Supervision of Families of Nonlinear Controllers*

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Abstract

A high-level controller called a "supervisor," is defined for the purpose of orchestrating the switching of a sequence of candidate controllers into feedback with an imprecisely modeled process so as to stabilize it. The controller selection is made by (i) continuously comparing in real time suitably defined "normed" output estimation errors or "performance signals" and (ii) placing in the feedback-loop, from time to time, that candidate controller whose corresponding performance signal is the smallest. The paradigm is a manifestation of the idea of certainty equivalence from parameter adaptive control.

1 Problem Formulation

The process Σ_P is modeled by a k-input, n-output, n-dimensional nonlinear dynamical system

$$\dot{x} = A_\omega(x, u), \qquad\qquad y = C_\omega(x) \tag{1}$$

where ω is an uncertain constant parameter taking values in a finite set $\mathcal{P} \triangleq \{p_1, \dots, p_m\}$. The functions $A_p(\cdot)$, $C_p(\cdot)$ $(p \in \mathcal{P})$ are given and it is assumed that for each $p \in \mathcal{P}$, $C_p(x)$ is invertible[1]. We take as given a family of candidate state feedback control laws $F_p(\cdot)$ $(p \in \mathcal{P})$ which, at least, stabilize the system $\dot{z} = A_p\big(z, F_p(z)\big)$. In fact, we demand a little bit more:

Assumption 1.1 *There exist constants $\lambda, \mu > 0$, and two families of nonnegative continuously differentiable functions $V_p(\cdot), W_p(\cdot)$ $(p \in \mathcal{P})$ such that for all $p \in \mathcal{P}$, $V_p(\cdot)$ is positive definite and radially unbounded, $W_p(0) = 0$, and*

$$\frac{\partial V_p}{\partial x}(x) \Big(A_p\big(x - d, F_p(x-d)\big) - \lambda\, d\Big) \le -2\mu V_p(x) + W_p(d)\,, \quad d, x \in \mathbb{R}^n$$

This requirement not only stipulates that $F_p(\cdot)$ stabilizes the unperturbed system $\dot{z} = A_p\big(z, F_p(z)\big)$, but that it input-to-state stabilizes [1] the perturbed system

$$\dot{z} = A_p\big(z - d, F_p(z-d)\big) - \lambda\, d, \qquad p \in \mathcal{P}, \tag{2}$$

as well. Our reason for imposing this requirement will become apparent shortly. Suffice it to say that this requirement is satisfied, for example, by any globally feedback linearizable process model, provided $F_p(\cdot)$ is chosen properly [2]. Note that the control $u \triangleq F_\omega\big(C_\omega^{-1}(y)\big)$ asymptotically stabilizes (1). However, implementation of $u \triangleq F_\omega\big(C_\omega^{-1}(y)\big)$ is not possible, because the value of ω is not presumed to be know. What's to be used instead is the "multi-controller" $\Sigma_C(\sigma)$ defined by the feedback law

$$u \triangleq F_\sigma\big(C_\sigma^{-1}(y)\big) \tag{3}$$

where σ is a suitably defined piecewise-constant *switching signal* taking values in $\mathcal{P}$. The problem of interest then, is to develop a provably correct, "high-level" controller called a *supervisor* which, without a-priori knowledge of ω, is capable of choosing σ so as to cause x to tend to zero.

Remark. The fact that each $V_p(\cdot)$ is continuous, positive definite, and radially unbounded guarantees that there exists a family of positive definite functions $\Gamma_p(\cdot)$ $(p \in \mathcal{P})$ such that $\Gamma_p\big(V_p(x)\big) \ge \|x\|^2$ for all $x \in \mathbb{R}^n$.

2 Estimator-Based Supervisor

The supervisor proposed consists of a *multi-estimator* Σ_E, a *performance signal generator* Σ_Π, and a *switching logic* Σ_S. The underlying decision making strategy used by this supervisor is basically the following: from time to time, select for σ that candidate control index q whose corresponding performance signal π_q is the smallest among π_p, $p \in \mathcal{P}$.

2.1 Multi-Estimator

The multi-estimator Σ_E consists of a family of n-dimensional estimators of the form

$$\dot{x}_p = -\lambda\big(x_p - C_p^{-1}(y)\big) + A_p\big(C_p^{-1}(y), u\big), \quad p \in \mathcal{P} \tag{4}$$

$$e_p = x_p - C_p^{-1}(y) \tag{5}$$

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[1] This is not tantamount to assuming that x can be measured since ω is not presumed to be known.

Because of (1) and the defining equations (4)-(5), $\dot{e}_\omega = -\lambda e_\omega$. Hence, for any input u, the estimation error associated with ω converges to zero exponentially fast.

Suppose q is the value of σ on some interval $[t_1, t_2)$. During this period, the closed-loop differential equations that result from application of the certainty equivalence control law (3) to (1) and (4), together with the equation for e_q, can be written in the form

$$\dot{\bar{x}} = \bar{A}(\bar{x}), \qquad e_q = \bar{C}(\bar{x}) \tag{6}$$

where $\bar{x} \stackrel{\triangle}{=} [x' \; x'_{p_1} \; \cdots \; x'_{p_m}]'$, and $\bar{A}(\cdot)$, $\bar{C}(\cdot)$ are suitably defined functions. Under the assumption that the functions $A_p(\cdot)$, $C_p(\cdot)$, $C_p^{-1}(\cdot)$, $F_p(\cdot)$, $\Gamma_p(\cdot)$ have polynomial growth rate[2], it can be shown that the system (6) is detectable[3] through the estimation error e_q [2]. In particular, because of the input-to-state stability requirement, one can construct positive definite, radially unbounded, continuously differentiable functions $\bar{V}, \bar{W}$ on $\mathbb{R}^{(m+1)n}$ and $\mathbb{R}^n$ respectively and a positive number γ, such that

$$\frac{\partial \bar{V}}{\partial \bar{x}}(\bar{x})\bar{A}(\bar{x}) \leq -\gamma \bar{V}(\bar{x}) + \bar{W}\big(\bar{C}(\bar{x})\big)$$

It is worth noting that detectability of (6) does *not* depend on ω being an element of $\mathcal{P}$. In other words detectability has nothing to do with "exact matching."

2.2 Performance Signals Generator

The performance signal generator Σ_Π to be considered consists of the following family of one-dimensional differential equations

$$\dot{\pi}_p = \bar{W}(e_p), \qquad \pi_p(0) = 0, \qquad p \in \mathcal{P}$$

Under appropriate assumptions, state-sharing [3, 5] can be used to generate the performance signals more efficiently and extend this formulation to situations in which $\mathcal{P}$ has infinitely many elements.

2.3 Switching Logic

To specify the switching logic Σ_S it is necessary to first pick a number $h > 0$ called the *hysteresis constant*. The hysteresis switching logic Σ_S is then defined by the computer diagram[4] shown in Figure 1 where $\rho(t) = \min_{i \in \{1,\ldots,m\}}\{i : \pi_{p_i}(t) \leq \pi_p(t),\; p \in \mathcal{P}\}$. The functioning of Σ_S is roughly as follows: Suppose that at some time t_i, Σ_S has just changed the value of σ to $q \in \mathcal{P}$. The switching signal σ is then held fixed at this value unless, and until, there is a time $t_{i+1} > t_i$ at which $\pi_p + h \leq \pi_q$ for some $p \in \mathcal{P}$. If this occurs, σ is set equal to p and so on.

[2] A function $f : \mathbb{R}^n \to \mathbb{R}^m$ has *polynomial growth rate* if there exist constants $a_1, a_2 > 0, k \geq 1$ such that for all $x \in \mathbb{R}^n$, $\|f(x)\| \leq a_1\|x\| + a_2\|x\|^k$.

[3] With detectability defined as in [3] {see also [4]}.

[4] In interpreting this diagram it is understood that σ's value at each of its switching times $\bar{t}$ is equal to its limit from above as $t \downarrow \bar{t}$. Thus if t_i and t_{i+1} are any two successive switching times, then σ is constant on $[t_1, t_{i+1})$.

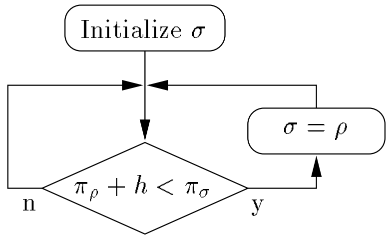

Figure 1: Computer Diagram of Σ_S.

3 Main Result

The following theorem [2] is a straightforward consequence of the Hysteresis Switching Lemma [6] and detectability of (6) through the estimation error.

Theorem 3.1 *There exists a time T^* beyond which the value of σ becomes constant and $\bar{x} \to 0$ as $t \to +\infty$.*

4 Concluding Remarks

The use of supervisory control to feedback stabilize an imprecisely modeled process seems to have important advantages over conventional adaptive techniques: Unlike conventional adaptive control, supervisory control can deal with nonlinearly parameterized models. Supervisory control makes use of off-the-shelf {non-adaptive} controllers rather than highly specialized controllers tailored to the specific needs of continuously-tuned conventional adaptive algorithms.

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