Noncooperative Game Theory — ECE594D: Homework #2

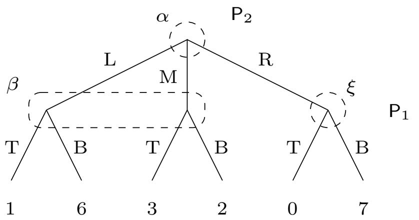

Figure 1. Game in extensive form for Exercise 5. The player P_1 is the minimizer and P_2 the maximizer.

Exercise 5. Consider the game in extensive form in Figure 1. Find mixed saddle-point equilibria for this game using the graphical method and policy domination. □

Exercise 6. Compute pure or behavioral saddle-point equilibria for the games in extensive form depicted in Figure 2. □

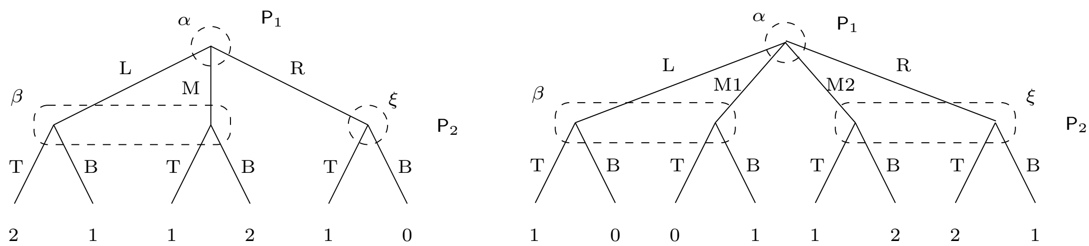

Figure 2. Single-stage games in extensive form for Exercise 6. For both games P_1 is the minimizer and P_2 the maximizer.

Exercise 7. Consider the games in extensive form depicted in Figure 2 and behavioral policies of the form

$$\mathsf{P}_1: \begin{array}{c|cccc} IS & 1 & 2 & \cdots & m \\ \hline \alpha & y_1^\alpha & y_2^\alpha & \cdots & y_m^\alpha \end{array} \qquad y_i^\alpha \geq 0,\ \forall i, \quad \sum_i y_i^\alpha = 1.$$

$$\mathsf{P}_2: \begin{array}{c|cc} IS & 1 & 2 \\ \hline \beta & z_1^\beta & z_2^\beta \\ \xi & z_1^\xi & z_2^\xi \end{array} \qquad \begin{array}{l} z_1^\beta, z_2^\beta \geq 0, \quad z_1^\beta + z_2^\beta = 1 \\ z_1^\xi, z_2^\xi \geq 0, \quad z_1^\xi + z_2^\xi = 1 \end{array}$$

1. Show that the expected cost in both games can be written in the form

$$J(y^\alpha, z^\beta, z^\xi) := y^{\alpha\prime} A^\beta z^\beta + y^{\alpha\prime} A^\xi z^\xi, \tag{1}$$

   where

$$y^\alpha := \begin{bmatrix} y_1^\alpha & y_2^\alpha & \cdots & y_m^\alpha \end{bmatrix}', \qquad z^\beta := \begin{bmatrix} z_1^\beta & z_2^\beta \end{bmatrix}', \qquad z^\xi := \begin{bmatrix} z_1^\xi & z_2^\xi \end{bmatrix}'.$$

2. Formulate the computation of the security policy for P_2 as a linear program. The matrices A^{β} and A^{ξ} in (1) should appear in this linear program.

3. Formulate the computation of the security policy for P_1 as two linear programs.

4. Use the two linear programs above to find numerically using Matlab behavioral saddle-point equilibria for the games in Figure 2.

Exercise 8 (Strategic equivalence for bimatrix games)**.** Consider two bimatrix games: the game G is defined by the $m \times n$ matrices $A := [a_{ij}]$, $B := [b_{ij}]$ and the game H is defined by the matrices $\bar{A} := [\bar{a}_{ij}]$, $\bar{B} := [\bar{b}_{ij}]$ with

$$\bar{a}_{ij} = \alpha(a_{ij}), \qquad \bar{b}_{ij} = \beta(b_{ij}), \qquad \forall i \in \{1, 2, \dots, m\},\ j \in \{1, 2, \dots, n\}. \tag{2}$$

for some monotone strictly increasing scalar functions $\alpha : \mathbb{R} \to \mathbb{R}$ and $\beta : \mathbb{R} \to \mathbb{R}$.

1. Show that G and H are strategically equivalent in pure policies.

2. Show that if the functions α and β are affine, then G and H are also strategically equivalent in mixed policies. □

Notation 1. A function $f : \mathbb{R} \to \mathbb{R}$ is called *affine* if the function $\bar{f}(s) := f(s) - f(0)$ is linear, i.e., if $\bar{f}(\sum_i c_i x_i) = \sum_i c_i \bar{f}(x_i)$ or equivalently if $f(\sum_i c_i x_i) - f(0) = \sum_i c_i \big(f(x_i) - f(0)\big)$.