Please explain all you answers.

1. For which values of $a$, $b$, and $c$ does the transfer function
\[
\frac{(s-1)(s-2)}{s^3 + as^2 + bs + c}
\]
has a two-dimensional realization that is both controllable and observable. Explain.

2. Consider the $n$-dimensional system
\[
\dot{x} = Ax, \quad y = Cx
\]
and suppose that $A$ has an eigenvector $v$ for which $Cv = 0$.

(a) Compute $Ov$, where $O$ denotes the observability matrix of the system.
(b) Is the system observable? Explain.

3. Find a state-space realization for the following transfer matrix:
\[
\hat{G}(s) = \begin{bmatrix}
\frac{s}{s+1} \\
\frac{1}{s+2}
\end{bmatrix}
\]

4. Consider the system
\[
\dot{x} = Ax + bu
\]
with
\[
A := \begin{bmatrix}
0 & 1 \\
-1 & -2
\end{bmatrix}, \quad b := \begin{bmatrix}
0 \\
1
\end{bmatrix}.
\]

(a) Is this system controllable? Explain.
(b) Compute a $1 \times 2$ matrix $f$ so that the eigenvalues of $A + bf$ are both at zero.
(c) For the matrix $f$ computed before, is the closed-loop system
\[
\dot{x} = (A + bf)x
\]
stable?

5. Consider the system
\[
\begin{align*}
\dot{x} &= \begin{bmatrix}
-2 & -1 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix} x + \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} u, \\
y &= \begin{bmatrix}
0 & 1 & 1
\end{bmatrix} x.
\end{align*}
\]
(a) Compute the system’s transfer matrix $T(s)$.
(b) Is (1) a minimal realization of $T(s)$? If not compute a minimal realization.

6. Give examples of a Jordan block $J$ with the following properties. Choose a $2 \times 2$ block whenever possible.

(a) The system $\dot{x} = Jx$ is asymptotically stable.
(b) The system $\dot{x} = Jx$ is unstable.
(c) The system $\dot{x} = Jx$ is stable but not asymptotically stable.