Distributed Fault Detection Using Relative Information in Linear Multi-Agent Networks ⋆

Daniel Silvestre * **Paulo Rosa** ** **João P. Hespanha** *** **Carlos Silvestre** ****

* *Dep. of Electrical and Computer Engineering, Instituto Superior Técnico, ISR, 1046-001 Lisboa, Portugal, (e-mail: dsilvestre@isr.ist.utl.pt).*
** *Deimos Engenharia, Lisbon, Portugal, (e-mail: paulo.rosa@deimos.com.pt).*
*** *Dept. of Electrical and Computer Eng., University of California, Santa Barbara, CA 93106-9560, USA, (e-mail: hespanha@ece.ucsb.edu)*
**** *Department of Electrical and Computer Engineering of the Faculty of Science and Technology of the University of Macau, Macao, China, on leave from Instituto Superior Técnico/Technical University of Lisbon, 1049-001 Lisbon, Portugal (e-mail: csilvestre@umac.mo)*

Abstract: This paper addresses the problem of fault detection in the context of a collection of agents performing a shared task and exchanging relative information over a communication network. A techniques in the literature is used to construct a meaningful observable system equivalent to the original unobservable system of systems. A solution involving Set-Valued Observers (SVOs) and a left-coprime factorization of the system is proposed to estimate the state in a distributed fashion and a proof of convergence of the estimates is given under mild assumptions. The performance of the detection algorithm against deterministic and stochastic faults; and when the system is subject to unmodeled disturbances is assessed through simulations.

Keywords: FDI for linear systems, FDI for discrete-event systems, FDI theory for networked systems.

1. INTRODUCTION

The problem of detecting faults in a group of dynamical systems cooperating over a network is considered in this paper. The motivation for this work is to provide tools to facilitate the distributed fault detection task in a system with sub-systems with a shared objective and only sharing relative information. The importance of this problem is reported in Fax and Murray (2004) and later in Massioni and Verhaegen (2009), where the detection is crucial as a single malfunctioning node can severely impact the overall network performance. Applications span the areas of mobile robots, cooperating unmanned vehicles tasks such as surveillance and reconnaissance, distributed state estimation, among others (see Menon and Edwards (2014) and the references therein).

In Menon and Edwards (2014), one of the main results is showing that the overall system is unobservable when only considering relative information of the states of the individual subsystems. A transformation is introduced that allows to perform fault detection and isolation by considering the observable subspace of the overall system. The algorithm requires a centralized detection scheme and in this paper, we aim at giving an alternative approach based on Set-Valued Observers (SVOs) which enables a distributed detection for the observable subspace.

In Silvestre et al. (2013), the use of SVOs for distributed fault detection were firstly introduced for the specific case of consensus. The overall system is modeled as a Linear Parameter-Varying (LPV) system where communications are seen as a parameter-dependent dynamics matrix. Even though, the whole system is not observable in every time instant, for a sufficiently long time interval, the system is observable as long as the underlying network topology is strongly connected. Whereas in Silvestre et al. (2013), each node has access to its own state and one of the neighbor states to which it communicates, in this paper, it is assumed that nodes have access only to relative information. The distributed detection can also be improved by resorting to exchanging state estimates whenever the systems communicate or take measurements by using a similar algorithm to the one presented in Silvestre et al. (2014).

The Set-Valued Observers (SVOs) framework, whose concept was introduced in Witsenhausen (1968) and Schweppe (1968) (further information can be found in Schweppe (1973) and Milanese and Vicino (1991) and references therein) is used as a way to represent and propagate the set-valued state estimates. The approach accommodates any kind of linear dynamics for the agents, and also incorporates disturbances and sensor noise.

The main contributions of this paper are as follows:

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- the use of SVOs to compute the set-valued state estimates for the observable subsystem in a distributed fashion;
- it is shown how the nodes can estimate only their neighbors or the whole system;
- we show two options for addressing the case where the system is unobservable but detectable with its corresponding features.

The remainder of this paper is organized as follows. In Section 2, we describe the problem of distributed detection using relative information and its motivation, as well as the main issues involved in its formulation. The proposed solution is discussed in Section 3. The main convergence result is given in Section 4 and illustrated in simulations through Section 5. Concluding remarks and directions of future work are provided in Section 6.

Notation : The transpose of a matrix A is denoted by $A^\intercal$. We let $\mathbf{1}_n := [1 \dots 1]^\intercal$ and $\mathbf{0}_n := [0 \dots 0]^\intercal$ indicate n-dimensional vector of ones and zeros, respectively, and I_n denotes the identity matrix of dimension n. Dimensions are omitted when clear from context. The symbol $\otimes$ denotes the kronecker product. The notation $||.||$ refers to $\|v\| := \sup_i |v_i|$ for a vector, and $\|A\| := \bar{\sigma}(A)$.

2. PROBLEM STATEMENT

We consider the same problem as described in Menon and Edwards (2014), namely, a group of N dynamic systems interacting according to a bidirectional network topology. The graph has N vertices representing each of the n-dimensional dynamic system. In this paper, each subsystem is described by a Linear Time-Invariant (LTI) model S_i of the form:

$$S_i : \begin{cases} x_i(k+1) = Ax_i(k) + Bu_i(k) + Df_i(k) + Ed_i(k) \\ y_{ij}(k) = C(x_i(k) - x_j(k)), j \in \mathcal{J}_i \end{cases}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, represent the state and input signal of the ith subsystem. The unknown sequences f_i and d_i represent the fault and disturbance signal. Without loss of generality, it is assumed that $|d_i| \leq 1$. In addition, it is considered that $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{n\times q}$ and $E \in \mathbb{R}^{n\times r}$.

Remark 1. An interesting observation is that the difficulty of this problem relies on the fact that the dynamics matrices are equal for all of the subsystems, which renders the overall system unobservable.

The output of the ith system depends on all its neighbors j, $j \in \mathcal{J}_i$:

$$y_i = \sum_{j\in\mathcal{J}_i} C(x_i - x_j)$$

which motivates the introduction of the graph laplacian matrix defined as

$$\mathcal{L}_{ii} = |\mathcal{J}_i|, \quad \mathcal{L}_{ij} = \begin{cases} -1, & \text{if } j \in \mathcal{J}_i \\ 0, & \text{if } j \notin \mathcal{J}_i \end{cases}$$

where $|\mathcal{J}_i|$ is the number of neighbors of node i. By combining the state equations, the overall system is described by

$$\begin{aligned} x(k+1) =& \underbrace{(I_N \otimes A)}_{A_N} x(k) + \underbrace{(I_N \otimes B)}_{B_N} u(k) \\ &+ \underbrace{(I_N \otimes D)}_{D_N} f(k) + \underbrace{(I_N \otimes E)}_{E_N} d(k) \\ y(k) =& \underbrace{(\mathcal{L} \otimes C)}_{C_N} x(k) \end{aligned}$$

where $x := [x_1^\intercal \cdots x_N^\intercal]^\intercal$, $u := [u_1^\intercal \cdots u_N^\intercal]^\intercal$, $f := [f_1^\intercal \cdots f_N^\intercal]^\intercal$, $d := [d_1^\intercal \cdots d_N^\intercal]^\intercal$ and $y := [y_1^\intercal \cdots y_N^\intercal]^\intercal$. As shown in Lemma 1 of Menon and Edwards (2014), such a system is always unobservable and a transformation is proposed to extract the observable subsystem in the following fashion.

Let

$$T := T_s^{-1} \otimes I_n$$

where

$$T_s^{-1} := \begin{bmatrix} 1 & \mathbf{0}_{N-1}^\intercal \\ -\mathbf{1}_{N-1} & I_{N-1} \end{bmatrix}.$$

The important step is that when applied to the laplacian matrix we get

$$T_s^\intercal \mathcal{L} T_s = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{L}_r \end{bmatrix}$$

and an observable system is now defined as

$$\begin{aligned} \bar{x}(k+1) =& \underbrace{(I_{N-1} \otimes A)}_{\bar{A}_N} \bar{x}(k) + \underbrace{(I_{N-1} \otimes B)}_{\bar{B}_N} \bar{u}(k) \\ &+ \underbrace{(I_{N-1} \otimes D)}_{\bar{D}_N} \bar{f}(k) + \underbrace{(I_{N-1} \otimes E)}_{\bar{E}_N} \bar{d}(k) \\ \bar{y}(k) =& \underbrace{(\mathcal{L}_r \otimes C)}_{\bar{C}_N} \bar{x}(k) \end{aligned}$$

where $\bar{x}_i := x_i - x_1$, $\bar{u}_i := u_i - u_1$, $\bar{f}_i := f_i - f_1$ and $\bar{d}_i := d_i - d_1$ for $2 \leq i \leq N$.

The main goal of this paper is to detect signals $f \neq 0$ in a distributed fashion, in the sense that each subsystem i will only have access to its own measurements, which are the differences between its own state and those of its neighbors. We intend to study the case when the system is detectable using two different alternatives and exploiting what are the key features in each one. The case when the system is unobservable but detectable can also be addressed with one of these strategies, which places mild conditions on the sub-systems and relaxes the assumptions made in Menon and Edwards (2014), although we do not address the problem of *fault estimation*, but rather only *fault detection*. The results provided in this paper, although focusing on a particular application example, can be generalized for a wide class of dynamic systems.

3. PROPOSED SOLUTION

For the design of the proposed fault detection solution, we use the Set-Valued Observers (SVOs) framework from Rosa and Silvestre (2011) and Rosa (2011) and define

$$X(k) := \text{Set}(M(k), m(k))$$

where $\text{Set}(M, m) := \{q : Mq \leq m\}$ represents a convex polytope, where the operator $\leq$ is a component-wise operation between the two vectors. The aim of an SVO is to find the smallest set containing all possible states of the system at time k, $X(k)$, with the knowledge that $\forall_{0\leq i<N}, x(k-i) \in X(k-i)$ and that the dynamics of the system are a general description of the type

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Df(k) + Ed(k) \\ y(k) &= Cx(k) + n(k) \end{aligned} \tag{1}$$

where $n(k)$ is the noise in the sensors and $|n(k)| \leq \bar{n}$ by assumption. We assume M_0 and m_0 can be selected such that the initial state satisfies $x(0) \in X(0)$ where $X(0) := \text{Set}(M_0, m_0)$. The set $X(k+1) := \text{Set}(M(k+$

$1), m(k+1))$, which contains all the possible states of the system at time $k+1$, can be described by the set of points, $\mathbf{x}$, satisfying

$$\underbrace{\begin{bmatrix} M(k)A^{-1} & -M(k)A^{-1}E \\ \bar{C} & 0 \\ 0 & \bar{I} \end{bmatrix}}_{M(k+1)} \begin{bmatrix} \mathbf{x} \\ d(k) \end{bmatrix} \leq \underbrace{\begin{bmatrix} m(k) + \tilde{u}(k,1) \\ \bar{y}(k+1) - \bar{n}\bar{I} \\ 1 \end{bmatrix}}_{m(k+1)}$$

where $\tilde{u}(k,H) := \sum_{\tau=1}^{H} M(k)A^{-\tau}Bu(k-\tau+1)$ This procedure assumes an invertible transmission matrix. When this is not the case, we can adopt the strategy in Shamma and Tu (1999) and solve the inequality

$$\begin{bmatrix} \bar{I} & -\bar{A} & -\bar{E} \\ 0 & 0 & \bar{I} \\ \bar{C} & 0 & 0 \\ 0 & M(k) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{x}^- \\ d(k) \end{bmatrix} \leq \begin{bmatrix} \bar{B}u(k) \\ 1 \\ \bar{y}(k+1) - \bar{n}\bar{I} \\ m(k) \end{bmatrix}$$

by applying the Fourier-Motzkin elimination method Keerthi and Gilbert (1987) to remove the dependence on $\mathbf{x}^-$ and obtain the set described by $M(k+1)\mathbf{x} \leq m(k+1)$.

The above computations assume a horizon value $H = 1$. However, they can be extended to include previous measurements of the system and improve detection. Due to the uncertainty in the initial state, including previous measurements may reduce the conservatism of the set-valued state estimate, as shown in Rosa et al. (2013). For the case of invertible matrix A we have inequality (2)

$$M_H(k+1) \begin{bmatrix} \mathbf{x} \\ d(k) \\ \vdots \\ d(k-H+1) \end{bmatrix} \leq \underbrace{\begin{bmatrix} m(k) + \tilde{u}(k,H) \\ \bar{y}(k+1) - \bar{n}\bar{I} \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{m(k+1)}. \quad (2)$$

where $M_H(k+1)$ is defined by

$$M_H(k+1) := \left[\begin{array}{c|ccc} A^{-H} & -M(k)A^{-1}E & \cdots & -M(k)A^{-H}E \\ \bar{C} & 0 & \cdots & 0 \\ \hline 0 & & I \otimes \bar{I} & \end{array}\right]$$

If A is non-invertible, then the following alternative inequality will hold

$$M_H(k+1) \begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}(k) \\ d(k) \\ \mathbf{x}(k-1) \\ d(k-1) \\ \vdots \\ \mathbf{x}(k-H+1) \\ d(k-H+1) \end{bmatrix} \leq m_H(k+1)$$

where

$$M_H(k+1) := \left[\begin{array}{c|cc} M_{H-1}(k+1) & 0 & \\ \hline \begin{matrix} \bar{I} & 0 & \cdots & 0 & -\bar{E} \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & \bar{C} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} -\bar{A}^H & -\bar{A}^{H-1}E \\ 0 & \bar{I} \\ 0 & 0 \\ M(k-H+1) & 0 \end{matrix} \end{array}\right]$$

and

$$m_H(k+1) := \begin{bmatrix} m_{H-1}(k+1) \\ \sum_{\tau=0}^{H-1} \bar{A}^{\tau}Bu(k-\tau) \\ 1 \\ \bar{y}(\eta) - \bar{n}\bar{I} \\ m(k-H+1) \end{bmatrix}.$$

An important issue is that the system must be observable or otherwise the produced set grows without bounds. A possible alternative is to use the concept of left-coprime factorization when the system is detectable which returns two stable and observable systems that represent the whole initial system. For these, it is always possible to design two SVOs and require that the sets intersect or a fault is detected. We also review the concept of left-coprime factorization to use with SVOs as proposed in Rosa et al. (2013) and include it here for completeness, since it will allow us to derive some interesting results regarding the convergence of the overall system state estimates.

Let us define

$$[N \quad M] = \left[\begin{array}{c|cc} zI - A + KC & -K & B - KD \\ \hline RC & R & RD \end{array}\right]$$

where z is the unit step delay, to be a left-coprime factorization for a system with transfer function

$$G(z) := D + C(zI - A)^{-1}B := \left[\begin{array}{c|c} zI - A & B \\ \hline C & D \end{array}\right]$$

for a nonsingular matrix R and K is a gain such that $A - KC$ is stable. Moreover, if the system is observable, one can pick K to be a deadbeat observer O'Reilly (1983). Using the left-coprime factorization, we get $G = N^{-1}M$.

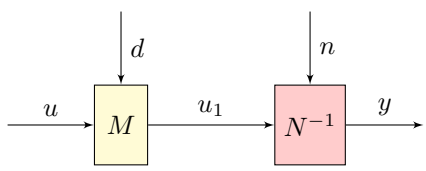

Fig. 1. Schematic representation of the two coprime systems.

Figure 1 depicts the block representation of the left-coprime factorization which creates two separate systems M and N. Let us define an SVO for the system M and for N. Then, a fault is detected if the computed set-valued estimates of u_1 (see Figure 1) for the two systems do not intersect. Using the factorization ensures that both systems M and N are stable even if the original system was not.

Returning to our problem, if the system is detectable, the left-coprime factorization can be used directly Ravi et al. (1990) and we construct an SVO for each of the resulting systems from the factorization and signal a fault whenever the two sets do not intersect. It is stressed that this method does not pose any additional assumptions on the system

dynamics. For instance, the plant to be considered may be unstable, as long as the unstable modes are observable.

As an alternative, we can resort to the transformation presented in Section 2 to obtain an observable subsystem, apply the factorization and follow the same detection procedure. In the next section, we evaluate these two approaches in terms of the set-valued estimates convergence.

4. CONVERGENCE RESULT

In this section, we establish a convergence result for the sets produced by the SVO which bounds the detection time by the number of states of the plant, in the case of an observable system, or that the rate of convergence is governed by the unobservable mode with the highest magnitude eigenvalue. The results imply that old observations are *less important* in the construction of the estimates and we can discard them. In order to formally pose the problem, the following definition is introduced.

Definition 1. A sequence of sets, $U(1), U(2), \cdots$, is said to *converge* if there exist $\epsilon > 0, k_o \geq 1$ such that $vol(U(k)) < \epsilon$ for all $k \geq k_o$. Moreover, if $vol(U(k)) < \Gamma_o \frac{1-\lambda^k}{1-\lambda}$, for some $\Gamma_o, \lambda > 0$, then the sequence of sets is said to have a *convergence governed* by $1/\lambda$.

The next theorem summarizes the convergence properties of the SVOs.

Theorem 2. (estimate convergence). Consider a system G as in (1), with $f \equiv 0$, where $x(k) \in \mathbb{R}^n$, which admits a left-coprime factorization such that $G = N^{-1}M$ and an SVO constructed for M and N providing estimates of u_1. Assume that $x(0) \in X(0)$, and both $|d(k)| \leq 1$, $|n(k)| \leq 1$. Then:

i) the set-valued estimates of u_1 have an infinite convergence rate, if G is observable;
ii) the set-valued estimates of u_1 convergence is governed by $\frac{1}{\lambda_{\max}}$, where $\lambda_{\max} := \max_{\lambda} |\lambda(A-KC)|$, if $\lambda_{\max} < 1$.

Proof.

i) The proof can be found in Rosa et al. (2013) and is due to the fact that for an observable pair (A, C), one can place the eigenvalues of $A-KC$ at the origin and get a deadbeat observer such that $(A-KC)^n = 0$.
ii) Since the system is detectable, one can build a state observer satisfying

$$\hat{x}(k+1) = (A-KC)\hat{x}(k) + [L \quad B] \begin{bmatrix} y(k) \\ u(k) \end{bmatrix}$$

which means that the state estimate can be written based on the previous time instants as

$$\hat{x}(k+1) = (A-KC)^k \hat{x}(0) + \sum_{\tau=0}^{k} (A-KC)^{k-\tau} \; [L \quad B] \; \begin{bmatrix} y(\tau) \\ u(\tau) \end{bmatrix}$$

and an overbound for the set-valued estimate can be written as

$$|\hat{x}(k+1)| \leq \sum_{\tau=0}^{k} \left\| (A-KC)^{k-\tau} \right\| \; \|[L \quad B]\| \; \left\| \begin{bmatrix} y(\tau) \\ u(\tau) \end{bmatrix} \right\|.$$

Since (A, C) is detectable, take $\lambda_{\max}$ as defined in the statement of the theorem, which means $||(A-KC)^k|| \leq c\lambda_{\max}^k$, for some positive constant c and we get

$$\begin{aligned} |\hat{x}(k+1)| &\leq \sum_{\tau=0}^{k} \left\| (A-KC)^{k-\tau} \right\| \, \|[L \quad B]\| \, \left\| \begin{bmatrix} y(\tau) \\ u(\tau) \end{bmatrix} \right\| \\ &\leq \sum_{\tau=0}^{k} c\lambda_{\max}^{k-\tau} \; \|[L \quad B]\| \; \left\| \begin{bmatrix} y(\tau) \\ u(\tau) \end{bmatrix} \right\| \\ &\leq \frac{c(1-\lambda_{\max}^k)}{1-\lambda_{\max}} \; \|[L \quad B]\| \; \max_{0 \leq \tau \leq k} \left\| \begin{bmatrix} y(\tau) \\ u(\tau) \end{bmatrix} \right\| \end{aligned}$$

which concludes the proof since the set-valued estimates is bounded and its worst-case is governed by $1/\lambda_{\max}$.

■

Therefore, by applying directly the design of SVOs with the left-coprime factorization to the detectable case yields a set-valued estimate of the state, whose convergence depends on the unobservable mode with the highest magnitude eigenvalue. The same procedure can apply to any observable system since the coprime factorization produces two stable sub-systems. On the other hand, by exploring the transformation introduced in Menon and Edwards (2014), it is possible to extract, for the particular example considered, an observable subsystem and then perform the design of the SVOs using the gain matrix K of the observer as that of a deadbeat observer to obtain detection within a time horizon of n. Notice that this case applies to any unobservable system as long as we apply a transformation and extract only the observable subspace.

5. SIMULATION RESULTS

In this section, we present a set of simulations illustrating the fault detection mechanism described in this article. In particular, we are interested in showing the distributed detection paradigm where each subsystem runs its own SVO to detect faults. This option reduces the dependability of a single point of detection, but increases the aggregated computational power, since each nodes acts as a detector.

We recover the example described in Menon and Edwards (2014), but, as previously mentioned, considering only the problem of fault detection. Each subsystem is a flexible link robot dynamical system modeled as:

$$\begin{bmatrix} \dot{\theta}_m^i \\ \dot{\omega}^{i_m} \\ \dot{\theta}_\ell^i \\ \dot{\omega}^{i_\ell} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_\ell}{J_m} & -\frac{B}{J_m} & \frac{K_\ell}{J_m} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_\ell}{J_\ell} & 0 & -\frac{L_\ell}{J_m} - \frac{mgh}{J_\ell} & 0 \end{bmatrix} \begin{bmatrix} \theta_m^i \\ \omega^{i_m} \\ \theta_\ell^i \\ \omega^{i_\ell} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_\tau}{J_m} \\ 0 \\ 0 \end{bmatrix} u^i + \begin{bmatrix} 0 \\ \frac{K_\tau}{J_m} \\ 0 \\ 0 \end{bmatrix} f^i + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{mgh}{J_\ell} \end{bmatrix} d^i$$

$$y_i = \sum_{j \in \mathcal{J}_i} C(x_i - x_j)$$

for $i \leq N$ and $C = [I_3 \; 0_{3\times 1}]$. The states represent the angular position and velocity of the motor shaft (θ_m^i and ω^{i_m}), and the angular position and velocity of the link (θ_ℓ^i and ω^{i_ℓ}). For further details of the subsystems model, the interested reader is referred to Menon and Edwards (2014) and the references therein. The network is selected at random with 25 nodes with a maximum degree of 3 and, the system is discretized using a sampling time of 0.01 seconds, and the simulations are run for 100 discrete time steps. The simulations are the result from the computation at node 1.

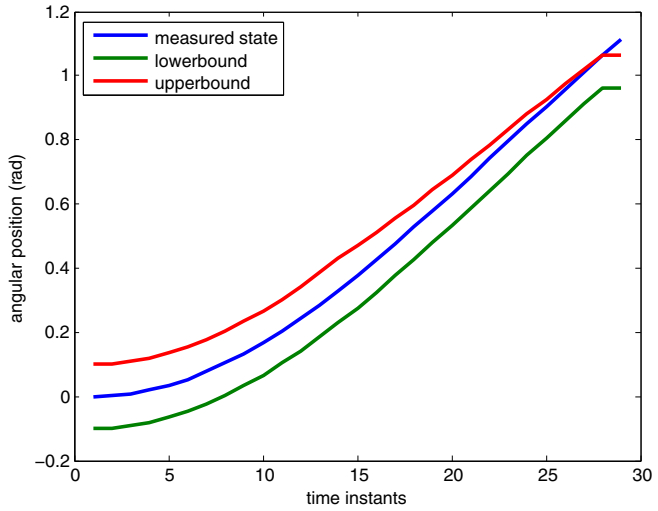

Fig. 2. Example of a simple fault detection where the state of the system (blue line) crosses the upperbound (red line) of the state given through the projection of the set-valued estimate onto the corresponding coordinate.

We consider three different scenarios: one where one of the subsystems has an actuator fault represented by a constant fault signal; a second where this fault is random across time; and, a last one where no fault is injected, but the predefined bounds for the disturbance are not satisfied. Each scenario aims to present a different aspect of the detection algorithm.

Figure 2 depicts the detection of the algorithm using the SVOs. The red and green lines represent the upper and lower bounds for the state, respectively. These are obtained by projecting the set of estimates onto the coordinate corresponding to this variable. When the state of the system crosses one of the bounds, the corresponding observation will produce an empty set, as none of the admissible state realizations is compatible with the input/output sequences.

In Figure 3, it is shown the reported detection time for the case of a constant actuator fault, as a function of the associated amplitude. As soon as the magnitude of the signal goes slightly over the bound considered for the disturbances, the fault is detected, as the model with $f = 0$ is not capable of generating state realizations compatible with this fault.

Based on the previous results, we investigated harder fault profiles. In a sense, the faults that are harder to detect are likely behaving as the modeled disturbances. Following this reasoning, we consider the case where the fault is stochastic with normal distribution and we vary the maximum magnitude to see its impact. Figure 4 shows the detection time for the simulated case. It is noticeable that the detection requires a higher magnitude than the constant case, due to the fact that the fault signal magnitude is going to be lower most of the times.

It is stressed that the SVOs provide means to tackle a wide range of models for the dynamic system. We take advantage of this fact to further evaluate the proposed method in a more demanding scenario. By definition, every fault is going to be detected as long as the measurements do not comply with the assumed fault-free model. For this reason, we introduced unmodeled stochastic disturbances in the state of the system. Notice that in the dynamics of the subsystems, the disturbances only affect the variable $\omega^{i\ell}$ which makes the detection troublesome.

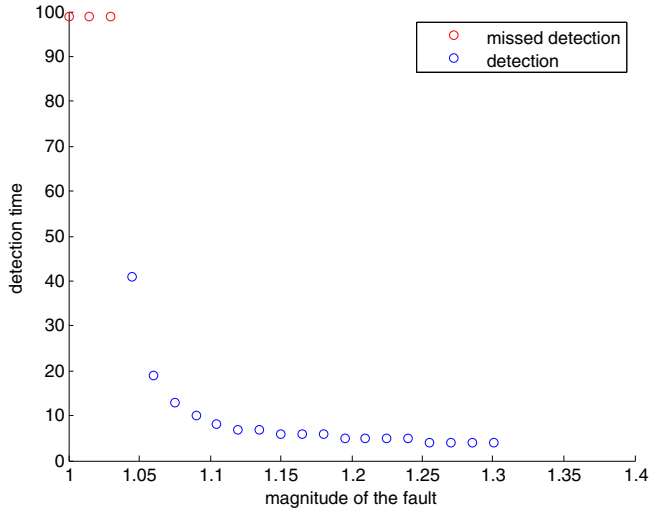

Fig. 3. Reported detection times when varying the magnitude of a constant fault.

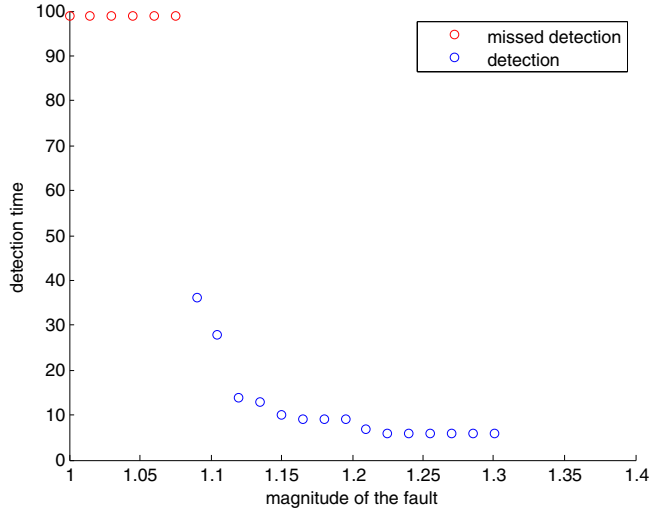

Fig. 4. Reported detection times when varying the maximum magnitude of a random fault.

Figure 5 shows the detection times for the unmodeled disturbances case. The fault is only detected when its maximum magnitude reaches 2, which is clearly above the previous required magnitude values for the fault signal.

A last important feature of the algorithm is its convergence, which we showed to depend on the largest eigenvalue if the system is detectable. In Figure 6, we present the bounds given by the SVO when the disturbance and noise signals were equal to zero and without performing the coprime factorization. The tight bounds mean the SVOs produce sets that converge when the system is observable. We also tested performing the coprime factorization for the observable subspace and without removing the unobservable modes using the left-coprime factorization proposed in Ravi et al. (1990) and obtained hypervolumes very close to zero which showed that, in this particular example, the SVOs converge, in the sense of Definition 1, much faster than the rate in Theorem 2.

6. CONCLUSIONS AND FUTURE WORK

This paper addressed the problem of detecting faults in a distributed system environment which is composed of multiple dynamic subsystems. Moreover, it is assumed

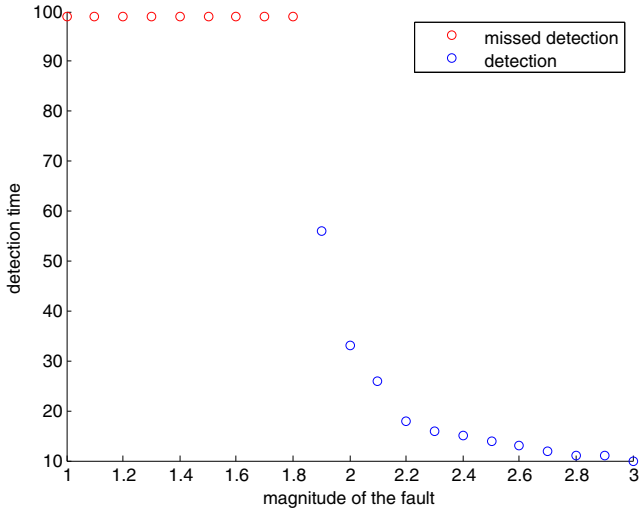

Fig. 5. Reported detection time for a fault free system but with unmodeled disturbances.

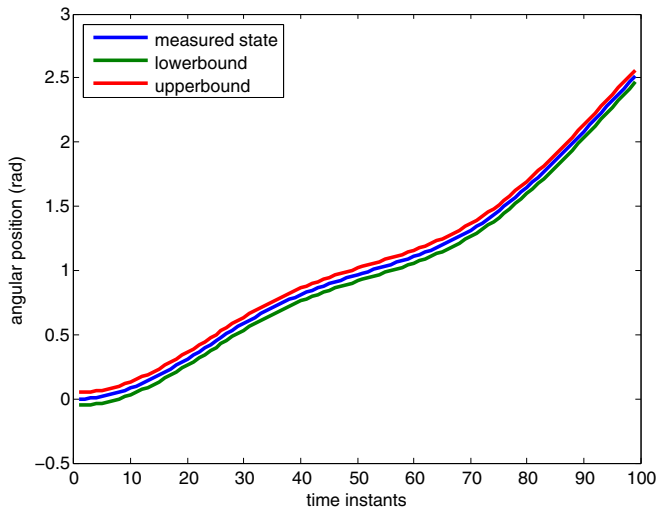

Fig. 6. Lower and upper bounds of the set-valued estimates when not in the presence of disturbances.

that only relative measurements are available. From recent literature, this problem is difficult since the overall system is not observable and a technique for extracting its observable subspace has been introduced.

The solution adopted herein revolves around the concept of Set-Valued Observers, due to their ability to cope with asynchronous measurements and allow general models for the subsystems. The generated sets converge according to a given convergence definition and do not require any type of design choice such as defining threshold faults.

Simulations have shown that when the maximum magnitude of a fault exceeds the disturbance bounds, the detection occurs and the time before declaring the faulty state goes near to 1 discrete time instant. Both constant and stochastic faults are simulated using a flexible link robot model for each subsystem. In addition, the SVOs are capable of detecting unmodeled disturbances and declare faults whenever the model is not compliant with the measurements. The main shortcoming of the proposed method is its computation time in the sense that other observers can be computed faster. Thus, for time-sensitive applications, SVOs are not suitable unless the discretization interval can be made larger.

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