P_q, k_{pq}, \delta, \epsilon, U_q, W_q$

Consider multiple Lyapunov functions:

$V_q(z) := \begin{bmatrix} -z^T & I_n \end{bmatrix} P_q \begin{bmatrix} z \\ 1 \end{bmatrix} \quad q \in Q$

1st On the boundary between $\chi_q$ and $\chi_p$:

$f_{pq} \begin{bmatrix} z \\ 1 \end{bmatrix} = 0 \Rightarrow V_q(z) = V_p(z)$
Constructing multiple Lyapunov functions

\[ \dot{x} = A_\sigma x \quad x = x^- \quad (\sigma, x) \in \mathcal{S} \subset \mathcal{S}[x] \]

\[ \chi_q := \left\{ z \in \mathbb{R}^n : E_q \begin{bmatrix} z \\ 1 \end{bmatrix} \geq 0 \right\} \quad \bar{\chi}_q \cap \bar{\chi}_p \subset \left\{ z \in \mathbb{R}^n : f'_{pq} \begin{bmatrix} z \\ 1 \end{bmatrix} = 0 \right\} \]

\[ P_q - P_p = k_{pq} f'_{pq} + f_{pq} k'_{pq} \]

\[ P_q - E_q^T W_q E_q \geq \delta \Pi^T \Pi^* \]

\[ \Pi^T A_q^T \Pi P_q + P_q \Pi^T A_q \Pi + E_q^T U_q E_q \leq -c \Pi^T \Pi \]

Why?
Consider multiple Lyapunov functions

\[ V_q(z) := \begin{bmatrix} z^T & 1 \end{bmatrix} P_q \begin{bmatrix} z \\ 1 \end{bmatrix} \quad q \in \mathcal{Q}, \]

2nd On \( \chi_q \):

\[ V_q(z) = \begin{bmatrix} z^T & 1 \end{bmatrix} \begin{bmatrix} P_q & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z \\ 1 \end{bmatrix} = \left[ z^T \quad 1 \right] \left( P_q - E_q^T W_q E_q \right) \begin{bmatrix} z \\ 1 \end{bmatrix} + \left[ z^T \quad 1 \right] E_q^T W_q E_q \begin{bmatrix} z \\ 1 \end{bmatrix} \geq \delta \left[ z^T \quad 1 \right] \Pi^T \Pi \begin{bmatrix} z \\ 1 \end{bmatrix} \geq 0 \]

\[ \geq \delta \left[ z^T \quad 1 \right] \Pi^T \Pi \begin{bmatrix} z \\ 1 \end{bmatrix} = \delta \|z\|^2 \]

3rd On \( \chi_q \):

\[ \frac{\partial V_q(z)}{\partial z} A_q z = \begin{bmatrix} z^T & 1 \end{bmatrix} \left( \Pi^T A_q^T \Pi P_q + P_q \Pi^T A_q \Pi + E_q^T U_q E_q \right) \begin{bmatrix} z \\ 1 \end{bmatrix} \]

\[ = \left[ z^T \quad 1 \right] \left( \Pi^T A_q^T \Pi P_q + P_q \Pi^T A_q \Pi + E_q^T U_q E_q \right) \begin{bmatrix} z \\ 1 \end{bmatrix} - \left[ z^T \quad 1 \right] E_q^T U_q E_q \begin{bmatrix} z \\ 1 \end{bmatrix} \leq -c \left[ z^T \quad 1 \right] \Pi^T \Pi \begin{bmatrix} z \\ 1 \end{bmatrix} = -c \|z\|^2. \]
**Example: Vision-based control of a flexible manipulator**

4th dimensional small bending approximation

\[
m_{\text{tip}} \ell^2 \ddot{\theta}_{\text{tip}} = \ell k_{\text{flex}} (\dot{\theta}_{\text{base}} - \dot{\theta}_{\text{tip}}) + \ell b_{\text{flex}} (\dot{\theta}_{\text{base}} - \dot{\theta}_{\text{tip}})
\]

\[
l_{\text{base}} \dot{\theta}_{\text{base}} = -b_{\text{base}} \dot{\theta}_{\text{base}} + \ell k_{\text{flex}} (\theta_{\text{tip}} - \dot{\theta}_{\text{base}}) + \ell b_{\text{flex}} (\dot{\theta}_{\text{tip}} - \dot{\theta}_{\text{base}}) + k_{\text{motor}} u
\]

Control objective: drive \(\theta_{\text{tip}}\) to zero, using feedback from

- \(\theta_{\text{base}}\) → encoder at the base
- \(\theta_{\text{tip}}\) → machine vision (essential to increase the damping of the flexible modes in the presence of noise)

---

**Example: Vision-based control of a flexible manipulator**

To achieve high accuracy in the measurement of \(\theta_{\text{tip}}\), the camera must have a small field of view

Feedback output:

\[
y := \begin{cases} 
[\theta_{\text{base}} \theta_{\text{tip}}]' & |\theta_{\text{tip}}| \leq \theta_{\text{max}} \\
\theta_{\text{base}} & |\theta_{\text{tip}}| > \theta_{\text{max}}
\end{cases}
\]

Control objective: drive \(\theta_{\text{tip}}\) to zero, using feedback from

- \(\theta_{\text{base}}\) → encoder at the base
- \(\theta_{\text{tip}}\) → machine vision (essential to increase the damping of the flexible modes in the presence of noise)
Switched process

\[ [\theta_{\text{base}}, \theta_{\text{tip}}]' \]

\[ |\theta_{\text{tip}}| \leq \theta_{\text{max}} \]

\[ u \rightarrow \text{manipulator} \]

\[ \theta_{\text{base}} \]

\[ y \]

controller 1 optimized for feedback from \( \theta_{\text{base}} \) and \( \theta_{\text{tip}} \) and
controller 2 optimized for feedback only from \( \theta_{\text{base}} \)

E.g., LQG controllers that minimize

\[
\lim_{T \to \infty} \frac{1}{T} E \left[ \int_0^T \theta_{\text{tip}}^2 + \dot{\theta}_{\text{tip}}^2 + \rho u^2 \, dt \right]
\]
Switched system

|θ_{tip}| > θ_{max}?

\[ \dot{x} = A_1 x \]
\[ θ_{tip} = c_{tip} x \]

|θ_{tip}| ≤ θ_{max}?

\[ \dot{x} = A_2 x \]
\[ θ_{tip} = c_{tip} x \]

Feedback connection with controller 1
(θ_{base} and θ_{tip} available)

Feedback connection with controller 2
(only θ_{base} available)

\[ f_{12} := [c_{tip} \quad θ_{max}] \]
\[ f_{12} := [c_{tip} \quad -θ_{max}] \]

\[ E_2 := [-c_{tip} \quad -θ_{max}] \quad E_1 := [-c_{tip} \quad θ_{max}] \quad E_2 := [c_{tip} \quad -θ_{max}] \]

\[ θ_{max} \quad θ_{max} \quad θ_{tip} \]

\[ -θ_{max} \]

\[ \chi_q := \{ z \in \mathbb{R}^n : L_q [z] \geq 0 \} \]
\[ \bar{\chi}_q \cap \chi_p \subset \{ z \in \mathbb{R}^n : f_{pq} [z] = 0 \} \]

Closed-loop response

No switching
(feedback only from θ_{base})

With switching
(close to 0 feedback also from θ_{tip})
Visual servoing

**Goal:** Drive the position $x$ of the end-effector to a reference point $r$

**encoder-based control**
- reference $r$ in pre-specified position
- end-effector position $x$ determined from the joint angles

**mixed vision/encoder-based control**
- reference $r$ determined from the camera measurement $r_c$
- end-effector position $x$ determined from the joint angles

**Typically guarantees zero error, even in the presence of manipulator and camera modeling errors**
Prototype problem

**Goal:** Drive the position \((x_1, x_2)\) of the end-effector to the reference point \((r_1, r_2)\), using visual feedback

![End-effector and reference points](image)

For simplicity:
1. Normalized pin-hole camera moves along a straight line perpendicular to its optical axis
   
   \[
   \begin{align*}
   y_1 &= \frac{x_1 - x_3}{x_2} \\
   y_2 &= \frac{r_1 - x_3}{r_2}
   \end{align*}
   \]

   camera measurements (robot & reference)

2. Feedback linearized kinematic model for the motions of robots and camera
   
   \[
   \begin{align*}
   \dot{x}_1 &= u_1 \\
   \dot{x}_2 &= u_2 \\
   \dot{x}_3 &= u_3
   \end{align*}
   \]

Stabilizability by output-feedback

Can we design an output-feedback controller that asymptotically stabilizes the equilibrium point

\[
\begin{align*}
   x_1 &= r_1 \\
   x_2 &= r_2 \\
   x_3 &= \alpha
\end{align*}
\]

for some \(\alpha \in \mathbb{R}\) ?

Local linearization around equilibrium yields:

\[
\begin{bmatrix}
\dot{\tilde{x}}_1 \\
\dot{\tilde{x}}_2 \\
\dot{\tilde{x}}_3
\end{bmatrix} =
\begin{bmatrix}
   u_1 \\
   u_2 \\
   u_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tilde{y}_1 \\
\tilde{y}_2
\end{bmatrix} =
\begin{bmatrix}
1 & \frac{\alpha - r_1}{r_2} & -1 \\
0 & r_2 & -1
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2 \\
\tilde{x}_3
\end{bmatrix}
\]

**Manifestation of a fundamental limitation on the class of controllers that asymptotically stabilize the system...**
Stabilizability by output-feedback

\[
\begin{align*}
\dot{x}_1 &= u_1 \\
\dot{x}_2 &= u_2 \\
\dot{x}_3 &= u_3 \\
y_1 &= \frac{x_1 - x_3}{x_2} \\
y_2 &= \frac{r_1 - x_3}{r_2} \\
\end{align*}
\tag{1}
\]

Can we design an output-feedback controller that asymptotically stabilizes the equilibrium point

\[
x_1 = r_1 \\
x_2 = r_2 \\
x_3 = \alpha
\]

for some \( \alpha \in \mathbb{R} \)?

Lemma: There exist no locally Lipschitz functions

\[
F : \mathbb{R}^n \times \mathbb{R}^2 \times [0, \infty) \rightarrow \mathbb{R}^n \\
G : \mathbb{R}^n \times \mathbb{R}^2 \times [0, \infty) \rightarrow \mathbb{R}^3
\]

for which the closed-loop system defined by (1) and

\[
\dot{z} = F(z, y_1, y_2, t) \\
\dot{u} = G(z, y_1, y_2, t)
\]

has an asymptotically stable equilibrium point with

\[
x_1 = r_1 \\
x_2 = r_2
\]

This limitation is not caused by lack of controllability/observability

Lemma 2: For every \( r_1, r_2 \in \mathbb{R} \), the system (1) is observable. Moreover, there exists a constant input which allows one to uniquely determine the initial state by observing the output over any finite interval \( (t_0, t_0 + T) \). Indeed, for

\[
u_1 = u_2 = 0 \quad \Rightarrow \quad \text{robot stopped}
\]

\[
u_3 = 1 \quad \Rightarrow \quad \text{camera moving at constant speed}
\]

there exists a state reconstruction function \( h_T \) for which

\[
\begin{bmatrix}
    x_1(t_0 + T) \\
    x_2(t_0 + T) \\
    x_3(t_0 + T)
\end{bmatrix} =
\begin{bmatrix}
x_1(t_0) \\
x_2(t_0) \\
x_3(t_0)
\end{bmatrix} +
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}
\int_{t_0}^{t_0 + T} dt
\]

where \( h_T \) is the measured outputs

relative position with respect to reference
Limitation on asymptotic stabilization

**Goal:** Drive the position \((x_1, x_2)\) of the end-effector to the reference point \((r_1, r_2)\), using visual feedback

To disambiguate between cases I and II, the camera must move away from \((\alpha, 0)\).

This would violate the stability of the equilibrium point in case I.

---

Limitation on asymptotic stabilization

**Goal:** Drive the position \((x_1, x_2)\) of the end-effector to the reference point \((r_1, r_2)\), using visual feedback

There is no need to stabilize the camera and the state of the controller to a fixed equilibrium. It is sufficient to make the set

\[ \mathcal{G} := \{ (r_1, r_2, \alpha, z') \in \mathbb{R}^{n+3} : \alpha \in \mathbb{R}, z \in \mathbb{R}^n \} \]

asymptotically stable.

The set \(\mathcal{G}\) is asymptotically stable if \(\forall \varepsilon > 0, \exists \delta > 0\) such that \(\forall t_0 \geq 0\)

\[ d(x_0, \mathcal{G}) < \delta \implies d(x(t), \mathcal{G}) < \varepsilon \]

and \(d(x(t), \mathcal{G}) \to 0\) as \(t \to \infty\)

solution to closed-loop

(required to exist globally)
Hybrid controller

look mode:
\[ u_1 = u_2 = 0, \quad u_3 = 1 \]
\[ \dot{z} = [y_1 \quad t y_1 \quad y_2 \quad t y_2] \]
\[ \dot{\tau} = 1 \quad \text{timer} \]

move mode:
\[ \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}' = \frac{\mu h T_{\text{look}}(z)}{T_{\text{move}}} \]
\[ \dot{z} = 0 \quad \dot{\tau} = 1 \]

- robot stopped, camera moving
- collect information for reconstruction
- move towards reference at constant speed \((\mu \in (0,1])\)

Relies on the camera motion to estimate the robot’s position w.r.t. the target.

Theorem:
For any \(T_{\text{look}}, T_{\text{move}} > 0, \mu \in (0,1]\), the hybrid controller renders the set
\[ G := \{ [r_1 \quad r_2 \quad \alpha \quad z]' \in \mathbb{R}^{n+3} : \alpha \in \mathbb{R}, z \in \mathbb{R}^n \} \]
asymptotically stable for the closed-loop system.
For \(\mu = 1\), the the set \(G\) is reached in finite time.
Stateflow simulation

\[(r_1, r_2) = (0, 1)\]

\[(x_1, x_2) = (0, 1)\]

\[(x_3, 0)\]

Next lecture…

Applications: Supervisory control