1. Consider the nonlinear systems
\[ \ddot{y} + \dot{y} + y = u^2 - 1. \]

(a) Compute a state-space representation for the system with input \( u \) and output \( y \).

(b) Linearize the system around the solution \( y(t) = 0, u(t) = 1, \forall t \geq 0 \).

2. Consider the system
\[ \dot{x} = (A - bk)x + bu, \]
\[ y = cx, \]

where
\[
A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad k = [k_1 \ k_2], \quad c = [0 \ 1]
\]

where \( k_1 \) and \( k_2 \) are constant scalars.

(a) Compute the system’s transfer function.

(b) Determine values for \( k_1 \) and \( k_2 \) such that the transfer function is equal to
\[
\frac{s}{s^2 + s + 1}
\]

Hint: Your answer to (a) should appear as a function of the constants \( k_1 \) and \( k_2 \).

3. Consider the matrix
\[
A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}
\]

(a) Compute the characteristic polynomial of \( A \). Is \( A \) diagonalizable?

(b) Compute \( e^{At} \).

(c) Compute a value for the initial state \( x(0) := [x_1(0) \ x_2(0) \ x_3(0)]' \) such that the solution to
\[ \dot{x} = Ax, \]
\[ y = [1 \ 1 \ 1] x \]
is equal to
\[ y(t) = te^{-2t}, \quad \forall t \geq 0. \]
Hint: To solve (c), start by writing \( y(t) \) as a function of \( x_1(0), x_2(0), x_3(0) \) and then determine values for these constants to get the desired output.

4. Compute a matrix \( A(t) \) such that

\[
\Phi(t, t_0) = \begin{bmatrix}
1 & e^{t \cdot 3^2} - 1 \\
0 & e^{t \cdot 3^2 - 2^2}
\end{bmatrix}
\]

is the state transition matrix of the homogeneous ODE

\[
\dot{x} = A(t)x
\]