Linear Parametrically Varying Systems with Brief Instabilities: An Application to Vision/Inertial Navigation

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This paper addresses the problem of nonlinear filter design to estimate the relative position and velocity of an unmanned air vehicle (UAV) with respect to a point on a ship using infrared (IR) vision, inertial, and air data sensors. Sufficient conditions are derived for the existence of a particular type of complementary filters with guaranteed stability and performance in the presence of so-called out-of-frame events that arise when the vision system loses its target temporarily. The results obtained build upon new developments in the theory of linear parametrically varying systems (LPVs) with brief instabilities—also reported in the paper—and provide the proper framework to deal with out-of-frame events. Field tests with a prototype UAV illustrate the performance of the filter and the scope of applications of the new theory developed.

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I. INTRODUCTION

This paper addresses the problem of estimating the relative position and velocity of an unmanned air vehicle (UAV) with respect to a reference point on a ship. The sensor suite considered in this application includes an infrared camera (IR), an inertial measurement unit (IMU) and a pressure altitude sensor. These passive sensors, installed on board the UAV, provide data on the aircraft's inertial position and attitude as well as on the ship's position in the image frame of the camera. The ship's reference point is represented by the centroid of the image of its smokestack (referred to as a hot spot) in the IR camera plane. The primary application of the proposed solution is for the autonomous landing of an UAV on a ship in an adversarial environment, hence the reliance on passive sensors. In this environment, the ship will not expose itself by transmitting its position to the UAV.

This problem has been previously addressed in [1], where sufficient conditions for the existence of nonlinear filters to estimate the relative position and velocity of a UAV with respective to a ship using passive sensors were obtained. The structure of the filters derived is appealing because it embodies straightforward kinematic relationships and allows for an intuitive interpretation in terms of rotation matrices between the different reference frames involved. Stability and performance of the filters developed in an H_∞ setting were established by solving a set of linear matrix inequalities (LMIs). However, the results reported in [1] did not address the critical issue of out-of-frame events, inherent to the operation of the IR camera and that arise when the camera loses sight of the image it is tracking.

The key contribution of this work is the extension of the results in [1] to deal explicitly with out-of-frame events. Sufficient conditions are obtained for the existence of filters with guaranteed regional stability and performance in the presence of out-of-frame events. In addition, an LMI-based design technique is developed that allows an engineer to study the trade-off between filter performance, the size of a suitable spatial "work" region where good performance is desired, and the maximum expected duration of out-of-frame events.

The results obtained in [1] used the theory of linear parameter varying systems (LPVs), which are defined as linear systems whose dynamics depend on time-varying parameters. In its simplest form, an LPV admits the representation

$$\dot{x} = A(p(t))x, \qquad x \in \Re^n \tag{1}$$

where p is an arbitrary signal taking values in a parameter set P. Further assumptions may include restrictions on the parameter rates of variation. The representation (1) may also be extended to include exogenous inputs and outputs. In some cases, the parameter p may itself be a function of the state x (see [2, 3] and the references therein). By casting the dynamics of a general (possibly nonlinear, time-varying) system in the form of an LPV, similarities with linear systems can be exploited to yield powerful results on stability and input-output behavior characteristics. The results are established for all possible parameter variations, irrespective of which specific parameter trajectory (not known a priori) affects the system dynamics. This is in striking contrast with standard linear time-varying system theory, where the model description is assumed to be known for all times. LPVs provide thus an elegant and powerful set-up for the analysis of some classes of linear time-varying and even nonlinear systems.

In order to extend the results obtained in [1] to include out-of-frame events, a new mathematical machinery has been developed for the analysis of LPVs with so-called brief instabilities. This can be simply explained by considering an LPV of the form (1). If one assumes that any piecewise-continuous signal p is allowed in (1), then stability of all the matrices $A(p)$, $p \in P$ is necessary to guarantee boundedness of any solution to (1) (here, stability is equivalent to requiring that all eigenvalues of $A(p)$ have negative real parts). However, this is no longer the case when the time variation of p is such that the resulting $A(p)$ is only temporarily unstable. The work presented here shows how, with an appropriate notion of "brief instability," it is possible to prove that the state space trajectories of an LPV system converge to zero exponentially even when some of the matrices $A(p)$ are unstable for brief periods of time. Necessary conditions for this to occur are cast in terms of a parameterized family of LMIs, which have become the tool par excellence for controller and filter system analysis and design. For LPV systems described by (1), together with additional input and output equations, the paper analyzes the impact of brief instabilities on the performance of an LPV system, as measured in terms of its input-output L_2-induced operator norm. In particular, a parameterized set of LMIs is derived that, when feasible, provides an upper bound on the L_2-induced norm of an LPV system with brief instabilities.

The analysis of LPV systems with brief instabilities is inspired by previous work of the first author on switched systems [4, 5] as well as by the work reported in [6] and [7]. Switched systems can be viewed as a form of LPV systems where the signal $p(t)$ in (1) is restricted to be constant between two consecutive discontinuities. The idea of brief instabilities was introduced in [6] for switched systems,[1] where the authors provide conditions for exponential stability of switched system with brief instabilities. These results were extended in [7] for L_2 disturbance attenuation.

The work reported here is also closely related to that described in [8], where the authors provide conditions for the stability of asynchronous dynamical systems (ADSs). The latter can also be viewed as a particular form of switched systems for which the system dynamics change in response to external asynchronous events. These events may make the system become unstable for certain periods of time. In [8] the authors provide a set of LMIs that, if feasible, guarantee exponential convergence of the state of ADSs. Feasibility of the LMIs requires that the periods of instability occur for a small fraction of the time. Because the authors of [8] only consider asymptotic rates for the occurrence of the events that trigger changes in the dynamics, their results are only asymptotic and do not provide uniform bounds on the state.

In this paper, new results of the analysis of LPV systems with brief instabilities are shown to provide an appropriate framework for the design of filters that integrate IR, inertial, and air data to estimate the relative position and velocity of an UAV with respect to a given point on a ship and to yield guaranteed regional stability and performance in the presence of out-of-frame events. Field tests with a prototype UAV illustrate the performance of the filter and the scope of applications of the new theory developed.

The paper is organized as follows. Section II introduces basic theoretical results that play a key role in analyzing the stability and performance of LPV systems with brief instabilities. Section III applies the theory developed in Section II to the design of an integrated vision/inertial filter for an air vehicle. The experimental set up that was used to implement and assess filter performance is described. Finally, the paper ends with some conclusions and a brief description of problems that warrant further research.

II. LPVS WITH BRIEF INSTABILITIES

This section introduces the concept of LPV systems with brief instabilities. This is followed by the derivation of stability and performance analysis results for LPV-based systems. Consider the homogeneous LPV system

$$\Sigma_p := \begin{cases} \dot{x} = A(p)x \\ y = C(p)x \end{cases} \tag{2}$$

that is obtained from (1) by including an output equation for the variable y. In (2), p denotes a piecewise-continuous[2] time-varying parameter taking values in the set $P \subset \Re^k$, $k \geq 1$ and $A : P \rightarrow \Re^{n\times n}$ and $C : P \rightarrow \Re^{m\times m}$ are functions of the parameter set P.

[1]Although in [4] the authors consider a slightly more conservative definition of brief instabilities, their results seem to be easily extendable to the definition given in Section II.

[2]We say that a signal $v : [0,\infty) \rightarrow \Re^k$ is piecewise-continuous if v has a finite number of discontinuities on any finite interval.

Let P_{stable} denote the subset of P for which $A(p)$ is a stability matrix, i.e., $A(p)$ is stable if and only if $p \in P_{\text{stable}}$. The remaining elements of P form the set P_{unstable}. We assume that P is a compact (closed and bounded) subset of a finite-dimensional space and that A and C are continuous functions in P. Because of these assumptions, it is straightforward to show that P_{unstable} is also compact. In the sequel we derive conditions on p that are sufficient to guarantee that x converges to zero exponentially fast. We also compute an upper bound on the transient response of the output y.

For a given time-varying parameter p and $t > \tau > 0$, let $T_p(\tau,t)$ denote the amount of time in the interval (τ,t) that p remains in P_{unstable}. Formally,

$$T_p(\tau,t) := \int_\tau^t \chi(p(s))ds \tag{3}$$

where $\chi : P \to \{0,1\}$ denotes the characteristic function of P_{unstable} defined as

$$\chi(p) := \begin{cases} 0 & p \in P_{\text{stable}} \\ 1 & p \in P_{\text{unstable}} \end{cases}.$$

The integral in (3) is well defined because piecewise-continuity of p and compactness of P_{unstable} guarantee that $\chi(p)$ is also piecewise-continuous. We will say that Σ_p has brief instability if

$$T_p(\tau,t) \le T_0 + \alpha(t-\tau), \qquad \forall \quad t \ge \tau \ge 0$$

for some $T_0 \ge 0$, $\alpha \in [0,1]$. The scalar $T_0$is called the instability bound and α is the asymptotic instability ratio.

A. Stability

The following result gives conditions under which system (2) is stable in the presence of brief instabilities.

LEMMA 1 *Consider the LPV system Σ_p defined by* (2) *and assume there exist positive definite matrices $R \in \Re^{m\times m}$ and $X \in \Re^{n\times n}$ and positive scalars λ_0, μ such that*

$$A(p)^T X + XA(p) \le -\lambda_0 X, \qquad \forall \quad p \in P_{\text{stable}} \tag{4}$$

$$A(p)^T X + XA(p) \le \mu X, \qquad \forall \quad p \in P_{\text{unstable}} \tag{5}$$

and

$$X \ge C(p)^T RC(p), \qquad \forall \quad p \in P. \tag{6}$$

Further assume that Σ_p has brief instability with instability bound T_0 and asymptotic instability ratio $\alpha < \alpha^* = \lambda_0/(\lambda_0 + \mu)$. Then, x and y converge to zero exponentially and $y(t)^T Ry(t) \le e^{(\lambda_0+\mu)T_0} x(0)^T Xx(0)$, $\forall t \ge 0$, along solutions of (2).

Note 1 When (4) holds, (5) will always hold for sufficiently large μ. Moreover, X can always be scaled so that (6) also holds.

PROOF OF LEMMA 1 Given an arbitrary solution $x(t)$ of (2), let

$$V(t) := x(t)^T Xx(t).$$

From (4)–(5) it follows that $\dot V \le -\lambda_0 V$ while $p \in P_{\text{stable}}$ and $\dot V \le \mu V$ while $p \in P_{\text{unstable}}$. Therefore,

$$V(t) \le e^{-\lambda_0(t-\tau-T_p(\tau,t))+\mu T_p(\tau,t)} V(\tau), \qquad \forall \quad t \ge \tau \ge 0. \tag{7}$$

Let $\lambda := \lambda_0 - \alpha(\lambda_0 + \mu)$. From the assumptions of the theorem, $\lambda > 0$ and

$$-\lambda_0(t-\tau-T_p(\tau,t)) + \mu T_p(\tau,t) \le (\lambda_0 + \mu)T_0 - \lambda(t-\tau), \qquad \forall \quad t \ge \tau \ge 0. \tag{8}$$

Using (7) and (8) yields

$$V(t) \le e^{(\lambda_0+\mu)T_0 - \lambda(t-\tau)} V(\tau), \qquad \forall \quad t \ge \tau \ge 0.$$

Furthermore, (6) implies that

$$y(t)^T Ry(t) \le x(t)^T Xx(t) \le e^{(\lambda_0+\mu)T_0-\lambda t} x(0)^T Xx(0) \tag{9}$$

for every $t \ge 0$, thus completing the proof.

LPV models such as (2) are often used to model nonlinear systems where the time-varying parameter p is a function of the state, e.g.,

$$p(t) := f(x(t),t)$$

where $f : \Re^n \times [0,\infty) \to P$. When this happens, further care must be taken to ensure that the parameter p does indeed lie in P for all possible trajectories of the state $x(.)$. This can be done by restricting the set of initial states in (2), thus yielding a local version of Lemma 1. To that effect, take a positive definite matrix $R \in \Re^{m\times m}$ and consider the set of states for which the output y is guaranteed to be in the ellipsoid defined by $y^T Ry \le 1$, i.e., in the set

$$\Omega := \{w \in \Re^n : w^T C(p)^T RC(p)w \le 1,\ \forall p \in P\}.$$

We now consider a version of Lemma 1 that is local to the set Ω. Suppose that there exists a symmetric positive definite matrix $X \in \Re^{n\times n}$ and positive scalars λ_0, μ for which

$$A(p)^T X + XA(p) \le -\lambda_0 X, \qquad \forall \quad t:\ x(t) \in \Omega \quad \text{and} \quad p(t) \in P_{\text{stable}} \tag{10}$$

$$A(p)^T X + XA(p) \le \mu X, \qquad \forall \quad t:\ x(t) \in \Omega \quad \text{and} \quad p(t) \in P_{\text{unstable}} \tag{11}$$

$$X \ge C(p)^T RC(p), \qquad \forall \quad t:\ x(t) \in \Omega \quad \text{and} \quad p(t) \in P. \tag{12}$$

By requiring that the initialization of (2) satisfy $e^{(\lambda_0+\mu)T_0} x(0)^T Xx(0) < 1$, it is straightforward to prove by contradiction (see (9)) that $x(t)$ will always remain inside Ω along solutions to (2). The following corollary of Lemma 1 is thus proved.

COROLLARY 1 *Assume that* (10)–(12) *hold. Suppose that* Σ_p *has brief instabilities with instability bound* T_0 *and asymptotic instability* $\alpha < \alpha^*$, *and assume that* $x(0)^T X x(0) \le e^{-(\lambda_0+\mu)T_0}$. *Then, x converges to zero exponentially along solutions of* (2), *without leaving* Ω.

B. Performance

Suppose now that an input u and an extra output z are added to the LPV system considered in Section IIA to obtain the new system

$$\Sigma_p := \begin{cases} \dot{x} = A(p)x + B(p)u \\ y = C(p)x \\ z = D(p)x \end{cases}. \tag{13}$$

The reason why two outputs are included will become clear in the sequel. The variable z denotes the true output of the system under study, whereas y is a fictitious output aimed at restricting the state x to a desired region. The next result quantifies the input/output behavior of the above system. Namely, it provides conditions under which an input signal of bounded energy u will generate an output signal of bounded energy z and sets an upper bound on the ratio of their energy contents. The following standard notation is required.

Given a signal ρ in L_2, we denote by $\|\rho\|_2$ its L_2-norm, i.e., $\|\rho\|_2 = \sqrt{\int_0^\infty \|\rho(t)\|^2 dt}$. Given the LPV system Σ_p of (13), we say that Σ_p is finite gain stable if it maps L_2 to L_2 and the induced operator norm

$$\|\Sigma_p\|_{2,i} = \sup_{p\in P}\sup_{f\in L_2}\left\{\frac{\|\Sigma_p f\|_2}{\|f\|_2} : f \in L_2,\ \|f\|_2 \neq 0\right\}$$

from u to z is well defined and finite. This induced norm captures the worst case of input-output energy amplification when the parameter p undergoes arbitrary trajectories in the parameter space P.

The lemma below shows how to upper-bound the L_2-induced norm of (13) from u to z when p has brief instabilities.

LEMMA 2 *Consider the LPV system* Σ_p *defined by* (13) *and assume there exist positive definite matrices* $R \in \Re^{m\times m}$ *and* $X \in \Re^{n\times n}$ *and positive scalars* λ_0, μ *and* γ *such that*

$$\Im \le -\lambda_0 X, \qquad \forall \quad p \in P_{\text{stable}} \tag{14}$$

$$\Im \le \mu X, \qquad \forall \quad p \in P_{\text{unstable}} \tag{15}$$

and (6) *holds, where*

$$\Im = A(p)^T X + XA(p) + XB(p)B(p)^T X + \frac{D(p)D(p)^T}{\gamma^2}. \tag{16}$$

Suppose p has brief instability with instability bound T_0 *and asymptotic instability ratio* $\alpha < \alpha^*$ *and that u is bounded. Then x and y remain bounded along solutions of* (13) *with*

$$y(t)^T R y(t) \le e^{(\lambda_0+\mu)T_0}\left(x(0)^T X x(0) + \int_0^t \|u(s)\|^2 ds\right), \quad \forall \quad t \ge 0.$$

Moreover, the L_2*-induced norm from u to z is no larger than* $\gamma\sqrt{e^{(\lambda_0+\mu)T_0}\lambda_0/(\lambda_0 - \alpha(\lambda_0+\mu))}$ *and both x and y converge to zero if* $u \in L_2$.

PROOF OF LEMMA 2 Given an arbitrary solution $x(t)$ of (13), let $V(t) := x(t)^T X x(t)$. Consider now an interval (t_1, t_2) on which $p \in P_{\text{stable}}$. The existence of such an interval follows from the fact that p has brief instability. Inequality (14) implies that

$$\dot{V} \le -\lambda_0 V + \|u\|^2 - \|z\|^2\gamma^{-2}$$

on that interval and therefore

$$V(t) \le \frac{V(t_1)}{e^{\lambda_0(t-t_1)}} + \int_{t_1}^t \frac{\|u(\tau)\|^2 - \|z(\tau)\|^2\gamma^{-2}}{e^{\lambda_0(t-\tau)}} d\tau, \quad t \in [t_1, t_2]. \tag{17}$$

Similarly, it follows from (15) that on any interval (t_2, t_3) on which $p \in P_{\text{unstable}}$ the function V satisfies

$$V(t) \le \frac{V(t_2)}{e^{-\mu(t-t_2)}} + \int_{t_2}^t \frac{\|u(\tau)\|^2 - \|z(\tau)\|^2\gamma^{-2}}{e^{-\mu(t-\tau)}} d\tau, \quad t \in [t_2, t_3]. \tag{18}$$

Iterating (17) and (18) over consecutive intervals yields

$$V(t) \le \frac{V(\tau)}{e^{\lambda_0(t-\tau-T_p(\tau,t))-\mu T_p(\tau,t)}} + \int_\tau^t \frac{\|u(s)\|^2 - \|z(s)\|^2\gamma^{-2}}{e^{\lambda_0(t-s-T_p(s,t))-\mu T_p(s,t)}} ds, \quad \forall \quad t \ge \tau \ge 0.$$

Using the above relationship, the two following inequalities are also obtained for $\forall t \ge \tau \ge 0$:

$$V(t) \le \frac{V(\tau)}{e^{\lambda_0(t-\tau-T_p(\tau,t))-\mu T_p(\tau,t)}} + \int_\tau^t \frac{\|u(s)\|^2}{e^{\lambda_0(t-s-T_p(s,t))-\mu T_p(s,t)}} ds \tag{19}$$

and

$$\int_\tau^t \frac{\|z(s)\|^2}{\gamma^2 e^{\lambda_0(t-s-T_p(s,t))-\mu T_p(s,t)}} ds \le \frac{V(\tau)}{e^{\lambda_0(t-\tau-T_p(\tau,t))-\mu T_p(\tau,t)}} + \int_\tau^t \frac{\|u(s)\|^2}{e^{\lambda_0(t-s-T_p(s,t))-\mu T_p(s,t)}} ds. \tag{20}$$

Let $\lambda := \lambda_0 - \alpha(\lambda_0+\mu)$. From the assumptions of the theorem, $\lambda > 0$ and (8) holds true. From (8) and (19) it can be concluded that

$$V(t) \le e^{(\lambda_0+\mu)T_0 - \lambda(\tau - t)} V(\tau) + \int_\tau^t \frac{\|u(s)\|^2}{e^{-(\lambda_0+\mu)T_0 + \lambda(t-s)}} ds, \quad \forall \quad t \ge \tau \ge 0.$$

Furthermore, (6) implies that

$$y(t)^T Ry(t) \leq x(t)^T Xx(t) \leq \frac{x(0)^T Xx(0)}{e^{-(\lambda_0+\mu)T_0+\lambda t}} + e^{(\lambda_0+\mu)T_0}\int_0^t \frac{\|u(s)\|^2}{e^{\lambda(t-s)}}ds.$$

Using (8) in (20) and the fact that $(\lambda_0 + \mu)T_p(s,t) \geq 0$ yields

$$\int_\tau^t \frac{\|z(s)\|^2}{\gamma^2 e^{\lambda_0(t-s)}}ds \leq \frac{V(\tau)}{e^{-(\lambda_0+\mu)T_0+\lambda(t-\tau)}} + \int_\tau^t \frac{\|u(s)\|^2}{e^{-(\lambda_0+\mu)T_0+\lambda(t-s)}}ds, \quad \forall \quad t \geq \tau \geq 0. \tag{21}$$

Integrating both sides of (21) over the interval (τ,∞) gives

$$\frac{1}{\gamma^2}\int_\tau^\infty \int_\tau^t \frac{\|z(s)\|^2}{e^{\lambda_0(t-s)}}ds\,dt \leq \frac{V(\tau)}{e^{-(\lambda_0+\mu)T_0}} + \int_\tau^\infty \int_\tau^t \frac{\|u(s)\|^2}{e^{-(\lambda_0+\mu)T_0+\lambda(t-s)}}ds.$$

Exchanging the order of integration, it is easy to show that

$$\frac{1}{\lambda_0\gamma^2}\int_\tau^\infty \|z(s)\|^2 ds \leq \frac{e^{(\lambda_0+\mu)T_0}}{\lambda}\left(V(\tau) + \int_\tau^\infty \|u(s)\|^2 ds\right),$$

thus completing the proof.

A local version of Lemma 3 is derived next. Take a positive definite matrix $R \in \Re^{m\times m}$ and consider the set Ω of states for which the output y is guaranteed to be in the ellipsoid defined by $y^T Ry < 1$, i.e.,

$$\Omega := \{w \in \Re^n : w^T C(p)^T RC(p)w \leq 1, \ \forall p \in P\}.$$

Suppose now that there exist symmetric positive definite matrices $R \in \Re^{m\times m}$ and $X \in \Re^{n\times n}$ and positive scalars λ_0, μ, and γ such that

$$\mathfrak{J} \leq -\lambda_0 X, \qquad \forall \quad t: \ x(t) \in \Omega \quad \text{and} \quad p(t) \in P_{\text{stable}} \tag{22}$$

$$\mathfrak{J} \leq \mu X, \qquad \forall \quad t: \ x(t) \in \Omega \quad \text{and} \quad p(t) \in P_{\text{unstable}} \tag{23}$$

$$X \geq C(p)^T RC(p), \qquad \forall \quad t: \ x(t) \in \Omega \quad \text{and} \quad p(t) \in P \tag{24}$$

where $\mathfrak{J}$ is defined in (16). The following Corollary of Lemma 3 is then straightforward to derive.

COROLLARY 2 *Assume that* (22)–(24) *hold. Suppose that $\underline{\Sigma}_p$ has brief instability with instability bound T_0 and asymptotic instability ratio $\alpha < \alpha^*$, u is bounded, and* $x(0)^T Xx(0) + \int_0^t \|u(s)\|^2 ds \leq e^{-(\lambda_0+\mu)T_0}$. *Then, x converges to zero along solutions of* (2) *without leaving the set Ω. Furthermore, the L_2-induced norm from u to z is no larger than $\gamma\sqrt{e^{(\lambda_0+\mu)T_0}\lambda_0/\lambda}$.*

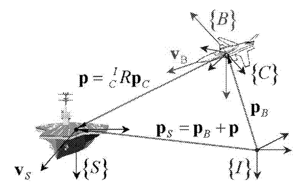

Fig. 1. Coordinate systems.

III. APPLICATION. DESIGN OF INTEGRATED VISION/INERTIAL FILTERS

This section contains the main results of the paper. Specifically, the theoretical results of Section II (on LPV systems with brief instabilities) are applied to the synthesis of a nonlinear filter that estimates the relative position and velocity of a UAV with respect to a given point on a ship. These results extend the work reported in [1] to explicitly account for out-of-frame events.

A. Process Model

Consider an aircraft equipped with an IR vision camera operating in the vicinity of a ship that is moving with a constant velocity (this is a general requirement during shipboard landing operations). The situation is depicted in Fig. 1. Let $\{I\}$ denote an inertial reference, $\{B\}$ a body-fixed frame attached to the aircraft, and $\{C\}$ a camera-fixed frame (we assume without loss of generality that the origins of $\{B\}$ and $\{C\}$ are coincident). The symbol $\{S\}$ denotes a ship-fixed body frame.

Suppose that: 1) the ship's inertial velocity is constant, 2) the ship is always located in front of the UAV's camera, and 3) the height of the ship's deck above the sea surface is negligible when compared with the altitude of the UAV. Following the notation introduced in [1], let $\mathbf{p}_C = [x_C \ \ y_C \ \ z_C]^T$ denote the relative position of the center of $\{C\}$ with respect to $\{S\}$ and let ${}^I_C R$ and ${}^I_B R$ denote rotation matrices from $\{C\}$ to $\{I\}$ and from $\{B\}$ to $\{I\}$, respectively, that we assume are available from the onboard attitude measurement system. Further let $\mathbf{p} = {}^I_C R\mathbf{p}_C$ and ${}^B\mathrm{a}_m$ be the inertial acceleration of the UAV (expressed in body frame $\{B\}$), as available from the onboard inertial navigation system. Using this notation, the underlying process model developed in [1] can be described as

$$G = \begin{cases} \dot{\mathbf{p}} = \mathbf{v} \\ \dot{\mathbf{v}} = -{}^I_B R({}^B\mathbf{a}_m + \mathbf{w}_a) \\ \mathbf{y}_m = g(\mathbf{p}_C) + \mathbf{w}_y \end{cases} \tag{25}$$

where $\mathbf{y}_m \in \Re^3$ denotes the vector of measurements available from the onboard IR camera and altimeter,

$g(\mathbf{p}_C) : \Re^3 \to \Re^3$ is given by

$$g(\mathbf{p}_C) = \begin{bmatrix} \dfrac{f}{x_C} \begin{bmatrix} y_C \\ z_C \end{bmatrix} \\ -\sin\theta x_C - \cos\theta \sin\phi y_C + \cos\theta\cos\phi z_C \end{bmatrix}, \tag{26}$$

and $\mathbf{w}_a$ and $\mathbf{w}_y$ denote process and measurement noise, respectively. In (26), f denotes the focal length of the IR camera, and ϕ and θ represent the roll and pitch angles, respectively, of the IR camera with respect to $\{I\}$.

B. Problem Definition

The practical problem considered in [1] consisted of determining the relative position and relative velocity of an aircraft with respect to a landing site using IR vision and other onboard passive sensors. In [1], the structure of a nonlinear filter was proposed and its stability and performance characteristics were assessed in the presence of measurement noise, but in the absence of out-of-frame events.

However, a realistic scenario suggests that the image of the ship smokestack can be lost temporarily by the onboard camera due, for example, to excessive aircraft rotational motions. It is thus crucial that the results of [1] be extended to deal explicitly with out-of-frame events.

As a first step in this direction, define a binary signal $s : [0,\infty) \to \{0,1\}$ as follows:

$$s(t) := \begin{cases} 0 & \text{out-of-frame event at time } t \\ 1 & \text{camera tracks the smokestack at time } t. \end{cases} \tag{27}$$

Furthermore, for a given binary signal s and $t > \tau > 0$, denote by $T_s(\tau,t)$ the amount of time in the interval (τ,t) that $s = 0$. Formally, $T_s(\tau,t) := \int_\tau^t (1 - s(l))dl$.

The signal s is said to have brief out-of-frame events if $T_s(\tau,t) \le T_0 + \alpha(t-\tau)$, $\forall t \ge \tau \ge 0$, for some $T_0 \ge 0$ and $\alpha \in [0,1]$. Notice the parallel with the definition of brief instability introduced before.

In what follows, due to operational reasons, the vector $\mathbf{p}_C$ is assumed to lie in the compact set

$$P_C = \{\mathbf{p}_C : x_{\min} \le x_C \le x_{\max},\ y_{\min} \le y_C \le y_{\max},\ z_{\min} \le z_C \le z_{\max}\}$$

where the positive numbers $x_{\min},\ldots,z_{\max}$ are determined from the geometry of the problem. As will become clear, filter design will aim at ensuring, among other objectives, that the estimates $\hat{p}_C$ of $\mathbf{p}_C$ also lie in a compact set

$$\begin{aligned} \hat{P}_C = \{\hat{\mathbf{p}}_C = [\hat{x}_C \quad \hat{y}_C \quad \hat{z}_C]^T : &|\hat{x}_C - x_C| \le x_{\max} - x_{\min} + dx, \\ &|\hat{y}_C - y_C| \le y_{\max} - y_{\min} + dy, \\ &|\hat{z}_C - z_C| \le z_{\max} - z_{\min} + dz\} \end{aligned}$$

where dx, dy, and dz are positive real numbers, and $dx < x_{\min}$. Equipped with the above notation, we now introduce a mathematical formulation of the practical problem that is a natural extension of the one in [1].

Problem F1: Regional Stability. Consider the process model (25) and assume that $\mathbf{w}_a = \mathbf{w}_y = 0$. Given sets P_C and $\hat{P}_C$, find $\alpha_0 > 0$ and a causal dynamical system (filter) F that operates on $\mathbf{y}_m$ and ${}^B\mathbf{a}_m$ to produce estimates $\hat{\mathbf{p}}$ of $\mathbf{p}$ and $\hat{\mathbf{v}}$ of $\mathbf{v}$ in the presence of brief out-of-frame events, such that:

$$\hat{\mathbf{p}}_C(t) \in \hat{P}_C \qquad \text{for any} \quad t > 0$$
$$\|\hat{\mathbf{p}}_C - \mathbf{p}_C\| + \|\hat{\mathbf{v}} - \mathbf{v}\| \to 0 \qquad \text{as} \quad t \to \infty,$$

provided that

$$\|[(\hat{\mathbf{p}}_C(0) - \mathbf{p}_C)^T \quad (\hat{\mathbf{v}}(0) - \mathbf{v}(0))^T]^T\| < \alpha_0. \tag{28}$$

Problem F2: Regional Stability and Performance. Consider the process model (25) with the noise vector $\mathbf{w} := [\mathbf{w}_a^T \ \ \mathbf{w}_y^T]^T : \|\mathbf{w}\|_2 \le \bar{\omega}$, $\bar{\omega} > 0$. Let the sets P_C and $\hat{P}_C$ of allowable position and estimation vectors respectively, defined before, be given. Given $\gamma > 0$ and $\alpha_0 > 0$, find a stable filter F that operates on $\mathbf{y}_m$ and ${}^B\mathbf{a}_m$ to obtain estimates $\hat{\mathbf{p}}$ of $\mathbf{p}$ and $\hat{\mathbf{v}}$ of $\mathbf{v}$ in the presence of brief out-of-frame events, such that if (28) holds, the filter satisfies the following conditions for all $\mathbf{w} \in L_2$, $\|\mathbf{w}\|_2 \le \bar{\omega}$:

$$\hat{\mathbf{p}}_C(t) \in \hat{P}_C \qquad \text{for all} \quad t \ge 0$$
$$\|\hat{\mathbf{p}}_C - \mathbf{p}_C\| + \|\hat{\mathbf{v}} - \mathbf{v}\| \to 0 \qquad \text{as} \quad t \to \infty^3$$
$$\|T_{ew}\|_{2,i} < \gamma,$$

where $T_{e_1 w} : \mathbf{w} \to \mathbf{e}_1$ is a closed-loop operator from disturbances to the position estimation errors $\mathbf{e}_1 := \hat{\mathbf{p}}_C - \mathbf{p}_C$ (a bound on $\mathbf{e}_1$ will be computed later in the Proof of Theorem 1).

C. Proposed Solution

This section describes the solutions to problems F1 and F2. For technical reasons, it is necessary to assume that

$$r_x = \frac{x_{\max} - x_{\min} + dx}{x_{\min}} < 1. \tag{29}$$

We start by presenting a solution to problem F1.

THEOREM 1 *Let P_C and r_x be given. Assume that* (29) *holds. Suppose there exists a matrix $X = X^T \in \Re^{6\times 6}$ and positive constants T_0, α, α_0, λ_0, and μ such that*

[3] As long as $\mathbf{w} \in L_2$ we always obtain convergence to zero.

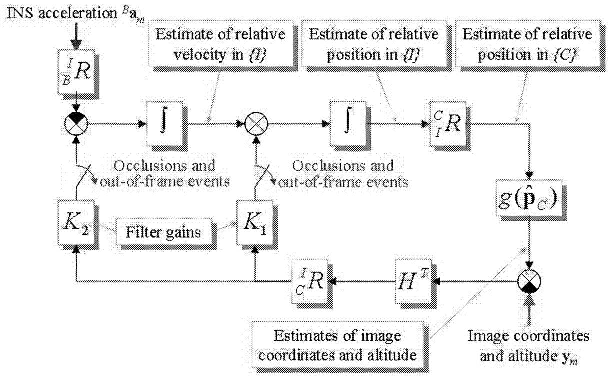

Fig. 2. Filtering structure: filters F_1 and F_2.

$\alpha < (\lambda_0/\lambda_0 + \mu)$ and

$$X > 0 \tag{30}$$

$$F^T X + XF - 2(1 - r_x)^2 \varepsilon C^T C \le -\lambda_0 X \tag{31}$$

$$F^T X + XF \le \mu X \tag{32}$$

$$X - \delta^{-2} C^T C \ge 0 \tag{33}$$

$$\alpha_0^{-2} e^{-(\lambda_0+\mu)T_0} I - X \ge 0 \tag{34}$$

where

$$F := \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$$

$$C := [I \ \ 0]$$

$$\delta := \min\{x_{\max} - x_{\min} + dx,\ y_{\max} - y_{\min} + dy,\ z_{\max} - z_{\min} + dz\}$$

and

$$\varepsilon := \min_{\hat{\mathbf{p}}_C \in \hat{P}_C} \lambda_{\min}(H^T(\hat{\mathbf{p}}_C) H(\hat{\mathbf{p}}_C)) = \min_{\hat{\mathbf{p}}_C \in \hat{P}_C} \lambda_{\min} \xi(\hat{\mathbf{p}}_C) \tag{35}$$

where $H(\mathbf{p}_C)$ denotes the Jacobian of $g(\mathbf{p}_C)$ with respect to $\mathbf{p}_C$ and $\xi(\hat{\mathbf{p}}_C) := H^T(\hat{\mathbf{p}}_C) H(\hat{\mathbf{p}}_C)$.

Define the filter (see Fig. 2)

$$F_1 := \begin{cases} \dot{\hat{\mathbf{p}}} = \hat{\mathbf{v}} + sK_1 {}^I_C R H^T(\hat{\mathbf{p}}_C)(g(\hat{\mathbf{p}}_C) - \mathbf{y}_m) \\ \dot{\hat{\mathbf{v}}} = -B^I R^B \mathbf{a}_m + sK_2 {}^I_C R H^T(\hat{\mathbf{p}}_C)(g(\hat{\mathbf{p}}_C) - \mathbf{y}_m) \\ \hat{\mathbf{p}}_C = {}^C_I R \hat{\mathbf{p}} \end{cases} \tag{36}$$

where s is defined by (27) and

$$K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} := -X^{-1}(1 - r_x) C^T. \tag{37}$$

Then, filter F_1 solves the filtering problem F1 for brief out-of-frame events characterized by the parameters T_o and α, provided that (28) holds.

REMARK 1 Theorem 1 is an extension of Theorem 4 in [1] to out-of-frame events. The key difference is the addition of inequality (32) and of the positive constants T_0, α, λ_0, and μ. As a result, the matrix inequalities (30)–(34) are nonlinear in the parameters X, T_0, λ_0, and μ. In contrast, the matrix inequalities obtained in Theorem 4 of [1] are linear.

REMARK 2 Notice how the proposed filter complements the information from the vision and air data sensors with that available from the inertial navigation system (INS) (see Fig. 2). In the presence of out-of-frame events, the filter simply integrates the inertial acceleration to obtain an estimate of the relative position (dead reckoning). Once a reliable image is reacquired the integrators are reset based on the new vision data.

REMARK 3 The solvability of inequality (31) is addressed in [1]. There, it is shown that the inequality has a solution if and only if $r_x < 1$.

PROOF OF THEOREM 1 Define the error state $\mathbf{e} = [\mathbf{e}_1^T \ \ \mathbf{e}_2^T]^T$ where $\mathbf{e}_1 := \hat{\mathbf{p}} - \mathbf{p}$ and $\mathbf{e}_2 := \hat{\mathbf{v}} - \mathbf{v}$. Simple algebra (see [1]) shows that the error dynamics can be written as

$$\frac{d}{dt}\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} = (F + sK^I_C R \pi(\hat{\mathbf{p}}_C, \mathbf{p}_C)^I_C R^T C)\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} \tag{38}$$

where the operator $\pi(\hat{\mathbf{p}}_C, \mathbf{p}_C)$ is defined by $\pi(\hat{\mathbf{p}}_C, \mathbf{p}_C) = H^T(\hat{\mathbf{p}}_C)\operatorname{diag}(\hat{x}_C x_C^{-1}, \hat{y}_C y_C^{-1}, 0) H(\hat{\mathbf{p}}_C)$. Notice that the error dynamics correspond to an LPV system with parameters s and $\mathbf{p}_C$. To show that $\hat{\mathbf{p}}_C \in \hat{P}_C$, it is sufficient to prove that $\|\mathbf{e}_1\| \le \delta$ or, equivalently, that $\mathbf{e}$ remains in the set $\Omega := \{\mathbf{e} \mid \|C\mathbf{e}\| \le \delta\}$. From Corollary 1, it can be concluded that this is true provided there exists a matrix $X > 0$ and constants λ_0 and μ such that (32) and (33) hold,

$$(F + K^I_C R \pi(\hat{\mathbf{p}}_C, \mathbf{p}_C)^I_C R^T C)^T X + X(F + K^I_C R \pi(\hat{\mathbf{p}}_C, \mathbf{p}_C)^I_C R^T C) \le -\lambda_0 X \tag{39}$$

is satisfied for all times for which $\mathbf{e} \in \Omega$ and $s = 1$, and

$$[\mathbf{e}(0)_1^T \ \ \mathbf{e}(0)_2^T] X [\mathbf{e}(0)_1^T \ \ \mathbf{e}(0)_2^T]^T \le e^{-(\lambda_0+\mu)T_0}. \tag{40}$$

Inequality (40) follows from $\|[\mathbf{e}(0)_1^T \ \ \mathbf{e}(0)_2^T]^T\| \le \alpha_0$ and (34). In the following, we focus on the solvability of (39). Given (37) it is straightforward to conclude that (39) is equivalent to

$$F^T X + XF + \begin{bmatrix} -2(1 - r_x)^I_C R \pi(\hat{\mathbf{p}}_C, \mathbf{p}_C)^I_C R^T & 0 \\ 0 & 0 \end{bmatrix} \le -\lambda_0 X. \tag{41}$$

In the set Ω we have $\|\mathbf{e}_1\| \le \delta$ and therefore

$$\operatorname{diag}(\hat{x}_C x_C^{-1}, \hat{y}_C y_C^{-1}, 1) := I + \begin{bmatrix} (\hat{x}_C - x_C)x_C^{-1} & 0 & 0 \\ 0 & (\hat{x}_C - x_C)x_C^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix} > 1 - r_x. \tag{42}$$

Thus,

$$-2(1 - r_x)^I_C R \pi(\hat{\mathbf{p}}_C, \mathbf{p}_C)^I_C R^T < -2(1 - r_x)^{2I}_C R \xi(\hat{\mathbf{p}}_C)^I_C R^T < -2(1 - r_x)^2 \varepsilon I. \tag{43}$$

Because of this and the fact that

$$C^TC = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},$$

we conclude that (31) implies (41) in the set Ω. Now, from Corollary 1 it follows that $\|\mathbf{e}_1(t)\| \le \delta$, $\forall t \ge 0$ and $\mathbf{e}_1(t), \mathbf{e}_2(t) \to 0$ as $t \to \infty$.

The next theorem provides a solution to the filtering problem F2.

THEOREM 2 *Let P_C be given. Assume that* (29) *holds. Given $\gamma > 0$, suppose there exists a matrix $X = X^T \in \Re^{6\times 6}$ and positive constants T_0, α, α_0, λ_0 and μ such that $\alpha < (\lambda_0/\lambda_0 + \mu)$ and*

$$X > 0 \tag{44}$$

$$\begin{bmatrix} F^TX + XF + \lambda_0 X + \left(\dfrac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda\gamma^2} I - (1-r_x)^2\varepsilon\right)C^TC & XF^T \\ FX & -I \end{bmatrix} \le 0 \tag{45}$$

$$F^TX + XF + XF^TFX + \frac{e^{(\lambda_0+\mu)T_0}}{\lambda\gamma^2}C^TC \le \mu X \tag{46}$$

$$X - \delta^{-2}C^TC \ge 0 \tag{47}$$

$$\alpha_0^{-2}(e^{-(\lambda_0+\mu)T_0} - \bar{\omega}^2)I - X \ge 0 \tag{48}$$

where

$$F := \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$$

$$C := [I \;\; 0]$$

$$\delta := \min\{x_{\max} - x_{\min} + dx,\; y_{\max} - y_{\min} + dy,\; z_{\max} - z_{\min} + dz\}$$

and

$$\varepsilon := \min_{\hat{\mathbf{p}}_C \in \hat{P}_C} \lambda_{\min}(\xi(\hat{\mathbf{p}}_C)).$$

Define the filter (see Fig. 2*)*

$$F_2 := \begin{cases} \dot{\hat{\mathbf{p}}} = \hat{\mathbf{v}} + sK_1{}^I_CRH^T(\hat{\mathbf{p}}_C)(g(\hat{\mathbf{p}}_C) - \mathbf{y}_m) \\ \dot{\hat{\mathbf{v}}} = -{}^I_BR^B\mathbf{a}_m + sK_2{}^I_CRH^T(\hat{\mathbf{p}}_C)(g(\hat{\mathbf{p}}_C) - \mathbf{y}_m) \\ \hat{\mathbf{p}}_C = {}^C_I R\hat{\mathbf{p}} \end{cases} \tag{49}$$

where

$$K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} := -X^{-1}(1-r_x)C^T. \tag{50}$$

Then, filter F_2 solves the filtering problem F2 for brief out-of-frame events characterized by the parameters T and α, provided that (28) *holds.*

REMARK 4 Theorem 2 is an extension of Theorem 5 in [1] to out-of-frame events. As in the case of Theorem 2, the key difference is the addition of inequality (46) and of the positive constants T_0, α, λ_0 and μ. As a result, the matrix inequalities (44)–(48) are nonlinear in the parameters X, T_0, λ_0 and μ. Again, this is in contrast to the matrix inequalities obtained in Theorem 5 in [1].

PROOF OF THEOREM 2 Using the same notation as in Theorem 1, the error dynamics can be written as (see [1])

$$\frac{d}{dt}\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} = (F + sK^I_CR\pi(\hat{\mathbf{p}}_C, \mathbf{p}_C){}^I_CR^TC)\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} + \left(\begin{bmatrix} 0 \\ {}^I_BR \end{bmatrix} - sK^I_CRH^T(\hat{\mathbf{p}}_C)\right)\mathbf{w}. \tag{51}$$

We now show that if inequalities (44)–(48) are satisfied and (28) holds, then $\hat{\mathbf{p}}_C \in \hat{P}_C$ for all $\mathbf{w} \in L_2$ for which $\|\mathbf{w}\|_2 \le \bar{\omega}$.

To prove that $\hat{\mathbf{p}}_C \in \hat{P}_C$, it is sufficient to show that $\|\mathbf{e}_1\| \le \delta$ or, equivalently, that $[\mathbf{e}_1^T \;\; \mathbf{e}_2^T]^T$ remains in $\Omega := \{\mathbf{e} \mid \|C\mathbf{e}\| \le \delta\}$. From Corollary 2 (with $\rho := \gamma\sqrt{\lambda/e^{(\lambda_0+\mu)T_0}\lambda_0}$) it can be concluded that this is true provided that there exists a matrix $X > 0$ and constants λ_0, μ such that (46) and (47) hold,

$$\begin{aligned}(F + K^I_CR\pi(\hat{\mathbf{p}}_C, \mathbf{p}_C){}^I_CR^TC)^TX + X(F + K^I_CR\pi(\hat{\mathbf{p}}_C, \mathbf{p}_C){}^I_CR^TC) \\ + X(F^TF + K^I_CRH^T(\hat{\mathbf{p}}_C)H(\hat{\mathbf{p}}_C){}^I_CR^TK^T)X + \frac{e^{(\lambda_0+\mu)T_0}}{\lambda\gamma^2}C^TC \le -\lambda_0 X\end{aligned} \tag{52}$$

is satisfied for all times for which $[\mathbf{e}_1^T \;\; \mathbf{e}_2^T]^T \in \Omega$ and $s = 1$, and

$$[\mathbf{e}(0)_1^T \quad \mathbf{e}(0)_2^T]X[\mathbf{e}(0)_1^T \quad \mathbf{e}(0)_2^T]^T + \int_0^\infty \|\mathbf{w}(s)\|^2 ds \le e^{-(\lambda_0+\mu)T_0}. \tag{53}$$

In (46) and to obtain (52) the following identity was used:

$$\begin{bmatrix} 0 & 0 \\ {}^I_BR & 0 \end{bmatrix}\begin{bmatrix} 0 & {}^I_BR \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} = F^TF.$$

Inequality (53) follows from $\|[\mathbf{e}(0)_1^T \;\; \mathbf{e}(0)_2^T]^T\| \le \alpha_0$, $\|\mathbf{w}\|_2 \le \bar{\omega}$, and (48). We now discuss the solvability of (52). Equation (50) yields

$$XK = -(1-r_x)C^T$$

and therefore (52) is equivalent to

$$F^TX + XF + XF^TFX + \begin{bmatrix} \dfrac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda\gamma^2}I + (1-r_x)^2{}^I_CR\xi(\hat{\mathbf{p}}_C){}^I_CR^T \\ -2(1-r_x){}^I_CR\pi(\hat{\mathbf{p}}_C, \mathbf{p}_C){}^I_CR^T & 0 \\ 0 & 0 \end{bmatrix} \le -\lambda_0 X.$$

Using the first inequality in (43), the above matrix inequality reduces to

$$F^TX + XF + XF^TFX + \begin{bmatrix} \dfrac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda\gamma^2}I - (1-r_x)^2{}^I_CR\xi(\hat{\mathbf{p}}_C){}^I_CR^T & 0 \\ 0 & 0 \end{bmatrix} \le -\lambda_0 X.$$

Furthermore, using the second inequality in (43) and the fact that

$$C^T C = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

it is straightforward to show that the latter inequality holds if

$$F^T X + XF + XF^T FX + \frac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda\gamma^2} - (1-r_x)^2\varepsilon \leq -\lambda_0 X. \tag{54}$$

By applying Schur complements [9] and from definition (35), (54) holds because of (45). The theorem then follows from Corollary 2.

The next theorem derives necessary and sufficient conditions under which (45) is satisfied.

THEOREM 3 *Let F, γ and ε and be defined in Theorem* 2. *Then, $\exists X = X^T > 0$ such that*

$$\begin{bmatrix} F^T X + XF + \lambda_0 X + \left(\frac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda\gamma^2} I - (1-r_x)^2\varepsilon\right) C^T C & XF^T \\ FX & -I \end{bmatrix}$$

$$\leq 0 \Leftrightarrow \frac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda\gamma^2} - (1-r_x)^2\varepsilon \leq 0.$$

Notice how Theorem 3 *entirely avoids searching over the parameter space in equations* (44)–(48). *This is done by exploring the geometry of the problem under consideration, namely using indentity* (19) *in* [1].

PROOF OF THEOREM 3 The results are obtained by rescaling γ in the proof of Theorem 5 in [1].

REMARK 5 Theorem 3 shows that the LMI (45) is feasible if and only if

$$\frac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda\gamma^2} I - (1-r_x)^2\varepsilon \leq 0 \Leftrightarrow \gamma^2 \geq \frac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda(1-r_x)^2\varepsilon}.$$

Recall that

$$\varepsilon = \min_{\hat{\mathbf{p}}_C \in \hat{P}_C} \{\|\lambda_{\min}(\xi(\hat{\mathbf{p}}_C))\|\} \leq \min_{\mathbf{p}_C \in P_C} \{\|\lambda_{\min}(H^T(\mathbf{p}_C)H_T(\mathbf{p}_C))\|\}$$
$$= \max_{\mathbf{p}_C \in P_C} \{\|(H^T(\mathbf{p}_C)H_T(\mathbf{p}_C))^{-1}\|\}^{-1}.$$

and therefore

$$\gamma^2 \geq \frac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda(1-r_x)^2} = \max_{\mathbf{p}_C \in P_C} \{\|(H^T(\mathbf{p}_C)H_T(\mathbf{p}_C))^{-1}\|\}. \tag{55}$$

This inequality imposes a lower bound on the achievable values of γ. Furthermore, since $\lambda := \lambda_0 - \alpha(\lambda_0 + \mu)$, it follows that

$$\lim_{T_0\to 0, \alpha\to 0} \frac{e^{(\lambda_0+\mu)T_0}\lambda_0}{\lambda(1-r_x)^2} = \frac{1}{(1-r_x)^2}.$$

The above expression shows that the lower bound on the achievable γ in the absence of out-of-frame events converges to the lower bound derived in [1].

The bound derived in (55) bears close affinity to the classical positional dilution of precision (PDOP) metric that is commonly used in navigation systems to determine a lower bound on the achievable error covariance as a function of geometry of the underlying navigation problem [10–12]. Using our notation, the PDOP for the problem at hand can be written as

$$\mathrm{PDOP} = \sqrt{\mathrm{tr}(H^T(\mathbf{p}_C)H(\mathbf{p}_C))^{-1}}.$$

We therefore see that the new bound derived here captures a worst case performance scenario and the estimate of x_C increases the lower bound on the achievable γ, since $1 > (1-r_x)^2 > 0$.

D. Numerical Implementation and Performance Studies

In the absence of out-of-frame events ($\alpha = 0$, $T_0 = 0$) the matrix inequalities developed in Theorem 2 can be reduced to the set

$$X > 0$$
$$\begin{bmatrix} F^T X + XF + \left(\frac{1}{\gamma^2} - (1-r_x)^2\varepsilon\right) C^T C & XF^T \\ FX & -I \end{bmatrix} < 0$$
$$\frac{1}{\alpha_0^2}(1-\bar{\omega}^2) - X > 0$$
$$X - \delta^{-2} C^T C > 0$$

where ϵ is inversely proportional to the "size" of $\hat{P}_C$, γ determines the filter's performance, and α_0 is the bound on the initial error in position and velocity estimates.

From a design standpoint, one would like to minimize ϵ and maximize α_0. In what follows we solve a related problem that allows for the study of tradeoffs involved in filter design. This is done by letting $w_1 = \gamma^2$, $w_2 = 1/\alpha_0^2$, $w_3 = \varepsilon$ and defining the cost functional $J = c_1 w_1 + c_2 w_2 + c_3 w_3$, where c_1, c_2, c_3 are positive weights to be selected by the designer. This leads naturally to the following convex optimization problem that can be solved numerically using MATLAB's LMI toolbox [13]:

$$\text{find} \quad \min J \quad \text{subject to}$$
$$X > 0$$
$$\begin{bmatrix} F^T X + XF - (1-r_x)^2 w_3 C^T C & C^T & XF^T \\ C & -w_1 & 0 \\ XF & 0 & -I \end{bmatrix} < 0$$
$$w_2(1-\bar{\omega}^2) - X > 0$$
$$X - \delta^{-2} C^T C > 0.$$

For a given choice of weights c_i; $i = 1,2,3$ the resulting values of X, α_0 and γ can then be used

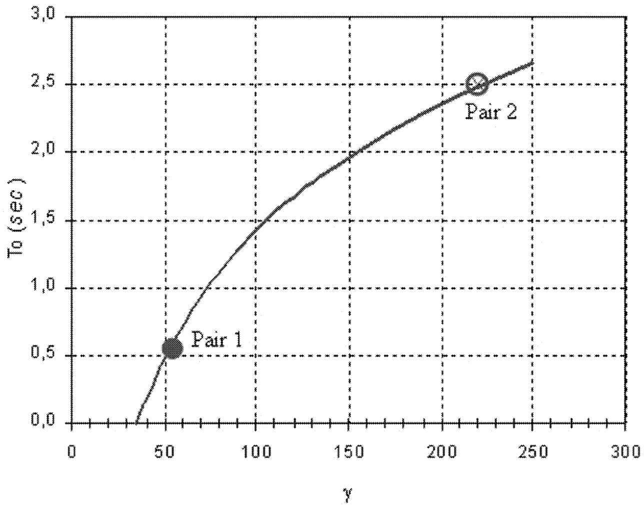

Fig. 3. Achievable γ versus T_0.

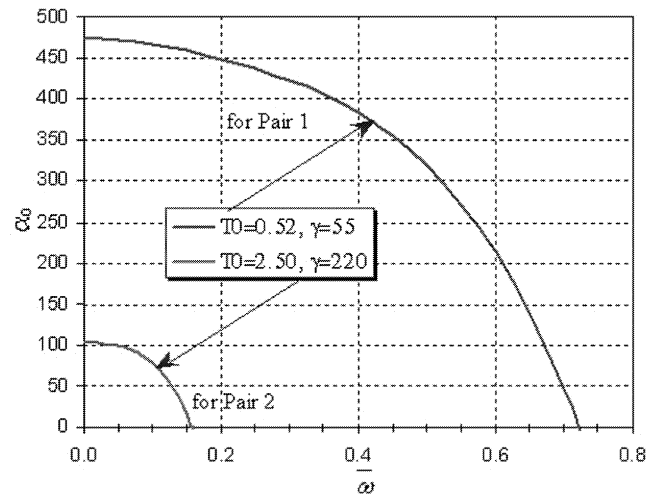

Fig. 4. Achievable α_0 versus $\bar{\omega}$.

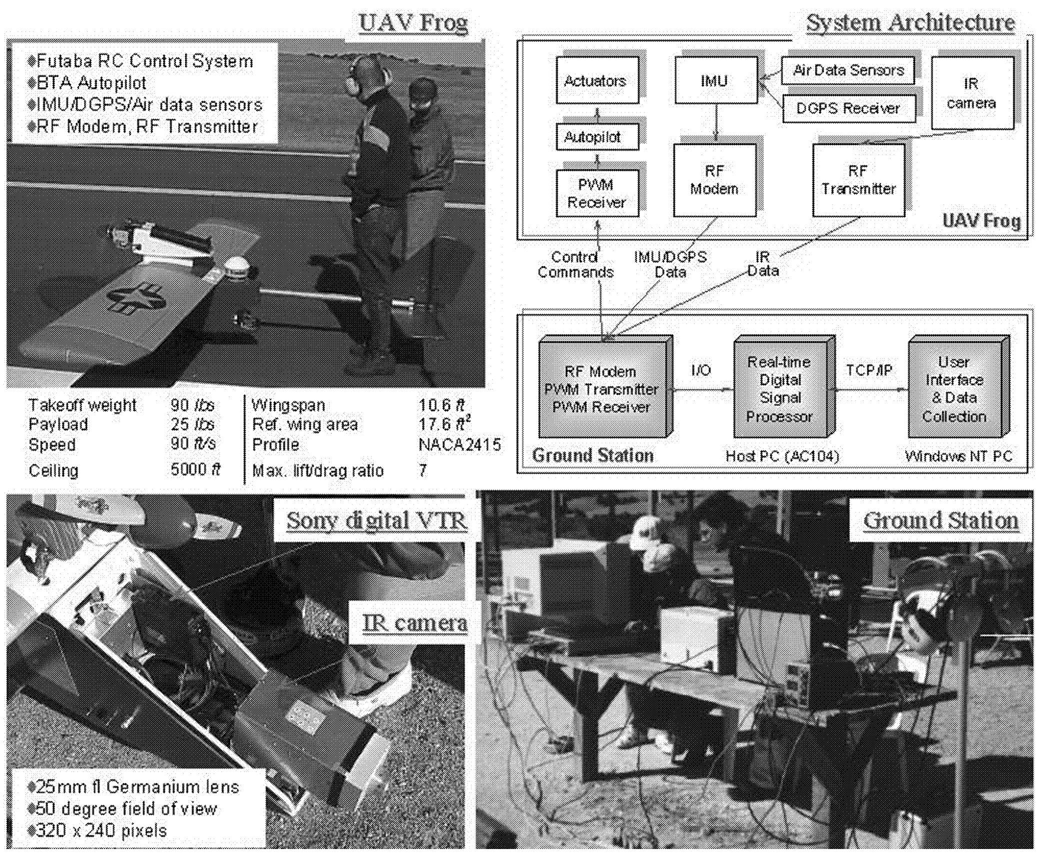

Fig. 5. Experimental setup.

to study the impact of out-of-frame events on filter performance using equations (45)–(48) in Theorem 2. This was done here by inserting the solutions X, α_0 obtained above into equations (45)–(48) and studying the evolution of γ as a function of out-of-frame parameters T_0 and α_0.

In a particular design example that we considered, and in the absence of out-of-frame events, the value of the performance bound γ achieved with the filter was 35. However, in the presence of out-of-frame events the value of γ increases as a function of T_0 as illustrated in Fig. 3. The numerical values of α obtained were on the order of $10^{-5}-10^{-4}$ and their impact on the levels of achievable γ is thus negligible. Furthermore, as the graph in Fig. 3 suggests, T_0 exhibits logarithmic dependence on γ. This implies that for values of $T_0 > 2.5$ s small increases in T_0 result in large increases in achievable γ, in other words, recovery from out-of-frame events that exceed 2.5 s becomes very difficult.

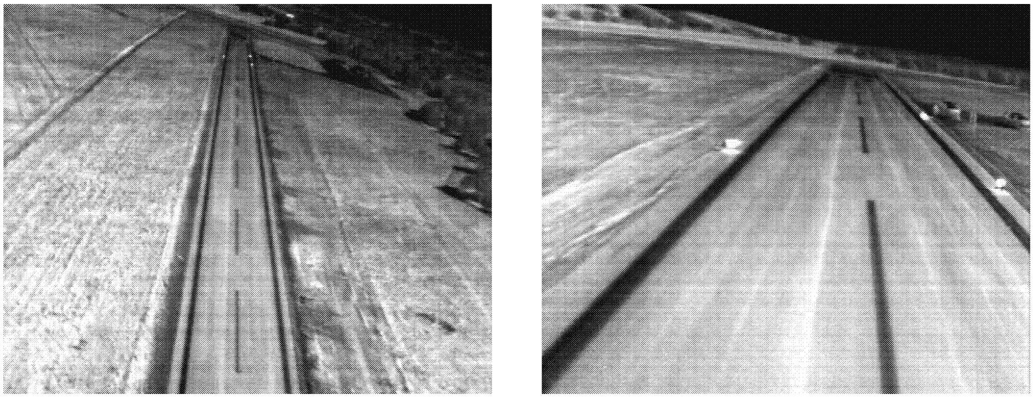

Fig. 6. Examples of IR images. (a) At range of 450 m. (b) 80 m.

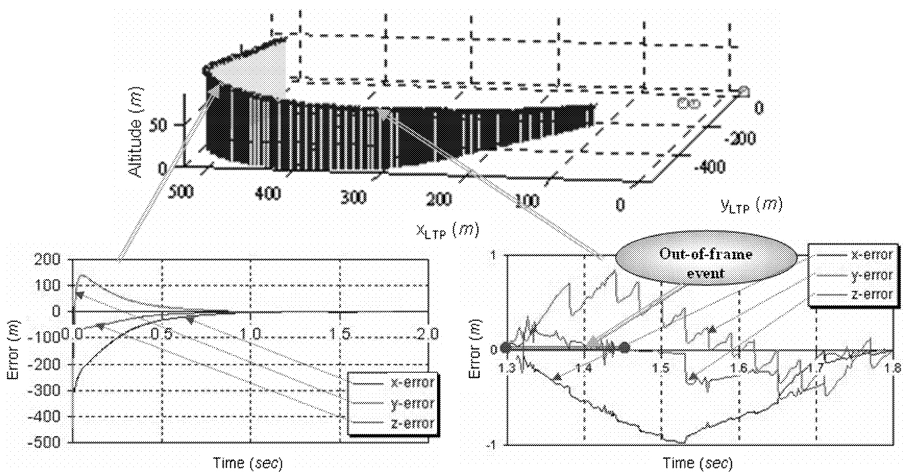

Fig. 7. Performance of filter during final approach.

Another interesting trade-off is shown in Fig. 4, where for two value pairs of $(T_0,\gamma) = (0.52,55)$, $(T_0,\gamma) = (2.5,250)$ the graphs of $\bar{\omega}$ versus α_0 are plotted. Recall, in this work α_0 defines the bound on the norm of the initial estimation error (28), while $\bar{\omega}$ defines the bound on the norm of the sensor noise. Fig. 4 shows the trade-off between the size of the initial estimation error tolerated by the filter and the bound on the sensor noise. Clearly, as T_0 increases the achievable values of $\bar{\omega}$ and α_0 decrease.

E. Experimental Setup and Flight-Test Results

This section describes briefly the experimental setup and the flight test experiments that were executed to evaluate the performance of the nonlinear filter obtained in the previous section. The Frog UAV operated by the controls lab at Naval Postgraduate School was equipped with an IR video camera (pixel resolution of 320×240), a DGPS receiver, an IMU, and a pressure altitude sensor. Fig. 5 illustrates the complete experimental setup that includes the UAV, onboard avionics schematics and ground station. (Reference [14] contains a complete description of the onboard sensor suite and the performance characteristics of the UAV.)

A charcoal grill was used to model the hot spot on a ship and an image-processing algorithm was developed to find and track the grill observed by the airborne IR camera. Fig. 6 includes examples of the images taken by the IR camera. A detailed description of the image-processing algorithm can be found in [14]. The image plane coordinates and GPS altitude were used by the integrated IR/Inertial filter to compute the relative position and velocity with respect to the hot spot.

Fig. 7 shows the results of applying the integrated IR/Inertial filter to the flight test data. In particular, the upper graph shows the DGPS landing approach trajectory. The bottom left graph shows estimation errors computed by comparing the DGPS position with the position estimates produced by the filter. It shows rapid convergence of the position estimation errors, down to a few centimeters in less than a second from the time the hot spot was acquired by the onboard vision system. The bottom right graph shows the filter's response to an out-of-frame event. It zooms in on the position estimation errors in the interval between 1.3 s and 1.8 s. The out-of-frame event took place in the interval of [1.3 s, 1.45 s] and resulted in the gradual increase in the x and y position errors. The filter quickly recovered once the image of the hot spot was reacquired. Notice that the errors in the z channel remained small due to uninterrupted availability of barometric altitude sensor.

IV. CONCLUSIONS

This paper introduced the concept of LPV systems with brief instabilities and derived new results for stability and performance analysis of such systems, where performance is evaluated in terms of L_2 induced norms. The main results show that stability and performance can be assessed by examining the feasibility of parameterized sets of nonlinear matrix inequalities. These results were applied to the design of an integrated IR/inertial/air data navigation filter with guaranteed stability and performance in the presence of out-of-frame events. Numerical trade-off studies were conducted to determine the filter's achievable performance versus the duration of the out-of-frame events. Finally, the filter was tested using flight test data collected by a UAV equipped with air data and inertial sensors as well as an IR camera. The results of the test showed that the filter performs well in the presence of out-of-frame events. Future work will aim at extending these results to address the problem of determining the relative position, velocity, and orientation of multiple vehicles flying in formation by using vision, passive sensors, and other navigational data available through the inter-vehicle network. Vision is expected to play a major role in situations where the network quality of service deteriorates.

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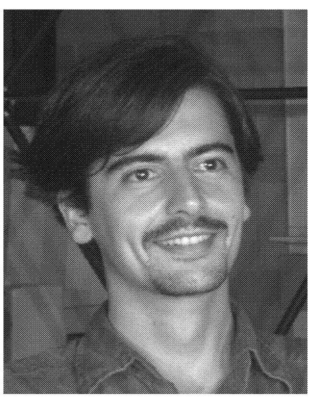

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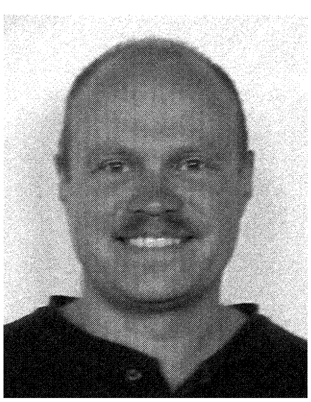

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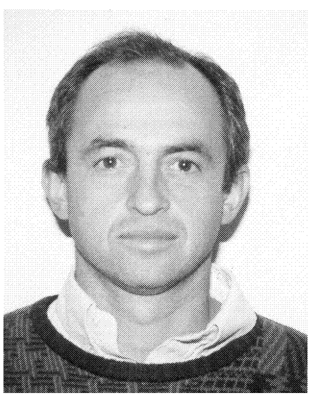

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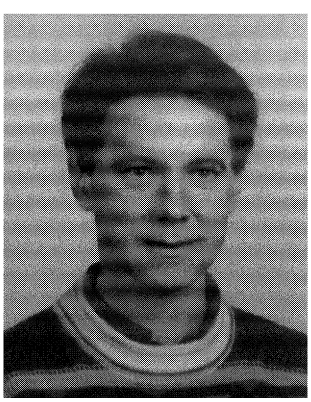

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