

Model-free Adaptive Switching Control of Uncertain Time-Varying Plants

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Abstract: This paper addresses the problem of controlling an uncertain time-varying plant by means of a finite family of linear candidate controllers supervised by an appropriate switching logic. It is shown that global stability of the closed-loop system can be guaranteed provided that (i) at every time there is at least one candidate controller that would stabilize the current time-invariant “frozen” plant model, and (ii) the changes in the plant model are infrequent.

Keywords: Control of uncertain plants; Adaptive control; Switching control.

1. INTRODUCTION

One of the approaches for controlling uncertain plants relies on the introduction of adaptation in the feedback loop. In recent years, *adaptive switching control* has emerged as an alternative to conventional continuous adaptation. In switching control, a so-called supervisor switches-on at any time in feedback with the plant one controller from a family of candidate controllers using recorded plant I/O data. These data are processed in such a way to enable the supervisor of deciding whether or not the currently switched-on controller is adequate, and, in the negative, replace it by another candidate controller. For an early overview of the topic, see Morse (1995).

The present paper deals with the design of adaptive switching control systems for uncertain time-varying plants. Switching control has been approached by several diversified techniques such as pre-routing, dwell-time and hysteresis switching. In this respect, hysteresis switching, compared to the other alternatives, offers definite advantages: In contrast with pre-routing schemes or alike (Zhivoglyadov et al. (2000); Angeli and Mosca (2002)), hysteresis switching allows one to predict the performance achievable by the use of each candidate controller and, hence, to select, at each time, the controller potentially yielding the best performance. Furthermore, unlike dwell-time schemes (Morse (1995); Narendra and Balakrishnan (1997)), hysteresis switching automatically chooses the waiting time between two consecutive switches, according to the real needs arising from detection of a performance loss.

As elaborated next in more detail, many hysteresis switching variants were considered for time-invariant plants, and shown capable of ensuring stability. However, no similar results are available for plants subject to time variations, and it is the objective of this paper to explore such a topic. We present an adaptive control scheme based on hysteresis switching, which, when combined with appropriate test functionals, makes it possible to ensure global stability

of the resulting closed-loop system despite possible time variations of the uncertain plant. The focus of this paper is on model-free adaptive control, as introduced by Safonov and Tsao (1997) in the *unfalsified control* framework.

The paper is organized as follows: Sect. 2 formulates the control problem; Sect. 3 recalls some relevant background on unfalsified control; The proposed adaptive control scheme is analyzed in Sect. 4, its application to time-varying plants being assessed in Sect. 5.

Notations. In the sequel, prime denotes transpose, $\|\cdot\|$ the Euclidean norm, and $\mathcal{S}(\mathbb{Z}_+)$ the space of all real-valued vector sequences on the set \mathbb{Z}_+ of nonnegative integers. For any $s \in \mathcal{S}(\mathbb{Z}_+)$, and $t_0, t \in \mathbb{Z}_+$, $t_0 \leq t$, we define $s|_{t_0}^t := \{s(t_0), \dots, s(t)\}$. If $t_0 = 0$, $s^t = s|_{t_0}^t$. Given λ , $0 < \lambda < 1$, we denote the λ -exponentially weighted ℓ_2 -norm of $s|_{t_0}^t$ by $\|s|_{t_0}^t\|_\lambda := \sqrt{\sum_{\tau=t_0}^t \lambda^{2(t-\tau)} |s(\tau)|^2}$ whenever $t \geq t_0$ and the zero number otherwise. The ℓ_∞ -norm of $s|_{t_0}^t$ is defined as $\|s|_{t_0}^t\|_\infty := \max_{\tau \in \{t_0, \dots, t\}} \max_i |s_i(\tau)|$ where s_i denotes the i -th component of s . From now on, $s \in \mathcal{S}(\mathbb{Z}_+)$ is said to be *bounded* if it is bounded in the ℓ_∞ -sense.

2. PROBLEM SETTING

We consider the adaptive control system depicted in Fig. 1. Specifically, the plant P to be controlled consists of a discrete-time strictly causal SISO linear time-varying dynamic system, described by

$$A_t(d)y(t) = B_t(d)(u(t) + n_u(t)) + A_t(d)n_y(t), \quad (1)$$

with input u , output y , input disturbance n_u and output disturbance n_y . $A_t(d)$ and $B_t(d)$ denote time-varying polynomials in the unit backward shift operator d .

It is assumed that at any time t the time-invariant system obtained by freezing the parameters of (1) at their

values at time t belongs to a set \mathcal{P} , which then will represent the set of all possible plant configurations. Now, let P_* denote a generic member of \mathcal{P} , and $B_*(d)/A_*(d)$ its corresponding transfer function. The set \mathcal{P} will also account for both the range of parametric uncertainty and the unmodelled dynamics. Indeed, as we will demonstrate later on in this paper, no assumption on \mathcal{P} will be required other than it is a compact set, *i.e.* for any P_* the orders of the polynomials A_* and B_* are bounded by some possibly unknown positive integer n and their coefficients belong to a possibly unknown compact set. In this respect, we shall denote by $\xi_P := [u(-1) \cdots u(-n) y(-1) \cdots y(-n)]'$ the vector composed by the plant initial conditions.

A so-called supervisory unit handles the plant I/O records in order to generate the sequence σ giving rise to a switching controller C_σ . Within the framework of supervisory control discussed here, the switching controller is of the form $u(t) = C_{\sigma(t)}(r(t) - y(t))$, where r is a reference signal, while $\sigma(t)$ identifies the candidate controller connected in feedback to the plant at time t . Specifically, each candidate controller belongs to a finite family $\mathcal{C} = \{C_i, i \in \underline{N}\}$, $\underline{N} := \{1, \dots, N\}$, of one-degree-of-freedom LTI candidate controllers with transfer functions $C_i(d) := S_i(d)/R_i(d)$. Accordingly, given $\sigma(t)$, the plant input $u(t)$ is given by

$$R_{\sigma(t)}(d) u(t) = S_{\sigma(t)}(d) (r(t) - y(t)) \quad (2)$$

In the sequel it is assumed that (2) is initialized at time 0 from zero initial conditions.

2.1 Hysteresis Switching Logic

Given a finite family \mathcal{C} of candidate controllers, $\mathcal{C}_S(P_*)$ will denote the subset of \mathcal{C} composed by the candidate controllers which (internally) stabilize P_* .

Def. 1. The switched system (1)-(2) is said to be (globally) *stable* if, for all initial conditions, any bounded exogenous input (r, n_u, n_y) produces a bounded output (u, y) . The problem is said to be *feasible* if $\mathcal{C}_S(P_*) \neq \emptyset, \forall P_* \in \mathcal{P}$. *

To decide whether or not, and, in the affirmative, how to change the controller, the supervisor embodies a family $\pi := \{\pi_i, i \in \underline{N}\}$ of test functionals such that, in broad terms, $\pi_i(t)$ quantifies the performance of the loop (P/C_i) given the data up to time t . In the hysteresis switching logic considered hereafter, at each step, one computes the least index $i_*(t)$ in \underline{N} such that $\pi_{i_*(t)}(t) \leq \pi_i(t), \forall i \in \underline{N}$. Then, the switching index sequence σ is given by

$$\left. \begin{aligned} \sigma(t+1) &= l(\sigma(t), \pi(t)), \quad \sigma(0) = i_0 \in \underline{N} \\ l(i, \pi(t)) &= \begin{cases} i, & \text{if } \pi_i(t) < \pi_{i_*(t)}(t) + h \\ i_*(t), & \text{otherwise} \end{cases} \end{aligned} \right\} \quad (3)$$

where $h > 0$ is the hysteresis constant.

2.2 Stability under Persistent Switching

Several hysteresis switching strategies have been considered for time-invariant plants, and shown capable of ensuring stability despite plant uncertainties and disturbances. However, no similar results are available for plants subject to time variations.

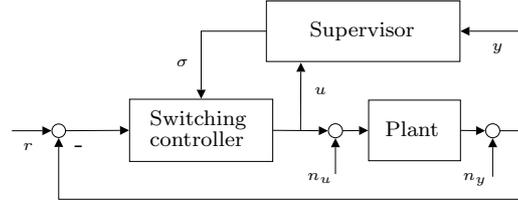


Fig. 1. Adaptive switching control scheme

To handle possible plant variations, one needs to adopt functionals with fading memory. However, the related potential risk is that the switched system become unstable due to persistent switching. To the best of the authors' knowledge, efforts to ensure stability without relying on a finite switching stopping time are limited to Hespanha and Morse (1999); Anderson et al. (2001); Hespanha et al. (2003); Vu and Liberzon (2010). However, such contributions generally provide conservative bounds on allowable unmodeled dynamics. This motivated in a certain number of papers (Pait and Kassab (2001); Angeli and Mosca (2004); Wang et al. (2007); Stefanovic and Safonov (2008); Baldi et al. (2010); Battistelli et al. (2010)) the use of a different type of supervisory scheme

$$\left. \begin{aligned} \Pi_i(t) &:= \|\pi_i^t\|_\infty \\ \sigma(t+1) &= l(\sigma(t), \Pi(t)), \quad \sigma(0) = i_0 \in \underline{N} \end{aligned} \right\} \quad (4)$$

where $l(\cdot, \cdot)$ is as in (3), and $\Pi := \{\Pi_i, i \in \underline{N}\}$. Such an approach provides a simple means for preventing the risk of instability caused by persistent switching. In fact, it forces the switching to stop, provided that at least one of the π_i 's be bounded. Unfortunately, such a strategy is invariably hampered in applications involving time-varying plants.

To overcome such limitations, we introduce a new class of algorithms based on hysteresis switching. The basic idea stems from the following observation: The supervisor based on (4) differs from the one based on (3) by the presence of a *memory* unit. This naturally leads to the idea of modifying (4) by *adaptively* selecting the memory length of $\Pi(t)$. In this respect, one simple scheme is to adopt a *resetting logic*, where resetting here denotes the mechanism according to which the supervisor resets all the Π_i 's to zero whenever suitable events (resetting conditions), to be specified next, occur. Specifically, the mentioned mechanism which will be considered throughout the paper takes the form

$$\left. \begin{aligned} \Pi_i(t) &:= \|\pi_i|_{t_k}^t\|_\infty, \quad t \in \mathbb{T}_k, \\ \mathbb{T}_k &:= \{t_k, \dots, t_{k+1} - 1\} \\ \sigma(t+1) &= l(\sigma(t), \Pi(t)), \quad \sigma(0) = i_0 \in \underline{N} \end{aligned} \right\} \quad (5)$$

where $\{t_k\}_{k \in \mathbb{Z}_+}$, $t_0 := 0$, is the sequence of resetting instants to be specified.

For clarity, from now on, we shall indicate (3) and (4) by HSL and, respectively, HSL- ∞ , while (5) will be referred to as HSL-R (Hysteresis Switching Logic with Resetting).

3. MODEL-FREE ADAPTIVE CONTROL

The resetting mechanism outlined above will be developed within the unfalsified control framework (Safonov and

Tsao (1997)), which is briefly recalled for the reader's benefit. At each time, and for each index $i \in \underline{N}$, one solves in real-time with respect to v_i the difference equation

$$S_i(d) (v_i(t) - y(t)) = R_i(d) u(t) \quad (6)$$

In practice, v_i^t equals the input sequence that reproduces the I/O sequence (u^t, y^t) of the plant P fed-back by the controller C_i . For all candidate indices, (6) is initialized at time 0 from zero initial condition.

Now, since (6) can be computed for all the indices in \underline{N} , it also makes it possible to evaluate the closed-loop performance achievable by each candidate loop (P/C_i) driven by the related v_i . In connection with the control objective specified by Def. 1, a convenient way to define such a performance is as follows

$$\pi_i(t) := \frac{\|\zeta_i^t\|_\lambda}{\mu + \|v_i^t\|_\lambda}, \quad t \in \mathbb{Z}_+ \quad (7)$$

where $\zeta_i := [u (v_i - y)]'$, and μ a positive constant. In case of noise-free LTI plant, π_i provides an estimate from below of the λ -weighted \mathcal{H}_∞ mixed-sensitivity norm of the loop (P/C_i) Datta (1993).

Remark 1. On-line computation of (6) requires that all the candidate controllers be stably causally invertible (SCI). This constraint can be removed by resorting to alternative virtual reference signals (Dehghani et al. (2007)). Then, we will not pursue this issue further in this paper. *

3.1 Assumptions

We make the following assumptions.

- A1:** The plant uncertainty set \mathcal{P} is compact. •
- A2:** For every $P_* \in \mathcal{P}$, there is at least a candidate controller $C_i \in \mathcal{C}$ such that the characteristic polynomial of the closed-loop (P_*/C_i)

$$\chi_{*/i}(d) := A_*(d) R_i(d) + B_*(d) S_i(d)$$

has no root in the closed disk of radius λ^{-1} of the complex plane. •

- A3:** Each controller $C_i \in \mathcal{C}$ is SCI, and $S_i(d)$ has no root in the closed disk of radius λ^{-1} of the complex plane. •
- A4:** The exogenous inputs r , n_u , and n_y are bounded. •

Remark 2. A2 implies feasibility, i.e. $\mathcal{C}_S(P_*) \neq \emptyset$, $\forall P_* \in \mathcal{P}$. A3 requires that, in addition to the SCI condition, the inverse of each controller has a large enough stability margin. Because \mathcal{C} is a finite set, this requirement can always be fulfilled by choosing λ close enough to one. *

3.2 Key Lemmas

In this subsection, we introduce certain key properties upon which the stability analysis depends. To this end, we first introduce some preliminary notations.

Notations: In the sequel, to simplify notations, we let Σ_σ denote the switched system (1)-(2) mapping the input $w := [r \ n_u \ n_y]'$ to the output $\zeta := [u (r - y)]'$ under an arbitrary switching sequence σ . *

Let

$$\Pi^k := \min_{i \in \underline{N}} \{\max_{t \in \mathbb{T}_k} \pi_i(t)\} + h, \quad k \in \mathbb{Z}_+ \quad (8)$$

The following results hold.

Lemma 1. Consider the HSL-R. Let \mathfrak{N}_k denote the number of switching times over the interval \mathbb{T}_k , and $\lceil \alpha \rceil$ denote the smallest integer greater than or equal to $\alpha \in \mathbb{R}_+$. Then,

$$\pi_{\sigma(t+1)}(t) < \Pi^k, \quad \forall t \in \mathbb{T}_k \quad (9)$$

$$\mathfrak{N}_k \leq N \lceil \Pi^k/h \rceil, \quad \forall k \in \mathbb{Z}_+ \quad (10)$$

hold for any resetting sequence $\{t_k\}_{k \in \mathbb{Z}_+}$. □

Lemma 2. Let the switched system Σ_σ be based on the HSL-R, with test functionals as in (7). Let A1, A3 and A4 hold. Then, there exists a bounded function $\gamma(\cdot) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ such that

$$\|\zeta^t\|_\lambda \leq \gamma(\Pi^k)(\mu + |\xi_P| \lambda^{t+1} + \|w^t\|_\lambda + \|\zeta^{t_k-1}\|_\lambda \lambda^{t-t_k+1}), \quad \forall t \in \mathbb{T}_k \quad (11)$$

holds for any resetting sequence $\{t_k\}_{k \in \mathbb{Z}_+}$. □

Before concluding this section some observations are in order. The bound in (11) depends on both the sequences $\{\Pi^k\}_{k \in \mathbb{Z}_+}$ and $\{\zeta^{t_k-1}\}_{k \in \mathbb{Z}_+}$. As will be addressed in the next sections, boundedness of $\{\Pi^k\}_{k \in \mathbb{Z}_+}$ depends on the fulfillment of A2 and on the type of time variations of the plant parameters (notice that in Lemma 2 the time variations of the plant parameters can be arbitrary; this is the reason by which, at the present stage, A2 is of no help). Boundedness of $\{\zeta^{t_k-1}\}_{k \in \mathbb{Z}_+}$ is a more intricate matter since it also depends on the particular choice for the resetting mechanism. In the sequel, we show that stability can be ensured by only imposing a simple but essential admissibility condition on resetting times.

4. ADMISSIBLE RESETTING TIMES

In this section we introduce the adaptive mechanism used to generate resetting times which do not destroy stability. To this end, let

$$\pi_*(t) := \frac{\|\zeta^t\|_\lambda}{\mu + \|r^t\|_\lambda}, \quad t \in \mathbb{Z}_+ \quad (12)$$

and consider the following definition.

Admissible Resetting: A sequence of reset times $\{t_k\}_{k \in \mathbb{Z}_+}$ is called *admissible* if, for every $k \geq 0$, we have

$$\pi_*(t_k - 1) \leq \pi_{\sigma(t_k)}(t_k - 1) + \epsilon, \quad \epsilon > 0 \quad (13)$$

In essence, (13) only allows the k -th reset to occur at the time t_k if (13) holds. Notice that by definition of the λ -exponentially weighted ℓ_2 -norm, $\pi_*(-1) = \pi_{\sigma(0)}(-1) = 0$, and, hence, $t_0 := 0$ is an admissible resetting time.

To understand the rationale for (13), we note that, when the plant is time-invariant, π_* can be viewed as an estimate of the *actual* reference-to-data induced gain, whereas π_σ can be viewed as an estimate of the *virtual* reference-to-data induced gain. To obtain stability it is important

that these two estimates do not differ significantly. The inequality (13) guarantees that π_* does not get much larger than π_σ , whereas, as shown hereafter, the selection of σ through HSL-R makes sure that π_σ remains bounded.

To see this, consider an admissible resetting sequence $\{t_k\}_{k \in \mathbb{Z}_+}$. From (9) and (12) we have that

$$\|\zeta^{t_k-1}\|_\lambda \leq (\Pi^{k-1} + \epsilon) (\mu + \|r^{t_k-1}\|_\lambda), \quad \forall k \in \mathbb{Z}_+$$

Notice that one can let $\Pi^{-1} := 0$ according to the fact the $\|\zeta^{-1}\|_\lambda = 0$. Since $\|r^{t_k-1}\|_\lambda \lambda^{t-t_k+1} \leq \|r^t\|_\lambda \leq \|w^t\|_\lambda$, one concludes that, under an admissible resetting sequence, Lemma 2 implies that

$$\|\zeta^t\|_\lambda \leq \gamma(\Pi^k) (|\xi_P| \lambda^{t+1} + (\Pi^{k-1} + \epsilon + 1)(\mu + \|w^t\|_\lambda)), \quad \forall t \in \mathbb{T}_k \quad (14)$$

It is therefore immediate to conclude the following.

Theorem 1. Consider the same assumptions as in Lemma 2 and further assume that $\Pi^k \leq \Pi^*$, $\forall k \in \mathbb{Z}_+$, for some finite constant Π^* . Then, the HSL-R switched system Σ_σ is stable for any admissible resetting sequence $\{t_k\}_{k \in \mathbb{Z}_+}$ and

$$\|\zeta^t\|_\lambda \leq \gamma(\Pi^*) |\xi_P| \lambda^{t+1} + \eta(\Pi^*) (\mu + \|w^t\|_\lambda), \quad \forall t \in \mathbb{Z}_+ \quad (15)$$

where $\eta(\Pi^*) := \gamma(\Pi^*) (\Pi^* + \epsilon + 1)$. \square

Notice that, in contrast with (11), the bound in (15) now depends only on $\{\Pi^k\}_{k \in \mathbb{Z}_+}$. The remainder of this paper is devoted to show that A2 is sufficient to ensure boundedness of $\{\Pi^k\}_{k \in \mathbb{Z}_+}$ when the plant is a time-invariant system or when the time variations of the plant parameters are infrequent.

4.1 Stability under Admissible Resetting

To begin with, we consider the following result.

Lemma 3. Let the HSL-R switched system Σ_σ be based on the test functionals (7). Let A1–A4 hold and further assume that P is time-invariant on the interval $\{\tau, \tau + 1, \dots, t\}$, $\tau \leq t$. Then, there exist positive constants g_0, g_1, g_2 and g_3 such that, for any $P_* \in \mathcal{P}$,

$$\pi_s(t) \leq g_0 + g_1 |\xi_P| \lambda^{t+1} + g_2 \|w^t\|_\lambda + g_3 \|\zeta^{\tau-1}\|_\lambda \lambda^{t-\tau+1} \quad (16)$$

holds true for some $s \in \underline{N}$. \square

From Lemma 3 one sees that when the plant is a time-invariant system ($\tau = 0$ and $t = \infty$), A2 is sufficient to ensure boundedness of $\{\Pi^k\}_{k \in \mathbb{Z}_+}$. Indeed, by letting $\kappa_0 := g_0$, $\kappa_1 := g_1$ and $\kappa_2 := (1 - \lambda^2)^{-1/2} g_2$ and recalling that $\|\zeta^{-1}\|_\lambda = 0$, one sees that (16) implies that

$$\pi_s(t) + h \leq \kappa_0 + \kappa_1 |\xi_P| + \kappa_2 \|w\|_\infty + h =: \Pi_{TI}^*$$

holds over \mathbb{Z}_+ , from which the following result follows.

Theorem 2. Let the switched system Σ_σ be based on the HSL-R, with test functionals as in (7). Then, if the plant is time-invariant, under assumptions A1–A4, Σ_σ is stable for any admissible resetting sequence $\{t_k\}_{k \in \mathbb{Z}_+}$. \square

In view of (13) and Th. 2, the following *resetting rule* ($t_0 := 0$) is introduced

$$t_{k+1} := 1 + \min \{t : t \geq t_k; \pi_*(t) \leq \pi_{\sigma(t+1)}(t) + \epsilon\}, \quad k \in \mathbb{Z}_+ \quad (17)$$

4.2 Finite-Time Resetting

In this section, we show that, under the same assumptions as in Theorem 2, the admissibility condition (13) is always attained in finite time. Hence, the HSL-R based on (17) always experiences at least one resetting. Notice that this property becomes crucial in the presence of plant variations for ensuring that the supervisor can react to changes in the plant dynamics.

Under the same assumptions as in Theorem 2, we obtain

$$\begin{aligned} \max_{t \in \mathbb{Z}_+} \|\zeta^t\|_\lambda &\leq \gamma(\Pi^*) |\xi_P| + \kappa(\Pi^*) (\mu + \|w\|_\infty) \\ &=: Z(\Pi^*, \xi_P, \mu, w) \end{aligned} \quad (18)$$

where $\kappa(\Pi^*) := (1 - \lambda^2)^{-1/2} \eta(\Pi^*)$.

Further, let

$$\Delta(\Pi^*) := (\mathfrak{N}_* + 1) \left\lceil \log_\lambda \frac{\epsilon \mu}{Z(\Pi^*, \xi_P, \mu, w)} \right\rceil \quad (19)$$

where $\mathfrak{N}_* := N \lceil \Pi^*/h \rceil$.

Lemma 4. Consider the HSL-R based on (17). Then, under the same assumptions as in Theorem 2, one has

$$t_{k+1} - t_k \leq \Delta(\Pi^*), \quad \forall k \in \mathbb{Z}_+ \quad (20)$$

\square

One sees from Lemma 4 that, if the plant is time-invariant, under A1–A4 hold, a resetting always occurs after at most $\Delta(\Pi_{TI}^*)$ time steps.

Remark 3. Lemma 4 indicates that stability does not rely on a finite switching stopping time, thus extending the stability analysis of Unfalsified switching control to logics other than HSL- ∞ . We also point out that these results encompass the stability analysis of plants subject to large modeling uncertainties, for which, to the best of the author's knowledge, global stability under persistent switching has remained so far an open problem (Anderson et al. (2001); Vu and Liberzon (2010)). $*$

5. STABILITY UNDER TIME VARIATIONS OF THE PLANT PARAMETERS

In this section, we show how HSL-R makes it possible to achieve stability properties similar to those derived for time-invariant plants in the presence of time variations

of the plant parameters. We first recall that, by virtue of Theorem 2, the switched system is stable if there exists a finite constant Π^* such that $\Pi^k \leq \Pi^*$ for every $k \in \mathbb{Z}_+$. As discussed next, this requirement can be met when the plant variations are infrequent.

5.1 Stability under Infrequent Plant Changes

Let $\{\ell_c\}_{c \in \mathbb{Z}_+}$ denote the sequence of time instants at which a plant variation occurs, with $\ell_0 := 0$ by convention. Accordingly, $\mathbb{L}_c := \{\ell_c, \dots, \ell_{c+1} - 1\}$, $c \in \mathbb{Z}_+$, will denote the c -th time interval over which the plant is constant. Although we can no longer use Π_{TI}^* in Theorem 2 to deduce stability of the closed-loop, Lemma 3 ensures that for every $c \in \mathbb{Z}_+$ there exists a candidate index $s \in \underline{N}$ such that

$$\pi_s(t) + h \leq \Pi_{TI}^* + g_3 \|\zeta^{\ell_c - 1}\|_\lambda \lambda^{t - \ell_c + 1}, \quad \forall t \in \mathbb{L}_c \quad (21)$$

where g_3 is as in (16). Thus, for any given accuracy ν and provided that \mathbb{L}_c be large enough, the right hand side of (21) will eventually enter a neighborhood of amplitude ν of Π_{TI}^* .

Let

$$\mathbb{L}_c^\nu := \{t \in \mathbb{L}_c; g_3 \|\zeta^{\ell_c - 1}\|_\lambda \lambda^{t - \ell_c + 1} \leq \nu\}$$

Then, if at least two resets occur over \mathbb{L}_c^ν , *i.e.* there is at least one k such that $\mathbb{T}_k \subseteq \mathbb{L}_c^\nu$ one can use equation (14) to conclude that at time ℓ_{c+1} the following upper bound holds

$$\|\zeta^{\ell_{c+1} - 1}\|_\lambda \leq Z(\Pi_{TI}^* + \nu, \xi_P, \mu, w) \quad (22)$$

with $Z(\cdot)$ as in (18). Notice that one single reset would not be sufficient since the bound in (14) depends on both Π^k and Π^{k-1} . Unfortunately, at the present stage, Theorem 2 cannot be invoked to conclude stability of the switched system since the existence of a finite upper bound for $\{\Pi^k\}$ is not evident from boundedness of $\{\zeta^{\ell_c - 1}\}$. Nonetheless, this property actually holds as stated below.

Lemma 5. Let the HSL-R switched system Σ_σ be based on the test functionals (7) and reset rule (17). Let A1–A4 hold. Further assume that

$$\forall c \in \mathbb{Z}_+ \quad \exists k \in \mathbb{Z}_+ \quad \text{such that} \quad \mathbb{T}_k \subseteq \mathbb{L}_c^\nu \quad (23)$$

Then, for all $k \in \mathbb{Z}_+$ one has $\Pi^k \leq \Pi_{TV}^*$ with

$$\Pi_{TV}^* := \Pi_{TI}^* + g Z(\Pi_{TI}^* + \nu, \xi_P, \mu, w) \quad (24)$$

where $g := \max\{g_3, 2\mu^{-1}\}$. Hence, Σ_σ is stable.

In the light of Lemma 5 one sees that a sufficient condition for stability of the switched system is that the minimum interval between two consecutive plant variations (or plant *dwell-time*) be large enough to allow the fulfillment of the condition (23). In this respect, let ℓ_c^ν denote the first time instant of \mathbb{L}_c^ν , *i.e.* $\ell_c^\nu := \min\{t : t \in \mathbb{L}_c^\nu\}$. Thus, condition (23) amounts to requiring that, for any $c \in \mathbb{Z}_+$, ℓ_{c+1} is always greater or equal to ℓ_c^ν plus the time needed for two resetting to occur. To this end, notice that using (22) in

the definition of \mathbb{L}_c^ν one obtains

$$\ell_c^\nu - \ell_c \leq \left\lceil \log_\lambda \frac{\nu}{g_3 Z(\Pi_{TI}^* + \nu, \xi_P, \mu, w)} \right\rceil$$

Moreover, by simple induction argument, if condition (23) is satisfied up to a certain ℓ_c then Π_{TV}^* is an upper bound on the smallest test functional over \mathbb{L}_c . In turns, in agreement with Lemma 4, this implies that after at most $2\Delta(\Pi_{TV}^*)$ steps subsequent to ℓ_c^ν the two required reset times occur. In particular, letting $\nu = \epsilon$ and using the fact that $g_3 \mu Z(\Pi_{TI}^* + \nu, \xi_P, \mu, w) < \mu \Pi_{TV}^* < Z(\Pi_{TV}^*, \xi_P, \mu, w)$ the following result can be claimed.

Theorem 3. Let the HSL-R switched system Σ_σ be based on the test functionals (7) and the resetting rule (17). Let A1–A4 hold. Then, Σ_σ is stable provided that

$$\ell_{c+1} - \ell_c \geq 3\Delta(\Pi_{TV}^*)$$

holds for any $c \in \mathbb{Z}_+$. \square

5.2 An example

Consider the continuous-time plant with transfer function

$$P(s) = e^{-0.1s} \frac{g}{s(s^2 + \omega^2)} \quad (25)$$

The plant is controlled by feeding its input via a zero-order holder and sampling its output every 0.1s. The gain g and the frequency ω are assumed to be uncertain, $g \in [0.2, 1]$, $\omega \in [0, 2]$. Four discrete-time candidate controllers are designed so as to achieve satisfactory performance over the whole uncertainty region. In particular, the controllers are as in (2), with $S_i(d) = \sum_{k=0}^2 s_{ik} d^k$, $R_i(d) = 1 + \sum_{k=1}^2 r_{ik} d^k$, the coefficients of S_i and R_i being reported in Table 1.

We compare HSL- ∞ and HSL-R, both with hysteresis $h = 0.01$ and with $\lambda := 0.995$ and $\mu = 1$ in the test functionals (7). In particular, λ has been selected large enough so as to satisfy A2 and A3 for the uncertain discrete-time plant and the candidate controllers. For HSL-R, we have adopted the reset rule (17) with $\epsilon := 0.01$.

Assume that $\omega = 0$ while g switches between 0.2 (only C_1 stabilizes the plant) and 2 (only C_2 and C_3 stabilize the plant) every 10^4 s. Figures 2 and 3 show the plant output response under: zero plant initial conditions; a sinusoidal reference of frequency 0.01 [rad/sec] and amplitude 2.5; disturbances modeled by taking n_u and n_y to be uniformly distributed on $[-0.01, 0.01]$.

As expected, due to the presence of infinite memory, the control self-reconfiguration capability progressively degrades under HSL- ∞ . As shown in Figure 3, this issue does not arise in the HSL-R because of resetting.

Table 1. Controllers coefficients

	s_{i0}	s_{i1}	s_{i2}	r_{i1}	r_{i2}
C_1	757.414	-1486.011	729.129	-0.818	0.715
C_2	83.844	-164.152	80.502	-1.078	0.340
C_3	53.874	-103.478	49.724	-1.002	0.292
C_4	28.036	-55.064	27.608	-1.314	0.470

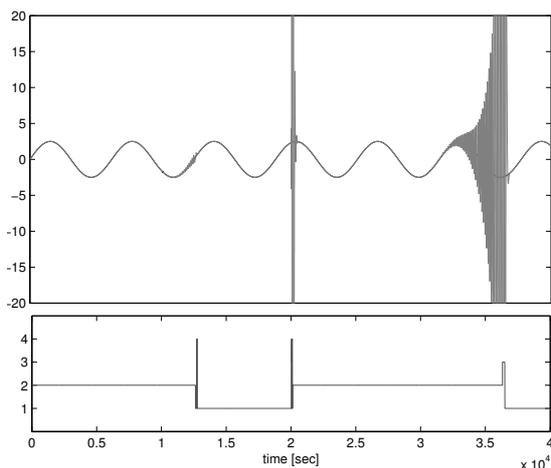


Fig. 2. Supervision based on HSL- ∞ in the presence of infrequent plant variations. Top: Plant Output; Bottom: Switching sequence.

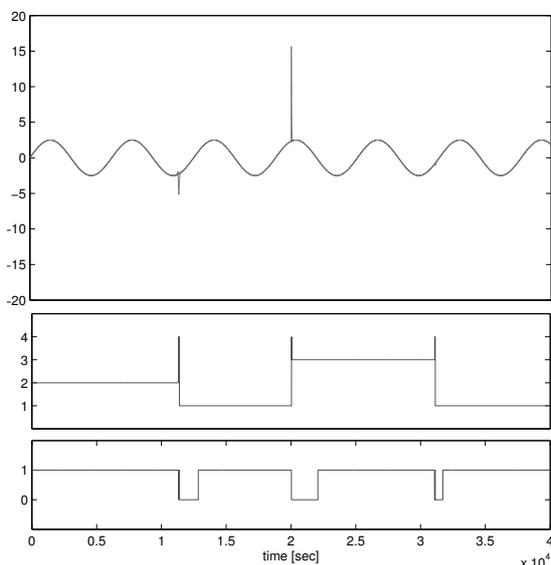


Fig. 3. Supervision based on HSL-R in the presence of infrequent plant variations. From top to bottom: Plant Output; Switching sequence; Resetting sequence [1 stands for resetting]

6. CONCLUSIONS

Consideration has been given to the control of uncertain time-varying plants by means of adaptive switching control techniques. To this end, we introduced a novel class of algorithms based on hysteresis switching, which, when combined with appropriate test functionals, make it possible to derive stability properties without relying on a finite switching stopping time. In particular, a resetting logic was devised so as to extend the stability results of Unfalsified Control, so far restricted to time-invariant plants, to the case of plants subject to time-variations.

As a final remark, we note that the scheme considered in this paper only allows one to address issues of robust stability. A further practical issue is concerned with how performance can be enhanced, by reducing both the time duration of “learning” transients as well as the chance that destabilizing controllers be switched-on. The development

of rules more sophisticated than a simple resetting logic as well as the adoption of test functionals based on multiple models may prove relevant in this regard.

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