Optimal Communication Logics in Networked Control Systems

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Typo in proceedings paper { eq. (22) and eq. above (11) }
Please download corrected version from http://www.ece.ucsb.edu/~hespanha

Distributed Control

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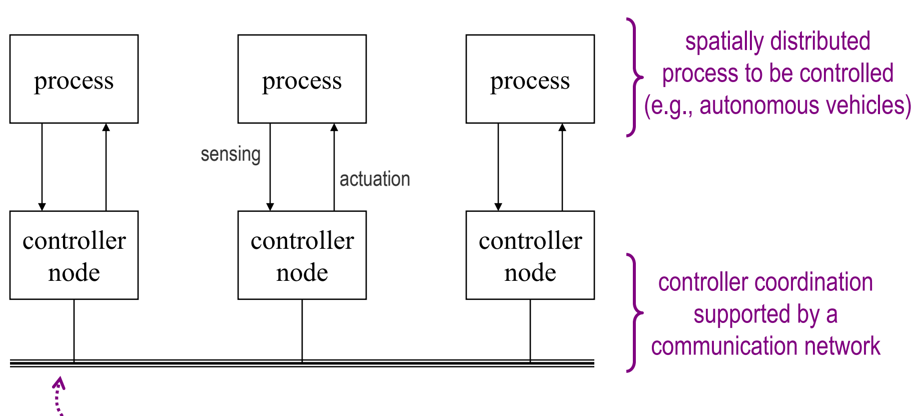
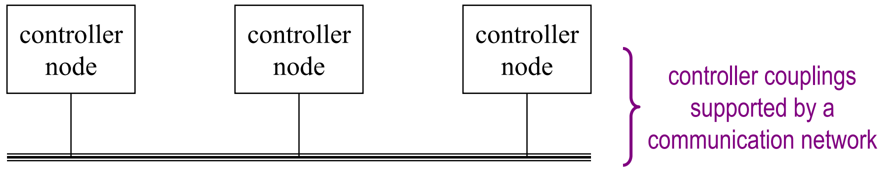


Goal:
- minimize controller communication (stealth, bandwidth)
- study the effect of non-ideal communication (delays, drops, blackouts)

Communication minimization



The "every bit-counts" paradigm…

Goal: Design each controller to minimize the number of *bits/second* that need to be exchanged between nodes (quantization, compression, …)

Domain: Media with little capacity and low-overhead protocols (bit at-a-time)
E.g., underwater acoustic comm. between a small number of nodes.

The "cost-per-message" paradigm…

paper focus

Goal: Design each controller to minimize the number of *message exchanges* between nodes (scheduling, estimation, …)

Domain: Media shared by a large number of nodes with nontrivial media access control (MAC) protocol (packet at-a-time)
E.g., 802.11 wireless comm. between a large number of nodes.

Scenarios

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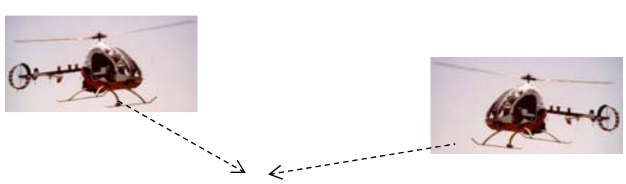
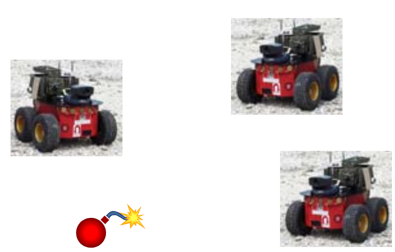
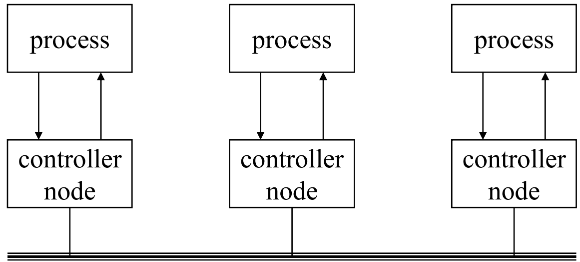


Rendezvous in minimum-time
or using minimum-energy
(in spite of disturbances)



Group of autonomous agents
cooperate in searching for a target
(perhaps mobile—search & pursuit)

Prototype problem



In this talk:

★ *decoupled* linear processes (with stochastic disturbance d)

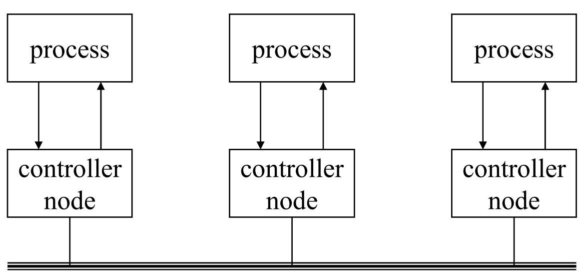
$$x^+ = \begin{bmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{bmatrix} x + \begin{bmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{bmatrix} u + d$$

★ *coupled* control objective

$$\sum_{k=0}^{\infty} x' \begin{bmatrix} \star & * & 0 \\ * & \star & * \\ 0 & * & \star \end{bmatrix} x + \|u\|^2$$

notation: $x^+ \equiv x(k+1)$

Prototype problem



In this talk:

★ *decoupled* linear processes (with stochastic disturbance d)

$$x^+ = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} x + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} u + d$$

★ *coupled* control objective

$$\sum_{k=0}^{\infty} \underbrace{x' \begin{bmatrix} C_1'C_1 & -C_1'C_2 \\ -C_2'C_1 & C_2'C_2 \end{bmatrix} x}_{\|C_1x_1 - C_2x_2\|^2} + \|u\|^2$$

E.g., rendez-vous of two vehicles

notation: $x^+ \equiv x(k+1)$

Prototype problem

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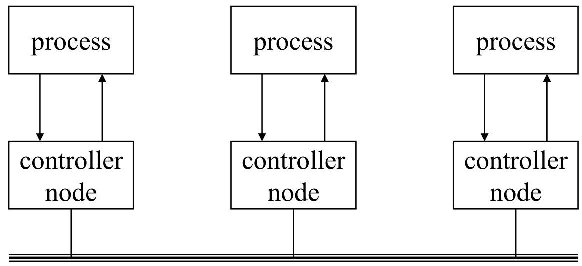

In this talk:

★ *decoupled* linear processes
(with stochastic disturbance *d*)

$$x^+ = \begin{bmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{bmatrix} x + \begin{bmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{bmatrix} u + d$$

Minimum-cost solution (centralized)

$$u = \begin{bmatrix} \star & * & * \\ * & \star & * \\ * & * & \star \end{bmatrix} x$$

Completely decentralized solution

$$u = \begin{bmatrix} \star & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & \star \end{bmatrix} x$$

Communication performance trade-off

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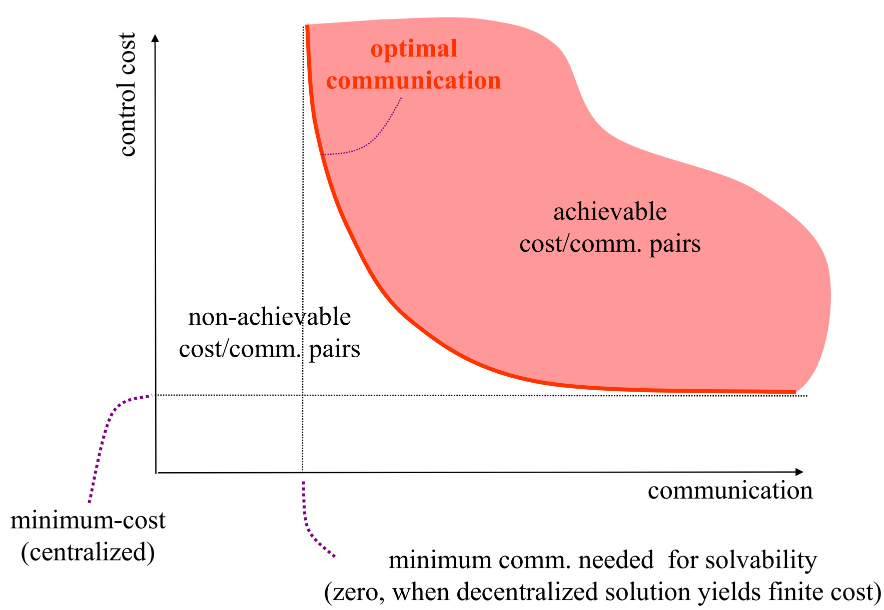
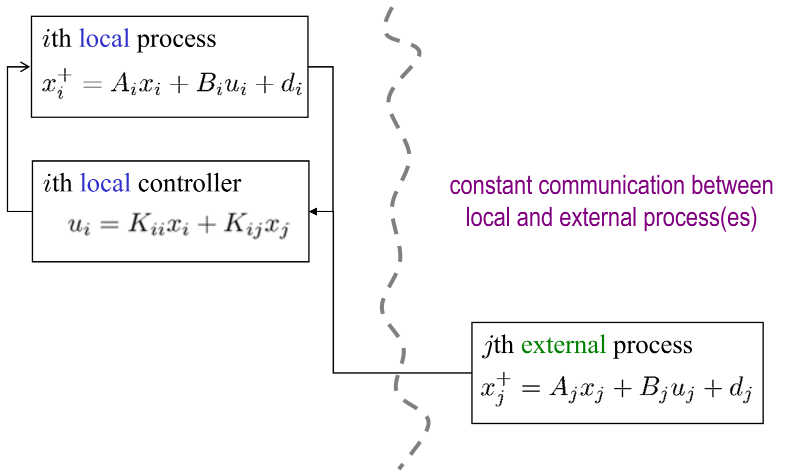


Centralized architecture



Closed-loop system

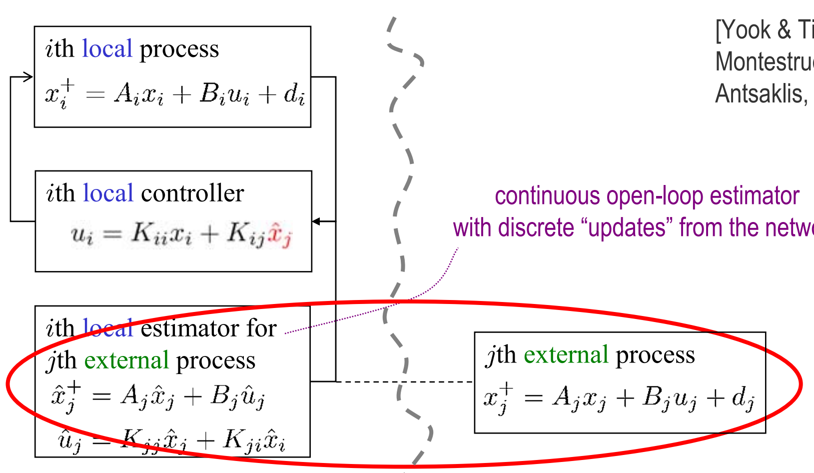
$$x_i^+ = (A_i + B_i K_{ii}) x_i + B_i K_{ij} x_j + d_i$$

$$x_j^+ = (A_j + B_j K_{jj}) x_j + B_j K_{ji} x_i + d_j$$

for simplicity here we assume only *two* processes

Estimator-based distributed architecture

[Yook & Tilbury, Montestruque & Antsaklis, Xu & JH]



Closed-loop system

$$x_i^+ = (A_i + B_i K_{ii}) x_i + B_i K_{ij} x_j + d_i - B_i K_{ij} e_j \qquad e_i := x_i - \hat{x}_i$$

$$x_j^+ = (A_j + B_j K_{jj}) x_j + B_j K_{ji} x_i + d_j - B_j K_{ji} e_i \qquad e_j := x_j - \hat{x}_j$$

additive perturbation w.r.t centralized equations

Communication logic

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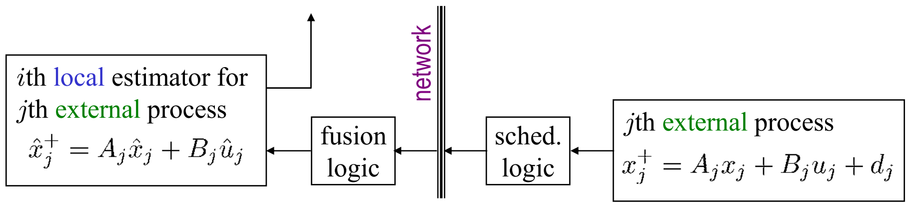

How to fuse data? *When to send data?*

With no noise & delay (for now…)

$x_j(k)$ received from network at time k

$$\Downarrow$$

$$\hat{x}_j(k+1) = A_j x_j(k) + B_j u_j(k)$$

"best-estimate" based on data received

$$\Downarrow$$

$$e_j(k+1) = -d_j(k)$$

Scheduling logic *action*

$$a_j(k) = \begin{cases} 1 & x_j(k) \text{ sent at time } k \\ 0 & \text{no transmission at } k \end{cases}$$

Options…
- periodically
  $a_j(k) = \{k \text{ divisible by } T?\}, \quad T \in \mathbb{N}$
- feedback policy
  $a_j(k) = F(x_j(k), \ldots)$
- *"optimal"* …

Optimal Scheduling Logic

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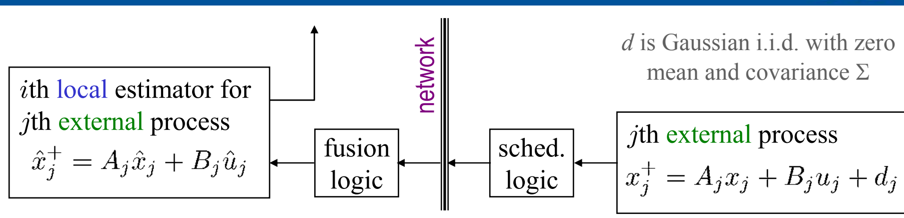

d is Gaussian i.i.d. with zero mean and covariance Σ

Goals:
minimize the estimation error $\Rightarrow$ minimize cost-penalty w.r.t. centralized
minimize the number of transitions $\Rightarrow$ minimize communication bandwidth

$$\limsup_{T\to\infty} \mathrm{E}\left[\frac{1}{T}\sum_{k=0}^{T-1} e_j(k)' Q e_j(k)\right] \qquad \limsup_{T\to\infty} \mathrm{E}\left[\frac{1}{T}\sum_{k=0}^{T-1} a_j(k)\right]$$

average L-2 norm   average transmission rate

$$\min_{a(k)} \limsup_{T\to\infty} \mathrm{E}\left[\frac{1}{T}\sum_{k=0}^{T-1} e_j(k)' Q e_j(k) + \lambda a_j(k)\right]$$

relative weight of two criteria
(will lead to Pareto-optimal solution)

Dynamic Programming solution

$$\min_{a(k)} \limsup_{T\to\infty} \mathrm{E}\left[\frac{1}{T}\sum_{k=0}^{T-1} e(k)'Qe(k) + \lambda\, a(k)\right]$$

$a(k)\in\{0,1\}$

undiscounted
average-cost problem

for simplicity of notation
1. dropped j/i subscripts
2. A denotes $A_j+B_jK_{jj}$
3. d is Gaussian with zero mean and covariance Σ

Theorem

$$(TV)(e) := \min_a \mathrm{E}\left[{e^+}'Qe^+ + \lambda\, a + V(e^+) \mid e\right]$$

dynamic programming
(DP) operator

1. There exists $J^* \in \mathbb{R}$ and bounded $h^* : \mathbb{R}^n \to \mathbb{R}$ such that
$$h^*(0) = 0, \qquad h^* + J = Th^*$$
2. J^* is the optimal cost and is achieved by the *(deterministic) static policy*
$$a(k) = \pi^*\big(e(k)\big), \quad \pi^*(e) := \begin{cases} 1 & \mathrm{E}[h^*(Ae+d)] + e'A'QAe \geq \mathrm{E}[h^*(d)] + \lambda \\ 0 & \text{otherwise} \end{cases}$$
3. h can be found by *value iteration*
$$h_{i+1} = Th_i - (Th_i)(0) \xrightarrow[i\to\infty]{exp.} h^*$$

Dynamic Programming solution

Proof outline:
1. *$e(k)$ is Markov and its transition distribution satisfies an Ergodic property* (requires a mild restriction on the set of admissible policies omitted here)
2. T is a span-contraction [Hernandez-Lerma 96]
3. Result follows using standard arguments based on Banach's Fixed-Point Theorem for semi-norms.

Theorem

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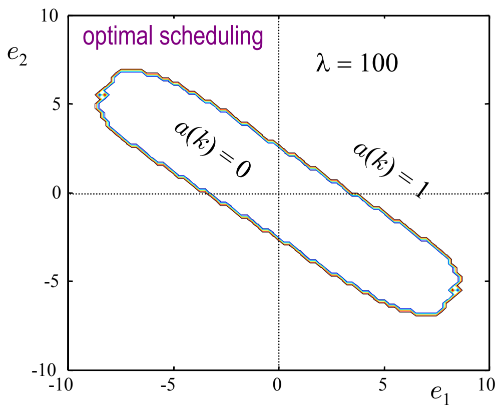
Example (2-dim)

$$\limsup_{T\to\infty} \mathrm{E}\left[\frac{1}{T}\sum_{k=0}^{T-1} e(k)' \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} e(k) + \lambda\, a(k)\right]$$

local process

$$A := A_j + B_j K_j = \begin{bmatrix} 1 & 1 \\ .1 & .9 \end{bmatrix}$$

$$\mathrm{E}\left[d(k)d(k)'\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



not ellipses!

Example (2-dim)

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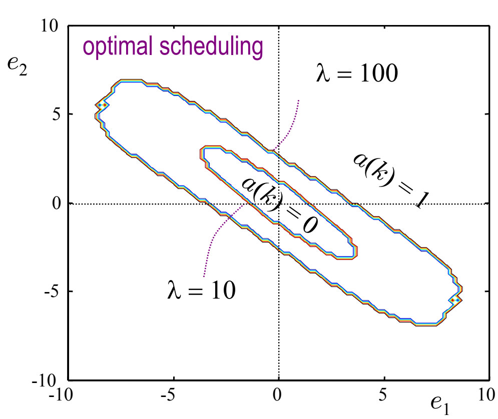


large weight in comm. cost

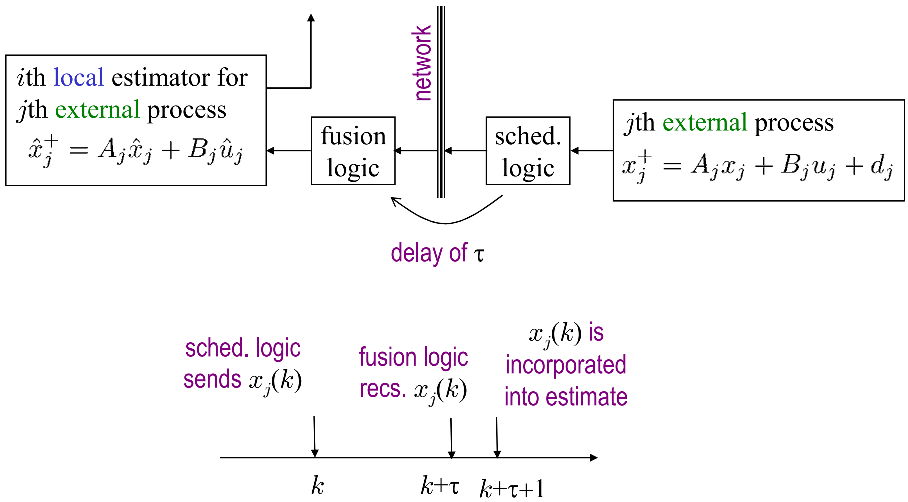
$\Downarrow$

large error threshold

$\Downarrow$

only communicate when error is very large

Communication with latency



$\tau = 0$ in previous case

Dynamic Programming solution

$$\min_{a(k)} \ \limsup_{T \to \infty} \mathrm{E}\left[\frac{1}{T}\sum_{k=0}^{T-1} e(k)'Qe(k) + \lambda\, a(k)\right] \qquad a(k) \in \{0,1\}$$

undiscounted average-cost problem

Theorem

$$(TV)(e) := \min_a \mathrm{E}\left[(1-a)e'A'^{\tau+1}QA^{\tau+1}e + \lambda a + \rho^2 + V((1-a)Ae + d) \,\middle|\, e\right]$$

$$\rho^2 := \mathrm{E}\left[\sum_{i=0}^{\tau} d'(t+i)A'^{\tau-i}QA^{\tau-i}d(t+i)\right]$$

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3. h can be found by *value iteration*

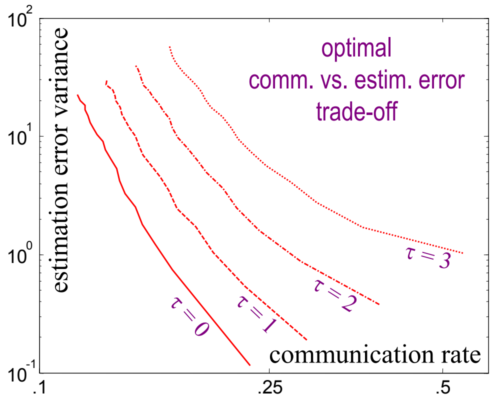
$$h_{i+1} = Th_i - (Th_i)(0) \xrightarrow[i \to \infty]{exp.} h^*$$

Example (1-dim)

$$\limsup_{T\to\infty} \mathrm{E}\left[\frac{1}{T}\sum_{k=0}^{T-1} e(k)^2 + \lambda\, a(k)\right]$$

$$A := A_j + B_j K_j = 2$$

$$\mathrm{E}[d(k)^2] = .01,\ \forall k$$



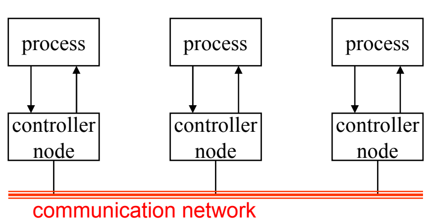
optimal scheduling

$$a(k) = \begin{cases} 1 & |\bar{e}(k)| \geq e_{\text{threshold}} \\ 0 & \text{otherwise} \end{cases}$$

estimation error, given all information enroute

with network latency
same error variance
requires more bandwidth

Conclusions



We constructed communications logics that minimize communication (measured in messages sending rate)

We considered networks with (fixed) latency

Study the effect of *packet losses*
(especially important in wireless networks)

Coupled control/communication-logic design

Nonlinear processes

Typo in proceedings paper { eq. (22) and eq. above (11) }
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