Noncooperative Game Theory — ECE594D: Homework #1

Exercise 1 (Pure security levels/policies)**.** The following matrix defines a zero-sum matrix game for which both players have 4 actions:

$$A = \underbrace{\begin{bmatrix} -2 & 1 & -1 & 1 \\ 2 & 3 & -1 & 2 \\ 1 & 2 & 3 & 4 \\ -1 & 1 & 0 & 1 \end{bmatrix}}_{\mathsf{P}_2 \text{ choices}} \left.\vphantom{\begin{bmatrix} 1\\1\\1\\1 \end{bmatrix}}\right\} \mathsf{P}_1 \text{ choices}.$$

Compute the security levels, all security policies for both players, and all pure saddle-point equilibria (if they exist). □

Exercise 2 (Mixed security levels/policies – Graphical method)**.** For each of the following two zero-sum matrix games compute the average security levels and all mixed security policies for both players.

$$A = \underbrace{\begin{bmatrix} 1 & 4 \\ 3 & -1 \end{bmatrix}}_{\mathsf{P}_2 \text{ choices}} \Big\} \mathsf{P}_1 \text{ choices}, \qquad B = \underbrace{\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}}_{\mathsf{P}_2 \text{ choices}} \left.\vphantom{\begin{bmatrix} 1\\1\\1 \end{bmatrix}}\right\} \mathsf{P}_1 \text{ choices},$$

$$C = \underbrace{\begin{bmatrix} 1 & 3 & -1 & 2 \\ -3 & -2 & 2 & 1 \\ 0 & 2 & -2 & 1 \end{bmatrix}}_{\mathsf{P}_2 \text{ choices}} \left.\vphantom{\begin{bmatrix} 1\\1\\1 \end{bmatrix}}\right\} \mathsf{P}_1 \text{ choices}, \qquad D = \underbrace{\begin{bmatrix} 2 & 1 & 0 & -1 \\ -1 & 3 & 1 & 4 \end{bmatrix}}_{\mathsf{P}_2 \text{ choices}} \Big\} \mathsf{P}_1 \text{ choices},$$

Use the graphical method.

Hint: for 3×2 *and* 2×3 *games start by computing the average security policy for the player with only two actions.* □

Exercise 3 (Mixed security levels/policies – LP method)**.** For each of the following two zero-sum matrix games compute the average security levels and a mixed security policiy

$$A = \underbrace{\begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}}_{\mathsf{P}_2 \text{ choices}} \left.\vphantom{\begin{bmatrix} 1\\1\\1 \end{bmatrix}}\right\} \mathsf{P}_1 \text{ choices}, \qquad B = \underbrace{\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}}_{\mathsf{P}_2 \text{ choices}} \left.\vphantom{\begin{bmatrix} 1\\1\\1\\1 \end{bmatrix}}\right\} \mathsf{P}_1 \text{ choices},$$

Solve this problem numerically using Matlab. □

Exercise 4 (Symmetric games)**.** A games defined by a matrix A is called *symmetric* if A is skew symmetric, i.e., if $A' = -A$. For such games show the following:

1. $V_m(A) = 0$,
2. if y^* is a mixed security policy for P_1, then y^* is also a security policy for P_2 and vice-versa,
3. if (y^*, z^*) is a mixed saddle-point equilibrium then (z^*, y^*) is also a mixed saddle-point equilibrium.

Hint: Make use of the facts that

$$\max_x f(x) = -\min_x \big(-f(x)\big), \qquad \min_w \max_x f(x) = -\max_w \min_x \big(-f(x)\big)$$

Note that the Rock-Paper-Scissors game is symmetric. □