Iterative algorithms for distributed leader-follower model predictive control

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***Abstract*— Two distributed algorithms to estimate the optimal control input sequence that solves a finite horizon quadratic optimization are proposed. The first algorithm utilizes information from 2-hop neighbors, whereas the second only considers 1-hop neighbors. The estimates obtained from both algorithms converge asymptotically, under appropriate assumptions, for any initialization of the algorithm. For the 2-hop algorithm, we show that the converged estimate is the optimal solution to the original optimization problem, while for the 1-hop algorithm the result is generally a suboptimal solution. We evaluate the methods with simulations for a leader-follower model predictive control problem with unstable linear agents dynamics.**

I. INTRODUCTION

Many engineering applications can benefit when single agent tasks are replaced by collective actions of coordinated agents, improving scalability, robustness, and avoiding a single-point of failure [2]. Examples of such applications include sensing and localization [22], search and rescue missions [13], astronomical observations [14], synthetic biology design [6], and surveillance [4]. However, controlling multi-agent systems in a distributed fashion brings new practical and theoretical challenges.

Because of the inherent interaction between agents in these types of problems, consensus based algorithms have shown to be a suitable tool for analyzing and designing distributed algorithms for multi-agent systems. Early works building on this approach such as [21], [24], and [12] introduced many important graph theoretical tools, but have mainly considered simple system dynamics and optimization criteria.

An optimization framework is an attractive way to explicitly address many practical considerations, such as noise models and actuators limits that exist in real world applications. Additionally, it offers desirable properties such as reduced sensitivity and robustness. Previous relevant works on distributed optimization for multi-agent systems consider problems where the agents are collectively minimizing an overall cost that is the sum of local convex functions [19], [1]. For problems with constraints, a distributed primal-dual subgradient method was proposed in [25] for connected and time varying graph topology. In [20] a similar problem setup was studied using a consensus-based subgradient method based on averaging. Local LQR-based algorithms for multi-agent systems that achieve consensus and are globally optimal for multi-agent systems were introduced in [17].

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Model predictive control provides a reliable framework for a range of applications, utilizing a dynamical model of the system and optimizing over a forecast window. An overview of distributed model predictive control strategies for various types of engineering problems can be found in [3]. For multi-agent coordination, most of the results have considered fully connected networks [9], [18], [15], which can be overly restrictive in some applications. For a vehicle formation control problem, [7] established stabilizing results, but no guarantees that the local performances match the performance resulting from a centralized implementation are given. More recently, for general linear systems with input constraints, [16] proposes algorithms that achieve consensus, highlighting the interplay between the graph structure and the system dynamics. A distributed output-feedback model predictive control that combines simultaneous state estimation with control computation has been studied in [5] providing practical consensus results. While most of the results guarantee synchronization or the stability of a predetermined agent formation, the analysis of the designed distributed algorithm performance in comparison to a centralized implementation has been overlooked.

We consider a multi-agent coordination problem where the objective is to design control inputs for a collection of agents to follow a single leader. The systems have identical dynamics, possibly unstable, and only a fraction of them communicate with the leader. Such agents can represent devices with identical hardware that are collectively involved in achieving a coordinated task. In our work, we formulate the problem as a finite horizon optimization of the type used in model predictive control [23], where the cost is given by the sum of local cost functions.

The main contributions of this paper are two iterative algorithms, in which each agent is assigned a cost function that it optimizes to compute its own control action. These algorithms have different cost functions that depend on the estimates of the control variables of the agents neighbors, which are obtained through inter-agent communication. In the first algorithm agents are required to get these estimates from 2-hop neighbors, whereas the second algorithm requires less communication, needing only estimates from 1-hop neighbors.

For the first algorithm, we show that under appropriate assumptions the local solutions converge to the optimal solution of the original optimization problem, regardless of how the algorithm is initialized. For the second algorithm the estimates generally converge to suboptimal solutions. Nevertheless, simulations show that the optimality gap is small and in some cases zero. The proposed methods are evaluated and compared in simulations for a leader-follower coordination problem using model predictive control.

This work is related to [8], where the authors presented a distributed approach for a maximum likelihood estimation problem. We introduce an alternative approach to the standard consensus tools described in [21], [12] that can be used to solve problems related to multi-agent coordination. By exploiting the inter-agent communication, we guarantee that a user defined performance criterion is satisfied. Additionally, by utilizing the optimization and model predictive control frameworks we benefit from the robustness and reduced sensitivity properties to unmodeled dynamics and noise.

Structure: This paper is organized as follows. In Section II we introduce the problem of leader-follower and set up the optimization problem. Section III presents a distributed algorithm that requires 2-hop neighbor information, and Section IV another algorithm that utilizes 1-hop neighbor information. Analysis of convergence results are presented for both algorithms. The algorithms are illustrated with a model predictive control problem in Section V, and concluding remarks are given in Section VI.

Notation: Given a vector $w_t^i \in \mathbb{R}^n$ representing a local variable at agent $i \in \mathcal{N} = \{1, 2, \ldots, N\}$ at time t, we represent the corresponding time sequence of length T as $w^i_{t:t+T-1} \coloneqq \begin{bmatrix} w_t^{i\top} & w_{t+1}^{i}{}^\top & \cdots & w_{t+T-1}^{i}{}^\top \end{bmatrix}^\top \in \mathbb{R}^{Tn}$, and the overall multi-agent time sequence as $w_{t:t+T-1} \coloneqq \begin{bmatrix} w^1_{t:t+T-1}{}^\top & w^2_{t:t+T-1}{}^\top & \cdots & w^N_{t:t+T-1}{}^\top \end{bmatrix}^\top \in \mathbb{R}^{NTn}$. The $n \times n$ identity matrix is denoted by I_n, the zero $n \times n$ matrix by 0_n (the subscript will be dropped when the dimension is clear from context). For a symmetric matrix $A \in \mathbb{R}^{n \times n}$, we denote by $\underline{\lambda}(A)$ and $\bar{\lambda}(A)$ its minimum and maximum eigenvalues, respectively. We write $A > 0$ ($A \geqslant 0$) if A is positive (semi) definite. The Kronecker product of two matrices is denoted by $\otimes$.

II. Problem Formulation

Consider the following linear discrete time multi-agent dynamical systems

$$x^0_{t+1} = Ax^0_t \quad x^i_{t+1} = Ax^i_t + Bu^i_t \quad \forall i \in \mathcal{N}, \quad t \geqslant 0, \quad (1)$$

where $x^0_t \in \mathbb{R}^{n_x}$ is the state of the leader agent at time t, $x^i_t \in \mathbb{R}^{n_x}$ is the state of the ith follower agent and $u^i_t \in \mathbb{R}^{n_u}$ its control input. All agents have identical dynamics defined by the matrices $A \in \mathbb{R}^{n_x \times n_x}$ and $B \in \mathbb{R}^{n_x \times n_u}$. The follower agents are able to exchange their state and control input via pair-wise communication restricted by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the set of agents and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of edges.

We are interested in designing control inputs u^i_t that use only local information to achieve synchronization with the leader, i.e., $\lim_{t \to \infty} \|x^i_t - x^0_t\| = 0, \ \forall i \in \mathcal{N}$.

To study this synchronization problem, we start by defining the neighborhood tracking error for each agent that we eventually want to drive to zero as

$$\varepsilon^i_t \coloneqq \sum_{j \in \mathcal{N}_i} (x^j_t - x^i_t) + g^i(x^0_t - x^i_t),$$

where $g^i = 1$ if agent i has access to the leader's state, and $g^i = 0$ otherwise. The dynamics of this error can be written as,

$$\varepsilon^i_{t+1} = A\varepsilon^i_t - (g^i + d^i)Bu^i_t + B \sum_{j \in \mathcal{N}_i} u^j_t, \quad (2)$$

where $\mathcal{N}_i \subset \mathcal{N}$ is the set of neighbors of agent i and $d^i \coloneqq |\mathcal{N}_i|$ is the degree of agent i.

Before we present a few strategies for the synchronization problem we state the following assumption and definition that will be used in our results.

Assumption 1: The graph $\mathcal{G}$ is connected and at least one follower agent is connected to the leader, i.e., there exists $i \in \mathcal{N}$ such that $g^i = 1$. □

Definition 1: For a leader-follower problem we define the Augmented Laplacian matrix $\bar{L} \in \mathbb{R}^{N \times N}$ of the graph $\mathcal{G}$ as

$$\bar{L} \coloneqq G + D - A_d,$$

where $G \coloneqq \operatorname{diag}(g^1, g^2, \ldots, g^N) \in \mathbb{R}^{N \times N}$ is the matrix that represents the connection to the leader, $D \coloneqq \operatorname{diag}(d^1, d^2, \ldots, d^N) \in \mathbb{R}^{N \times N}$ the degree matrix, and $A_d \in \mathbb{R}^{N \times N}$ the adjacency matrix of $\mathcal{G}$.

Our goal is to compute an optimal control sequence $u^*_{t:t+T-1}$ that solves the following optimization

$$\min_{u_{t:t+T-1}} J(u_{t:t+T-1}; \varepsilon_t), \quad (3)$$

with a quadratic criterion of the form $J(\cdot) \coloneqq \sum_{i \in \mathcal{N}} J^i(\cdot)$, which we specify as

$$\begin{aligned} &J^i(u^i_{t:t+T-1}; \varepsilon^i_t) \\ &\coloneqq \frac{1}{2} \sum_{k=t}^{t+T-1} (\varepsilon^i_k{}^\top Q \varepsilon^i_k + u^i_k{}^\top R u^i_k) + \frac{1}{2} \varepsilon^i_{t+T}{}^\top Q_f \varepsilon^i_{t+T} \\ &= \frac{1}{2} (\varepsilon^i_{t+1:t+T}{}^\top \mathcal{Q} \varepsilon^i_{t+1:t+T} + u^i_{t:t+T-1}{}^\top \mathcal{R} u^i_{t:t+T-1} + \varepsilon^i_t{}^\top Q \varepsilon^i_t) \end{aligned} \quad (4)$$

where $Q \in \mathbb{R}^{n_x \times n_x}$, $Q_f \in \mathbb{R}^{n_x \times n_x}$, and $R \in \mathbb{R}^{n_u \times n_u}$ are positive definite weighting matrices, and $\mathcal{Q} \coloneqq \operatorname{diag}(\begin{bmatrix} Q & \cdots & Q & Q_f \end{bmatrix}) \in \mathbb{R}^{Tn_x \times Tn_x}$, $\mathcal{R} \coloneqq \operatorname{diag}(\begin{bmatrix} R & R & \cdots & R \end{bmatrix}) \in \mathbb{R}^{Tn_u \times Tn_u}$. This optimization is to be solved subject to the dynamic constraints in (2) for all $i \in \mathcal{N}$.

In a model predictive control application, the terminal cost defined by the matrix Q_f would be selected to guarantee asymptotic stability [23], but this term could also be omitted when the horizon is sufficiently long [10].

Lemma 1: Suppose that Assumption 1 holds. Then (3) has a unique global minimum $u^*_{t:t+T-1}$ that is a solution to

$$\begin{aligned} (\bar{L}^2 \otimes (\mathcal{B}^\top \mathcal{Q} \mathcal{B}) + I_N \otimes \mathcal{R}) u^*_{t:t+T-1} \\ - (\bar{L} \otimes (\mathcal{B}^\top \mathcal{Q} \mathcal{A})) \varepsilon_t = 0, \end{aligned} \quad (5)$$

and the value of the minimum is given by

$$J^* \coloneqq J(u^*_{t:t+T-1};\varepsilon_t) = \frac{1}{2}\varepsilon_t^\top (I_N \otimes (\mathcal{A}^\top \mathcal{Q}\mathcal{A} + Q))\varepsilon_t - \frac{1}{2}\varepsilon_t^\top (\bar{L} \otimes (\mathcal{A}^\top \mathcal{Q}\mathcal{B}))(\bar{L}^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R})^{-1} (\bar{L} \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{A}))\varepsilon_t, \tag{6}$$

where $\mathcal{A} \in \mathbb{R}^{Tn_x \times n_x}$ and $\mathcal{B} \in \mathbb{R}^{Tn_x \times Tn_u}$,

$$\mathcal{A} \coloneqq \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^T \end{bmatrix}, \quad \mathcal{B} \coloneqq \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{T-1}B & A^{T-2}B & \cdots & B \end{bmatrix}. \quad \square$$

Proof. The closed form expression for the optimal control input sequence can be obtained by computing the first order optimality condition of the problem in (3). For that, we first write the dynamics for a sequence of tracking error of length T obtained from the local dynamics in (2)

$$\varepsilon^i_{t+1:t+T} = \mathcal{A}\varepsilon^i_t - (g^i + d^i)\mathcal{B}u^i_{t:t+T-1} + \mathcal{B}\sum_{j\in\mathcal{N}_i} u^j_{t:t+T-1}. \tag{7}$$

Then, by incorporating (7) into the cost function (4) we have that

$$\begin{aligned} J(u_{t:t+T-1};\varepsilon_t) = \frac{1}{2}\sum_{i\in\mathcal{N}} \Big[& {\varepsilon^i_t}^\top (\mathcal{A}^\top \mathcal{Q}\mathcal{A} + Q)\varepsilon^i_t \\ & + {u^i_{t:t+T-1}}^\top ((g^i + d^i)^2 \mathcal{B}^\top \mathcal{Q}\mathcal{B} + \mathcal{R})u^i_{t:t+T-1} \\ & + \big(\sum_{j\in\mathcal{N}_i} u^j_{t:t+T-1}\big)^\top (\mathcal{B}^\top \mathcal{Q}\mathcal{B})\big(\sum_{j\in\mathcal{N}_i} u^j_{t:t+T-1}\big) \\ & - 2(g^i + d^i){\varepsilon^i_t}^\top (\mathcal{A}^\top \mathcal{Q}\mathcal{B})u^i_{t:t+T-1} \\ & - 2(g^i + d^i)\big(\sum_{j\in\mathcal{N}_i} u^j_{t:t+T-1}\big)^\top \mathcal{B}^\top \mathcal{Q}\mathcal{B}u^i_{t:t+T-1} \\ & + 2{\varepsilon^i_t}^\top \mathcal{A}^\top \mathcal{Q}\mathcal{B}\big(\sum_{j\in\mathcal{N}_i} u^j_{t:t+T-1}\big)\Big]. \end{aligned}$$

By concatenation of the agents states and control inputs we obtain the compact form

$$\begin{aligned} J(u_{t:t+T-1};\varepsilon_t) &= \frac{1}{2}\varepsilon_t^\top (I_N \otimes (\mathcal{A}^\top \mathcal{Q}\mathcal{A} + Q))\varepsilon_t \\ &- \varepsilon_t^\top (\bar{L} \otimes (\mathcal{A}^\top \mathcal{Q}\mathcal{B}))u_{t:t+T-1} \\ &+ \frac{1}{2}u^\top_{t:t+T-1}(\bar{L}^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R})u_{t:t+T-1}. \end{aligned} \tag{8}$$

The optimal control input sequence $u^*_{t:t+T-1}$ that solves (3) satisfies the first order optimality condition

$$\nabla_{u_{t:t+T-1}} J(u_{t:t+T-1};\varepsilon_t)\big|_{u_{t:t+T-1}=u^*_{t:t+T-1}} = 0,$$

from which we obtain (5).

To prove uniqueness we show that $\bar{L}^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R}$ is positive definite and therefore invertible. From Assumption 1 we have that the graph is connected, resulting in the vector of 1s being the only eigenvector corresponding to the null eigenvalue of the positive semidefinite matrix $D - A_d$. Since at least one follower is connected to the leader, then G has at least one positive element in its diagonal. Hence, $\bar{L} > 0$ and so is $\bar{L}^2$. From the positive definiteness of the weights in the cost function, we have that $\mathcal{B}^\top \mathcal{Q}\mathcal{B} \geqslant 0$, and we conclude that $\bar{L}^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R} > 0$.

The optimal control sequence is then

$$u^*_{t:t+T-1} = (\bar{L}^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R})^{-1}(\bar{L} \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{A}))\varepsilon_t, \tag{9}$$

for which the minimum cost in (6) is obtained from a direct substitution of (9) in (8). ∎

III. 2-Hop local Optimization

For each $i \in \mathcal{N}$, consider the following optimization that is local to agent i

$$\min_{u^i_{t:t+T-1}} \bar{J}^i(u^i_{t:t+T-1};\varepsilon^i_t, \varepsilon^{-i_1}_t, u^{-i_1}_{t:t+T-1}, u^{-i_2}_{t:t+T-1}), \tag{10}$$

subject to the dynamic constraint in (2) for agent i and for every agent $\ell \in \mathcal{N}_i$. The local cost functions are of the form $\bar{J}^i(\cdot) \coloneqq J^i(\cdot) + \sum_{\ell\in\mathcal{N}_i} J^\ell(\cdot)$, with function $J^i(\cdot)$ defined as in (4).

The parameters to the optimization are: the local tracking error ε^i_t; the set of tracking errors associated with the 1-hop neighbors of i denoted by $\varepsilon^{-i_1}_t \coloneqq \{\varepsilon^\ell_t : \ell \in \mathcal{N}_i\}$; the control input sequence from the 1-hop neighbors $u^{-i_1}_{t:t+T-1}$; and the set of control input sequences from 2-hop neighbors denoted by $u^{-i_2}_{t:t+T-1} \coloneqq \{u^\ell_{t:t+T-1} : \ell \in \bigcup_{j\in\mathcal{N}_i} \mathcal{N}_j \backslash \{\mathcal{N}_i \cup i\}\}$. It is important to note that $u^i_{t:t+T-1}$ is the optimization variable for (10), but the optimization criterion also depends on the tracking error of the 1-hop neighbor and the control input sequences from 1-hop and 2-hop neighbors of i which can be viewed as optimization parameters. This dependence arises from the fact that we constraint the local and the neighbor's state sequences to be, a priori, a solution of (2). This observation will become evident below.

Denoting by $\bar{u}^{i*}_{t:t+T-1}$ the local optimal control input sequence that solves (10), the first order optimality condition for the optimization in (10) results in

$$\begin{aligned} &\nabla_{u^i_{t:t+T-1}} \bar{J}^i(u^i_{t:t+T-1};\varepsilon^i_t, \varepsilon^{-i_1}_t, u^{-i_1}_{t:t+T-1}, u^{-i_2}_{t:t+T-1})\Big|_{\substack{u^i_{t:t+T-1} \\ =\bar{u}^{i*}_{t:t+T-1}}} = 0 \\ &\Rightarrow ((g^i + d^i)^2 \mathcal{B}^\top \mathcal{Q}\mathcal{B} + \mathcal{R})\bar{u}^{i*}_{t:t+T-1} \\ &- (g^i + d^i)\mathcal{B}^\top \mathcal{Q}\mathcal{B}\sum_{j\in\mathcal{N}_i} u^j_{t:t+T-1} - (g^i + d^i)(\mathcal{B}^\top \mathcal{Q}\mathcal{A})\varepsilon^i_t \\ &+ \sum_{\ell\in\mathcal{N}_i} \Big[\mathcal{B}^\top \mathcal{Q}\mathcal{B}\sum_{j\in\mathcal{N}_\ell} u^j_{t:t+T-1} \\ &\qquad - (g^\ell + d^\ell)\mathcal{B}^\top \mathcal{Q}\mathcal{B}u^\ell_{t:t+T-1} + \mathcal{B}^\top \mathcal{Q}\mathcal{A}\varepsilon^\ell_t\Big] = 0. \end{aligned} \tag{11}$$

All the conditions (11), one for each agent $i \in \mathcal{N}$, can be expressed in vector form as

$$\begin{aligned} &\nabla_{u_{t:t+T-1}} \bar{J}(u_{t:t+T-1};\varepsilon_t)\big|_{u_{t:t+T-1}=\bar{u}^*_{t:t+T-1}} \coloneqq \\ &\Big[\nabla_{u^i_{t:t+T-1}} \bar{J}^i(u^i_{t:t+T-1};\varepsilon^i_t, \varepsilon^{-i_1}_t, u^{-i_1}_{t:t+T-1}, u^{-i_2}_{t:t+T-1})\Big]\Big|_{\substack{u_{t:t+T-1} \\ =\bar{u}^*_{t:t+T-1}}} = 0 \\ &\Rightarrow (\bar{L}^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R})\bar{u}^*_{t:t+T-1} \\ &\qquad - (\bar{L} \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{A}))\varepsilon_t = 0. \end{aligned} \tag{12}$$

This expression is the same as the one obtained in (5), which means that if all agents simultaneously optimize (10) they will be able to find the global optimum to (3).

A. Distributed Algorithm

For our first algorithm, we assume that each agent is able to store estimates of each of its neighbors optimal solution and re-transmit them to their 1-hop neighbors. In practice, this means that each agent has available the delayed values of the control sequence of their 1-hop and 2-hop neighbors and could therefore solve (10) based on these estimates. This leads to the following iterative algorithm, where $\hat{\bar{u}}^i_{t:t+T-1}(m)$ should be viewed as an estimate of $\bar{u}^{i*}_{t:t+T-1}$, at iteration m of the algorithm.

Algorithm 1: [2-hop distributed optimization]

At every agent $i \in \mathcal{N}$ and for a given tolerance σ,

1) Set $m = 0$, initialize local estimates $\hat{\bar{u}}^i_{t:t+T-1}(0)$ randomly and broadcast to neighbors.
2) Increment m and initialize $\hat{\bar{u}}^i_{t:t+T-1}(1)$ randomly.
3) Broadcast $\hat{\bar{u}}^i_{t:t+T-1}(m)$ and $\hat{\bar{u}}^j_{t:t+T-1}(m-1)\ \forall j \in \mathcal{N}_i$ to neighbors.
4) Compute the next estimate $\hat{\bar{u}}^i_{t:t+T-1}(m+1)$ as the solution to
$$\min_{u^i_{t:t+T-1}} \bar{J}^i(u^i_{t:t+T-1}; \varepsilon^i_t, \varepsilon^{-i_1}_t, \hat{\bar{u}}^{-i_1}_{t:t+T-1}(m), \hat{\bar{u}}^{-i_2}_{t:t+T-1}(m-1))$$
subject to the dynamics in (2) for i, and $\ell \in \mathcal{N}_i$.
5) If $\|\hat{\bar{u}}^i_{t:t+T-1}(m+1) - \hat{\bar{u}}^i_{t:t+T-1}(m)\| < \sigma$ stop, otherwise increment m and return to Step 3. □

To study the convergence of the estimates obtained from Algorithm 1 we treat the evolution of the estimates, $\hat{\bar{u}}^i_{t:t+T-1}(m),\ \forall i \in \mathcal{N}$ as a dynamical system. The optimization solved at Step 4 satisfies the following first order optimality condition

$$\begin{aligned}
&((g^i+d^i)^2\mathcal{B}^\top\mathcal{Q}\mathcal{B}+\mathcal{R})\hat{\bar{u}}^i_{t:t+T-1}(m+2)\\
&-(g^i+d^i)\mathcal{B}^\top\mathcal{Q}\mathcal{B}\sum_{j\in\mathcal{N}_i}\hat{\bar{u}}^j_{t:t+T-1}(m+1)\\
&-(g^i+d^i)(\mathcal{B}^\top\mathcal{Q}\mathcal{A})\varepsilon^i_t+\sum_{\ell\in\mathcal{N}_i}\Big[(\mathcal{B}^\top\mathcal{Q}\mathcal{B})(\sum_{j\in\mathcal{N}_\ell}\hat{\bar{u}}^j_{t:t+T-1}(m))\\
&\quad-(g^\ell+d^\ell)\mathcal{B}^\top\mathcal{Q}\mathcal{B}\hat{\bar{u}}^\ell_{t:t+T-1}(m+1)+\mathcal{B}^\top\mathcal{Q}\mathcal{A}\varepsilon^\ell_t\Big]=0
\end{aligned}$$

and for every agent we write this condition in the following vector form

$$\begin{aligned}
&((G+D)^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B})+I_N\otimes\mathcal{R})\hat{\bar{u}}_{t:t+T-1}(m+2)\\
&-((A_d(G+D)\\
&\qquad+(G+D)A_d)\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}))\hat{\bar{u}}_{t:t+T-1}(m+1)\\
&+(A_d^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}))\hat{\bar{u}}_{t:t+T-1}(m)-(\bar{L}\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{A}))\varepsilon_t=0. \quad (13)
\end{aligned}$$

The following result presents a sufficient condition for the convergence of the iterations given by (13).

Theorem 1: Consider the Algorithm 1 with dynamics given by (13). Assume that $\mathcal{R} = \rho I_{Tn_u}$ where $\rho > |\underline{\lambda}((G+D)^2 - A_d^2)\bar{\lambda}(\mathcal{B}^\top\mathcal{Q}\mathcal{B})|$ and suppose that Assumption 1 holds. Then the local estimates converge to the optimal solution in (9), i.e., $\lim_{m\to\infty}\hat{\bar{u}}_{t:t+T-1}(m) = u^*_{t:t+T-1}$. □

The following lemma addresses the stability of second order systems of difference equation like (13).

Lemma 2: Consider the following second order dynamical system,

$$Ax(m+2) - Bx(m+1) + Cx(m) - d = 0, \qquad (14)$$

where $x, d \in \mathbb{R}^n$, and $A, B, C \in \mathbb{R}^{n\times n}$. Assume that: *(A1)* $A-B+C > 0$, *(A2)* $A+B+C > 0$; *(A3)* $A > 0$, $C \geqslant 0$, and $A-C > 0$. Then $x^* := (A-B+C)^{-1}d$ is an asymptotically stable equilibrium point. □

Proof of Lemma 2. From *(A1)* we obtain the unique equilibrium x^*. Further, we have that the second order system in (14) is equivalent to the following first order system

$$w(m+1) = \begin{bmatrix} 0_n & I_n \\ -A^{-1}C & A^{-1}B\end{bmatrix} w(m) + \begin{bmatrix} 0 \\ A^{-1}d\end{bmatrix}, \quad (15)$$

where $w(m) \in \mathbb{R}^{2n}$. To show stability, we investigate the eigenvalues of the system matrix in (15). Let $v := \begin{bmatrix} v_1^\top & v_2^\top\end{bmatrix}^\top$ and λ be an arbitrary eigenvector and eigenvalues pair of the system matrix. Then,

$$\begin{aligned}
&\begin{bmatrix} 0_n & I_n \\ -A^{-1}C & A^{-1}B\end{bmatrix}\begin{bmatrix} v_1 \\ v_2\end{bmatrix} = \lambda\begin{bmatrix} v_1 \\ v_2\end{bmatrix}\\
&\Rightarrow (\lambda^2 A - \lambda B + C)v_1 = 0 \Rightarrow v_1^\top(\lambda^2 A - \lambda B + C)v_1 = 0\\
&\Leftrightarrow \lambda^2 v_1^\top A v_1 - \lambda v_1^\top B v_1 + v_1^\top C v_1 = 0.
\end{aligned}$$

Defining $F(\lambda) := \lambda^2 v_1^\top A v_1 - \lambda v_1^\top B v_1 + v_1^\top C v_1$, we obtain from the Jury stability test the following sufficient conditions for the eigenvalues to be inside the unit circle *(i)* $F(1) > 0$ *(ii)* $F(-1) > 0$ *(iii)* $v_1^\top A v_1 > |v_1^\top C v_1|$. Note that *(i)* $F(1) = v_1^\top A v_1 - v_1^\top B v_1 + v_1^\top C v_1$ and from *(A1)*, $F(1) > 0$. From *(A2)* it follows that *(ii)* $F(-1) = v_1^\top A v_1 + v_1^\top B v_1 + v_1^\top C v_1 > 0$. Finally, *(A3)* implies that *(iii)* $v_1^\top A v_1 > |v_1^\top C v_1|$, which concludes the proof. ■

Proof of Theorem 1. The theorem is a direct application of Lemma 2 to (13) and it suffices to show that the assumptions *(A1)*-*(A3)* hold true. We start with *(A1)*,

$$\begin{aligned}
&(G+D)^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N\otimes\mathcal{R}\\
&-(A_d(G+D)+(G+D)A_d)\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + A_d^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B})\\
&\qquad= (G+D-A_d)^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N\otimes\mathcal{R}\\
&\qquad= \bar{L}^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N\otimes\mathcal{R}.
\end{aligned}$$

From the discussion in Lemma 1 about the uniqueness of the optimal control input sequence, we conclude that $\bar{L}^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N\otimes\mathcal{R} > 0$ and *(A1)* holds.

Next, we consider *(A2)*, where the corresponding matrix expression reads

$$\begin{aligned}
&(G+D)^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N\otimes\mathcal{R}\\
&+(A_d(G+D)+(G+D)A_d)\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + A_d^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B})\\
&= (G+D+A_d)^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_n\otimes\mathcal{R}.
\end{aligned}$$

Since $\mathcal{Q} > 0$, we have that $\mathcal{B}^\top\mathcal{Q}\mathcal{B} \geqslant 0$. Also, $G+D+A_d$ is symmetric with real eigenvalues, then $(G+D+A_d)^2 \geqslant 0$. Therefore $(G+D+A_d)^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_n\otimes\mathcal{R} > 0$ and *(A2)* holds.

Finally for *(A3)*, we have that $(G+D)^2\otimes(\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N\otimes\mathcal{R} > 0$ since $G+D > 0$ and $\mathcal{B}^\top\mathcal{Q}\mathcal{B} \geqslant 0$. Also since A_d is symmetric, then $A_d^2 \geqslant 0$, and we have that $A_d^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) \geqslant 0$. Finally for the last inequality of (*A3*) we have that the corresponding matrix expression reads

$$\begin{aligned}(G+D)^2 &\otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R} - A_d^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) \\ &= ((G+D)^2 - A_d^2) \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R}. \quad (16)\end{aligned}$$

Let λ be an eigenvalue of (16). Then,

$$\lambda \geqslant \underline{\lambda}((G+D)^2 - A_d^2)\bar{\lambda}(\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + \rho > 0,$$

where the last inequality follows from the assumption of the theorem on the weight of the control input. ■

IV. 1-Hop Local Optimization

In this section, we consider local optimizations that could be solved with 1-hop neighbor information, rather than the 2-hop information required by Algorithm 1. For each $i \in \mathcal{N}$, we now consider the following local optimization

$$\min_{u^i_{t:t+T-1}} \tilde{J}^i(u^i_{t:t+T-1}; \varepsilon^i_t, u^{-i_1}_{t:t+T-1}), \quad (17)$$

subject to the dynamics in (2) for agent i, where the local cost functions are given by $\tilde{J}^i(\cdot) := J^i(\cdot)$ as defined in (4).

Note that $u^i_{t:t+T-1}$ is still the optimization variable for the local problem defined in (17), but the solution to this problem now only depends on the local tracking error ε^i_t and on the variables $u^{-i_1}_{t:t+T-1}$ that are associated with the 1-hop neighbors of i.

Denoting by $\tilde{u}^{i*}_{t:t+T-1}$ the solutions to (17), the first-order optimality condition for (17) is

$$\begin{aligned}&\nabla_{u^i_{t:t+T-1}} \tilde{J}^i(u^i_{t:t+T-1}; \varepsilon^i_t, u^{-i_1}_{t:t+T-1})\Big|_{u^i_{t:t+T-1} = \tilde{u}^{i*}_{t:t+T-1}} = 0 \\ &\Rightarrow ((g^i+d^i)^2\mathcal{B}^\top\mathcal{Q}\mathcal{B}+\mathcal{R})\tilde{u}^{i*}_{t:t+T-1} - (g^i+d^i)\mathcal{B}^\top\mathcal{Q}\mathcal{B}\sum_{j\in\mathcal{N}_i} u^j_{t:t+T-1} \\ &\qquad - (g^i+d^i)(\mathcal{B}^\top\mathcal{Q}\mathcal{A})\varepsilon^i_t = 0, \quad (18)\end{aligned}$$

and all the conditions (18), one for each agent $i \in \mathcal{N}$, can be expressed in vector form as

$$\begin{aligned}&\nabla_{u_{t:t+T-1}} \tilde{J}(u_{t:t+T-1}; \varepsilon_t)\Big|_{u_{t:t+T-1} = \tilde{u}^*_{t:t+T-1}} \\ &:= \Big[\nabla_{u^i_{t:t+T-1}} \tilde{J}^i(u^i_{t:t+T-1}; \varepsilon^i_t, u^{-i_1}_{t:t+T-1})\Big]\Big|_{u_{t:t+T-1} = \tilde{u}^*_{t:t+T-1}} = 0 \\ &\Rightarrow (((G+D)\bar{L}) \otimes (\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R})\tilde{u}^*_{t:t+T-1} \\ &\qquad - ((G+D) \otimes (\mathcal{B}^\top\mathcal{Q}\mathcal{A}))\varepsilon_t = 0. \quad (19)\end{aligned}$$

Unlike the 2-hop algorithm, these first order optimality conditions for the local optimization (17) now differ from the global optimality conditions in (5). However, the following result shows that (19) still has a unique solution.

Proposition 1: Assume that Assumption 1 holds and let $\mathcal{R} = \rho I_{Tn_u}$, $\rho > 0$. Then the optimal control sequence obtained from solving (17) is unique and equal to

$$\begin{aligned}\tilde{u}^*_{t:t+T-1} = (((G+D)\bar{L}) &\otimes (\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R})^{-1} \\ &\times ((G+D) \otimes (\mathcal{B}^\top\mathcal{Q}\mathcal{A}))\varepsilon_t. \quad (20)\end{aligned}$$

Proof. The proof is similar to that found in Lemma 1 and it suffices to show that the eigenvalues of $(G+D)\bar{L}$ are positive. Let v and λ be an arbitrary eigenvector and eigenvalue pair of $(G+D)\bar{L}$. Then,

$$\begin{aligned}&(G+D)\bar{L}v = \lambda v \Leftrightarrow \bar{L}v = \lambda(G+D)^{-1}v \\ &\Rightarrow \frac{v^\top \bar{L} v}{v^\top (G+D)^{-1} v} = \lambda > 0,\end{aligned}$$

where the inequality comes from the fact that $\bar{L} > 0$ and $G + D > 0$. Finally, we have that (20) follows from (19). ■

Remark 1: For the "cheap control" case, i.e., $R = 0_{n_u}$, and under the assumption that the matrix $\mathcal{B}$ is full column rank, one can show that (20) minimizes (8). This also happens in the case when $\rho \to \infty$, $R = \rho I_{n_u}$. In general, between these extremes (20) yields a suboptimal result, but as we see in the simulations, the optimality gap is often small. □

A. Distributed Algorithm

The local costs in (17) are the basis for the following algorithm that only requires 1-hop information, where $\hat{\tilde{u}}^i_{t:t+T-1}(m)$ should be viewed as an estimate of $\tilde{u}^{i*}_{t:t+T-1}$ computed at iteration m.

Algorithm 2: [1-hop distributed optimization]

At every agent $i \in \mathcal{N}$ and for a given tolerance σ,

1) Set $m = 0$ and initialize local estimates $\hat{\tilde{u}}^i_{t:t+T-1}(0)$ randomly.
2) Broadcast $\hat{\tilde{u}}^i_{t:t+T-1}(m)$ to neighbors.
3) Compute the next estimate $\hat{\tilde{u}}^i_{t:t+T-1}(m+1)$ as the solution to
$$\min_{u^i_{t:t+T-1}} \tilde{J}^i(u^i_{t:t+T-1}; \varepsilon^i_t, u^{-i_1}_{t:t+T-1}(m))$$
subject to the dynamics in (2) for i.
4) If $\|\hat{\tilde{u}}^i_{t:t+T-1}(m+1) - \hat{\tilde{u}}^i_{t:t+T-1}(m)\| < \sigma$ stop, otherwise increment m and return to Step 2. □

Similarly to the previous section, we revisit the first order optimality condition to investigate the convergence of the estimates computed using Algorithm 2.

At each iteration of the algorithm we have that the following equation must be satisfied

$$\begin{aligned}&((g^i+d^i)^2\mathcal{B}^\top\mathcal{Q}\mathcal{B}+\mathcal{R})\hat{\tilde{u}}^i_{t:t+T-1}(m+1) \\ &-(g^i+d^i)\mathcal{B}^\top\mathcal{Q}\mathcal{B}\sum_{j\in\mathcal{N}_i}\hat{\tilde{u}}^j_{t:t+T-1}(m) - (g^i+d^i)(\mathcal{B}^\top\mathcal{Q}\mathcal{A})\varepsilon^i_t = 0\end{aligned}$$

or, in vector form for every agent,

$$\begin{aligned}&((G+D)^2 \otimes (\mathcal{B}^\top\mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R})\hat{\tilde{u}}_{t:t+T-1}(m+1) \\ &- (((G+D)A_d) \otimes (\mathcal{B}^\top\mathcal{Q}\mathcal{B}))\hat{\tilde{u}}_{t:t+T-1}(m) \\ &\qquad - ((G+D) \otimes (\mathcal{B}^\top\mathcal{Q}\mathcal{A}))\varepsilon_t = 0 \quad (21)\end{aligned}$$

The next theorem establishes the convergence of the iterations given by (21).

Theorem 2: Consider Algorithm 2 with dynamics given by (21) and let Assumption 1 hold. Then the local estimates of the optimal control converge to (20). □

The following lemma addresses the stability of first order systems of a difference equation similar to (21).

Lemma 3: Consider the following discrete time system

$$Ax(m+1) - Bx(m) - c = 0$$

where $x, c \in \mathbb{R}^n$ $A, B \in \mathbb{R}^{n\times n}$. Let $A > 0$. If $2A \pm (B + B^\top) > 0$, then the system is asymptotically stable.

Proof of Lemma 3. Let v and λ be an arbitrary eigenvector and eigenvalue pair of $A^{-1}B$. Then,

$$A^{-1}Bv = \lambda v \Leftrightarrow Bv = \lambda Av$$
$$\Rightarrow \frac{v^\top Bv}{v^\top Av} = \lambda \Leftrightarrow \frac{v^\top (B+B^\top)v}{v^\top 2Av} = \lambda.$$

From the condition on the lemma, we have that $|\lambda| < 1$, and the state asymptotically converges to $x^* := (A-B)^{-1}c$. ■

Proof of Theorem 2. The proof is a direct application of Lemma 3 to the system in (21). We first note that $(G+D)^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R} > 0$. To check if the matrix inequality holds we write

$$\begin{aligned} &2((G+D)^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R}) \\ &\qquad - ((A_d(G+D) + (G+D)A_d) \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B})) \\ &= (\bar{L}(G+D) + (G+D)\bar{L}) \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + 2I_N \otimes \mathcal{R} > 0, \end{aligned}$$

where the inequality follows from the discussion in Proposition 1, where we concluded that the eigenvalues of $(G+D)\bar{L}$ are positive and $B^\top \mathcal{Q}\mathcal{B} \geqslant 0$. To show that

$$\begin{aligned} &2((G+D)^2 \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R}) \\ &\qquad + ((A_d(G+D) + (G+D)A_d) \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B})) \\ &= ((Ad + G + D)(G+D) \\ &\qquad + (G+D)(Ad + G + D)) \otimes (\mathcal{B}^\top \mathcal{Q}\mathcal{B}) + I_N \otimes \mathcal{R} \end{aligned}$$

is positive definite, we use the fact that the eigenvalues of $(A_d + G + D)(G+D)$ are positive and that $\mathcal{B}^\top \mathcal{Q}\mathcal{B} \geqslant 0$. ■

V. Example

We now illustrate our algorithms to solve optimizations associated with a model predictive control problem of the form of (3). At each time-step t, we execute Algorithms 1 and 2 until convergence to obtain a finite horizon optimal control input sequence. From this sequence we utilize only the first value of control input and repeat this process at the next time step.

For this problem, we generate random connected graphs with arbitrary number of nodes and topology. We use one such graph with 10 follower agents, where only one of them is connected to the single leader agent. The maximum node degree in this graph is 3.

We consider the synchronization of an unstable third order linear systema like (1) with

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

The initial state for each agent is initialized randomly and we select the weights on the cost as $Q = Q_f = 10I_3$, $R = 10$, and $T = 15$. The optimizations were solved using `TensCalc` [11], a toolbox that generates efficient solvers for large scale optimization problems.

Figure 1 shows a typical evolution of the norm of the tracking error (2) for one of the agents, comparing both algorithms, and showing that synchronization is achieved.

To illustrate Remark 1, we simulate various values of the weight R in the control input for a particular time instant of the simulation, comparing the cost from converged estimates obtained from Algorithm 2 versus the optimal cost in (6). Figure 2 confirms that the gap is small, and that in the extreme cases (i.e., $R = 0$ and $R = \rho$, with $\rho \to \infty$) the estimates obtained from the 1-hop algorithm are indeed optimal solutions to (3).

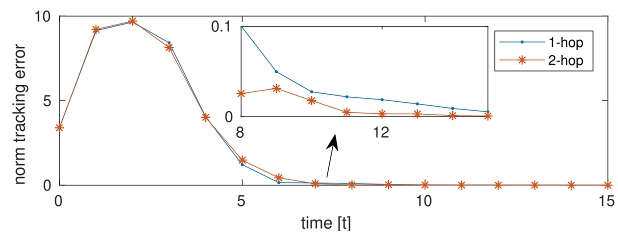

Fig. 1. Comparison between the tracking error for a given follower agent for each of the two algorithms.

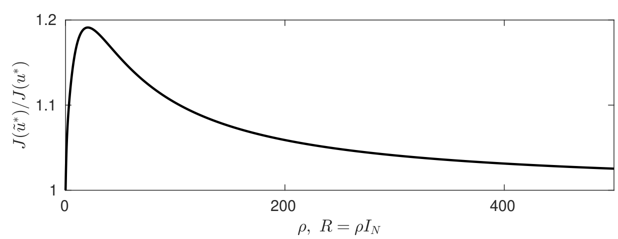

Fig. 2. Ratio between the cost from the converged estimated of the control input using 1-hop neighbor information, and the optimal cost for various values of the weighting matrix R.

VI. Conclusion

We presented two distributed algorithms to estimate the optimal control sequence obtained from a finite horizon quadratic optimization, of the type found in model predictive control problems for multi-agent coordination. The first algorithm utilizes 2-hop neighbor information, and we showed that it converges to the optimal solution under appropriate assumptions on the optimization parameters. When only 1-hop neighbor information is considered, a second algorithm was proposed that can still be shown to converge, but yields suboptimal results. The algorithms were compared in simulation for a leader-follower model predictive control problem. The simulations highlights that the optimality gap for the second algorithm is small and in some cases zero.

Future work will address the inclusion of input and state constraints on the optimization problem, extend the analysis to general nonlinear models and cost functions, and determine an upper bound on the optimality gap for Algorithm 2.

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