

# DECEPTION IN NON-COOPERATIVE GAMES

## WITH

## PARTIAL INFORMATION<sup>†</sup>

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### Abstract

*In this paper we explore how deception can be used by rational players in the context of non-cooperative stochastic games with partial information. We show that, when one of the players can manipulate the information available to its opponents, deception can be used to increase the player's payoff by effectively rendering the information available to its opponent useless. However, this is not always the case. Paradoxically, when the degree of possible manipulation is high, deception becomes useless against an intelligent opponent since it will simply ignore the information that has potentially been manipulated. This study is carried out for a prototype problem that arises in the control of military operations, but the ideas presented are useful in other areas of applications, such as price negotiation, multi-object auctioning, pursuit-evasion, etc.*

## 1 Introduction

Competitive games are usually classified as either having full or partial information. In *full-information* games both players know the whole state of the game when they have to make decisions. By state, we mean all information that is needed to completely describe the future evolution of the game, when the decision rules used by both players are known. Examples of full information games include Chess, Checkers, and Go. *Partial-information* games differ from these

in that at least one of the players does not know the whole state of the game. Poker, Bridge, and Hearts are examples of such games. In full information games, as a player is planning its next move, it only needs to hypothesize over its and the opponent's future moves to predict the possible outcomes of the game [1]. This is key to using dynamic programming to solve full-information games. Partial information games are especially challenging because this reasoning may fail. In many partial information games, to predict the possible outcomes of the game, a player must hypothesize not only on the future moves of both players, but also on the past moves of the opponent. This often leads to a tremendous increase in the complexity of the games. In general, partial information stochastic games are poorly understood and the literature is relatively sparse. Notable exceptions are games with lack of information for one of the players [2, 3] and games with particular structures such as the Duel game [4], the Rabbit and Hunter game [5], the Searchlight game [6, 7], etc.

Another issue that makes partial information games particularly interesting is the fact that a player can obtain future rewards by either one of two possible mechanisms:

1. Choosing an action that will take the game to a more favorable state;
2. Choosing an action that will make the other player act in our own advantage by making it believe that the game is in a state other than the actual one.

The latter corresponds to a deception move and is only possible in partial information games.

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The potential use of deception has been recognized in several areas, such as price negotiation [8, 9], multi-object auctioning [10], pursuit-evasion [11, 12], human relations [13], and card games [14]. In [8, 9], the authors analyze a negotiation where the players do not know each other’s payoffs, but receive estimates from their opponents. In order to increase its gain each player may bias the estimate given. In [9], an advising scheme is proposed to make deception mostly useless. In [10], it is analyzed how a bidder can use deception to lower the price of an item sold in a multi-object auction. A pursuit-evasion game is analyzed in [11], where the evader corrupts the information available to its opponent to gain an advantage. It is assumed that the evader can jam the pursuer’s sensor and therefore induce measurement errors, produce false targets, or interrupt the observations. Deception has also been studied in the context of military operations [11, 15, 16, 17, 18, 19]. A notable historical event was operation Overlord during the Second World War that culminated with the D-day invasion of France in June 1944 by the allied forces. The success of this operation relied heavily on deceiving the German command regarding the time and place of the sea-borne assault [19]. It is also widely recognized that, by the end of the cold war, the trend in Soviet naval electronic warfare was changing toward an independent type of combat action instead of a purely support role. This new role emphasized radio deception and misinformation at all levels of command [15]. The detection of false targets or decoys is now an important area of research in radar systems [16, 18].

In this paper, we analyze the use of deception in the framework of non-cooperative stochastic games with partial information. We take as a working example a prototype problem that arises in the control of military operations. In its simplest form, this game is played by an attacker that has to select one of several alternative targets and a defender that must distribute its defensive assets among them. This is a partial information game because the attacker has to make a decision without knowing precisely how the defense units have been distributed among the potential targets. We explore several variations of this game that differ in the amount of information available to the attacker. This can range from no information at all to perfect information provided, e.g., by intelligence, surveillance, or reconnaissance. The interesting cases happen between these two extremes because, in practice, the information available is not perfectly accurate and is often susceptible to manip-

ulation by the opponent. It turns out that when the defender can manipulate the information available to its opponents—e.g., by camouflaging some of its defensive units and not camouflaging others—deception can be used to increase its payoff by effectively rendering the information available to the attacker useless.

The remaining of this paper is organized as follows. In Section 2, we formally introduce the simplest version of the prototype game where both attacker and defender have no information available to use in their decisions. This will serve as the baseline to compare the deception games that follow. In Section 3, we consider the extreme situation where the defender completely controls the information available to the attacker. We show that none of the players profits from this new information structure and deception is useless. This situation changes in Section 4 where the defender may profit from using deception. Paradoxically, when the degree of possible manipulation is high, deception becomes useless against an intelligent opponent since it will simply ignore the information that has potentially been manipulated. Section 5 contains some concluding remarks and directions for future research. A full version of this paper is available as a technical report [20].

## 2 A Prototype Non-Cooperative Game

Consider a game between two players that pursue opposite goals. The *attacker* must choose one of two possible targets (A or B) and the *defender* must decide how to better defend them. We assume here that the defender has a finite number of assets available that can be used to protect the targets. To make these assets effective, they must be assigned to a particular target and the defender must choose how to distribute them among the targets. To raise the stakes, we assume that the defender only has three defense units and is faced with the decision of how to distribute them among the two targets. We start by assuming that both players make their decisions independently and execute them without knowing the choice of the other player. Although it is convenient to regard the players as “attacker” and “defender,” this type of games also arise in non-military applications. For example, the “attacker” could be trying to penetrate a market that the “defender” currently dominates.

The game described above can be played as a zero-

sum game defined by the cost below, which the attacker tries to minimize and the defender tries to maximize:

$$J := \begin{cases} c_0 & \text{no units defending the target attacked} \\ c_1 & \text{one unit defending the target attacked} \\ c_2 & \text{two units defending the target attacked} \\ c_3 & \text{three units defending the target attacked.} \end{cases}$$

Without loss of generality we can normalize these constants to have  $c_0 = 0$  and  $c_3 = 1$ . The values for the constants  $c_1$  and  $c_2$  are domain specific. Here, we consider arbitrary values for  $c_1$  and  $c_2$ , subject to the reasonable constraint that  $0 < c_1 \leq c_2 < 1$ . Implicit in the above cost is the assumption that both targets have the same strategic value. We only make this assumption for simplicity of presentation.

As formulated above, the attacker has two possible choices (attack A or attack B) and the defender has a total of four possible ways of distributing its units among the two targets. Each choice available to a player is called a *pure policy* for that player. We will denote the pure policies for the attacker by  $\alpha_i$ ,  $i \in \{1, 2\}$ , and the pure policies for the defender by  $\delta_j$ ,  $j \in \{1, 2, 3, 4\}$ . These policies are enumerated in Tables 1(a) and 1(b), respectively. In Table 1(b), each “o” represents one defensive unit. The defender policies  $\delta_1$  and  $\delta_2$  will be called *3-0 configurations*, whereas the policies  $\delta_3$  and  $\delta_4$  will be called *2-1 configurations*.

policy	target assigned
$\alpha_1$	A
$\alpha_2$	B

(a) Attacker policies

policy	target A	target B
$\delta_1$	o o o	
$\delta_2$		o o o
$\delta_3$	o o	o
$\delta_4$	o	o o

(b) Defender policies

Table 1: Pure policies

The game under consideration can be represented in its *extensive form* by associating each policy of the attacker and the defender with a row and column, respectively, of a matrix  $G \in \mathbb{R}^{2 \times 4}$ . The entry  $g_{ij}$ ,  $i \in \{1, 2\}$ ,  $j \in \{1, 2, 3, 4\}$  of  $G$  corresponds to the

cost  $J$  when the attacker chooses policy  $\alpha_i$  and the defender chooses policy  $\delta_j$ . The matrix  $G$  for this game is given by

$$G := \begin{bmatrix} & \delta_1 & \delta_2 & \delta_3 & \delta_4 \\ \alpha_1 & 1 & 0 & c_2 & c_1 \\ \alpha_2 & 0 & 1 & c_1 & c_2 \end{bmatrix} \quad (1)$$

In the context of non-cooperative zero-sum games, such as the one above, optimality is usually defined in terms of a saddle-point or Nash equilibrium [21]. A *Nash equilibrium* in pure policies would be a pair of policies  $\{\alpha_{i^*}, \delta_{j^*}\}$ , one for each player, for which

$$g_{i^*j} \leq g_{i^*j^*} \leq g_{ij^*}, \quad \forall i, j.$$

Nash policies are chosen by rational players since they guarantee a cost no worse than  $g_{i^*j^*}$  for each player, *no matter what the other player decides to do*. As a consequence, playing at a Nash equilibrium is “safe” even if the opponent discovers our policy of choice. They are also reasonable choices since a player will never do better by unilaterally deviating from the equilibrium. Not surprisingly, there are no Nash equilibria in pure policies for the game described by (1). In fact, all the pure policies violate the “safety” condition mentioned above for Nash equilibria. Suppose, for example, that the attacker plays policy  $\alpha_1$ . This choice is certainly not safe in the sense that, if the defender guesses it, he can then choose the policy  $\delta_1$  and subject the attacker to the highest possible cost. Similarly,  $\alpha_2$  is not safe and therefore cannot also be in a Nash equilibrium pair.

To obtain a Nash equilibrium, one needs to enlarge the policy space by allowing each player to randomize among its available pure policies. In particular, suppose that the attacker chooses policy  $\alpha_i$ ,  $i \in \{1, 2\}$ , with probability  $a_i$  and the defender chooses policy  $\delta_j$ ,  $j \in \{1, 2, 3, 4\}$ , with probability  $d_j$ . When the game is played repeatedly, the expected value of the cost is then given by

$$E[J] = \sum_{i,j} a_i g_{ij} d_j = a' G d.$$

Each vector  $a := \{a_i\}$  in the 2-dimensional simplex<sup>1</sup> is called a *mixed policy for the attacker*, whereas each vector  $d := \{d_j\}$  in the 4-dimensional simplex is called a *mixed policy for the defender*. It is well known that at least one Nash equilibrium in mixed policies always exists for finite matrix games (cf. Minimax

<sup>1</sup>We call the set of all vectors  $x := \{x_i\} \in \mathbb{R}^n$  for which  $x_i \geq 0$  and  $\sum_i x_i = 1$ , the *n-dimensional simplex*.

Theorem [22, p. 27]). In particular, there always exists a pair of mixed policies  $\{a^*, d^*\}$  for which

$$a^{*'}Gd \leq a^{*'}Gd^* \leq a'Gd^*, \quad \forall a, d.$$

Assuming that both players play at the Nash equilibrium the cost will then be equal to  $a^{*'}Gd^*$ , which is called the *value of the game*. It is straightforward to show that the unique Nash equilibrium for the matrix  $G$  in (1) is given by

$$a^* := \left[ \frac{1}{2} \quad \frac{1}{2} \right]', \quad (2)$$

$$d^* := \begin{cases} \left[ \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \right]' & c_1 + c_2 \leq 1 \\ \left[ 0 \quad 0 \quad \frac{1}{2} \quad \frac{1}{2} \right]' & c_1 + c_2 > 1 \end{cases} \quad (3)$$

with value equal to

$$a^{*'}Gd^* = \max \left\{ \frac{c_1 + c_2}{2}, \frac{1}{2} \right\}.$$

This equilibrium corresponds to the intuitive solution that the attacker should randomize between attacking targets A or B with equal probability, and the defender should randomize between placing most of its units next to A or next to B also with equal probability. The optimal choice between 3-0 or 2-1 configurations (policies  $\delta_1/\delta_2$  versus  $\delta_3/\delta_4$ ) depends on the parameters  $c_1$  and  $c_2$ . From (2) and (3) we conclude that 3-0 configurations are optimal when  $c_1 + c_2 \leq 1$ , otherwise the 2-1 configurations are preferable.

In the game described so far, there is no role for deception since the players are forced to make a decision without any information. We will change that next.

### 3 Full Manipulation of Information

Suppose now that the game described above is played in two steps. First the defender decides how to distribute its units. It may also disclose the position of some of its units to the attacker. On the second step the attacker decides which target to strike. To do this, it may use the information provided by the defender. For now, we assume that this is the only information available to the attacker and therefore the defender completely controls the information that the attacker uses to make its decision.

The rationale for the defender to voluntarily disclose the position of its units is to deceive the attacker. Suppose, for example that the attacker uses

two units to defend target A and only one to defend B. By disclosing that it has units next to B, the defender may expect the opponent to attack A and, consequently, suffer a heavier cost.

In this new game, the number of admissible pure policies for each player is larger than before. The attacker now has 8 distinct pure policies available since, for each possible observation (no unit detected, unit detected defending target A, or unit detected defending target B), it has two possible choices (strike A or B). These policies are enumerated in Table 2(a). In policies  $\alpha_1, \alpha_2$  the attacker ignores any available information and always attacks target A or target B. These policies are therefore called *blind*. In policies  $\alpha_3$  and  $\alpha_4$ , the attacker never selects the target where it detects a defense unit. These policies are called *naive*. In policies  $\alpha_5$  and  $\alpha_6$  the attacker chooses the target where it detects a defending unit. These policies are called *counter-deception* since they presume that a unit is being shown close to the least defended target.

The defender has ten distinct pure policies available, each one corresponding to a particular configuration of its defenses and a particular choice of which units to disclose (if any). These are enumerated in Table 2(b), where “o” represents a defense unit whose position has not been disclosed and “•” a defense unit whose position has been disclosed. Here, we are assuming that the defender will, at most, disclose the placement of one unit because more than that would never be advantageous. In policies  $\delta_1$  through  $\delta_4$  nothing is disclosed about the distribution of the units. These are called *no-information* policies. In policies  $\delta_9$  and  $\delta_{10}$  the defender shows units placed next to the target that has fewer defenses. These are *deception* policies. Policies  $\delta_5$  through  $\delta_8$  are *disclosure* policies, in which the defender is showing a unit next to the target that is better defended.

This game can be represented in extensive form by the following  $8 \times 10$  matrix

$$G := \begin{matrix} & \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & \delta_7 & \delta_8 & \delta_9 & \delta_{10} \\ \begin{matrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{matrix} & \begin{bmatrix} 1 & 0 & c_2 & c_1 & 1 & 0 & c_2 & c_1 & c_2 & c_1 \\ 0 & 1 & c_1 & c_2 & 0 & 1 & c_1 & c_2 & c_1 & c_2 \\ 0 & 1 & c_1 & c_2 & 0 & 0 & c_1 & c_1 & c_2 & c_2 \\ 1 & 0 & c_2 & c_1 & 0 & 0 & c_1 & c_1 & c_2 & c_2 \\ 1 & 0 & c_2 & c_1 & 1 & 1 & c_2 & c_2 & c_1 & c_1 \\ 0 & 1 & c_1 & c_2 & 1 & 1 & c_2 & c_2 & c_1 & c_1 \\ 1 & 0 & c_2 & c_1 & 0 & 1 & c_1 & c_2 & c_1 & c_2 \\ 0 & 1 & c_1 & c_2 & 1 & 0 & c_2 & c_1 & c_2 & c_1 \end{bmatrix} \end{matrix} \quad (4)$$

policy	target assigned when ...		
	no obs.	unit detected at A	unit detected at B
$\alpha_1$	A	A	A
$\alpha_2$	B	B	B
$\alpha_3$	B	B	A
$\alpha_4$	A	B	A
$\alpha_5$	A	A	B
$\alpha_6$	B	A	B
$\alpha_7$	A	B	B
$\alpha_8$	B	A	A

(a) Attacker policies

policy	target A	target B
$\delta_1$	o o o	
$\delta_2$		o o o
$\delta_3$	o o	o
$\delta_4$	o	o o
$\delta_5$	o o •	
$\delta_6$		o o •
$\delta_7$	o •	o
$\delta_8$	o	o •
$\delta_9$	o o	•
$\delta_{10}$	•	o o

(b) Defender policies

Table 2: Pure policies

Just as before, for this game to have Nash equilibria one needs to consider mixed policies. However, this particular game has multiple Nash equilibria, one of them being

$$a^* := \left[ \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \right]',$$

$$d^* := \begin{cases} \left[ \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \right]' & c_1 + c_2 \leq 1 \\ \left[ 0 \quad 0 \quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad 0 \right]' & c_1 + c_2 > 1 \end{cases}$$

with value equal to

$$a^{*'} G d^* = \max \left\{ \frac{c_1 + c_2}{2}, \frac{1}{2} \right\}.$$

This shows that (i) the attacker can ignore the information available and simply randomize among the two blind policies; and (ii) the defender gains nothing from disclosing information and can therefore randomize among its no-information policies. It should be noted that there are Nash equilibria that utilize different policies. For example, when  $c_1 + c_2 > 1$ , an

alternative Nash equilibrium is

$$\bar{a}^* := \left[ 0 \quad 0 \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad 0 \quad 0 \right]', \quad (5)$$

$$\bar{d}^* := \left[ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \right]'. \quad (6)$$

In this case, the defender randomizes between deception and disclosure policies with equal probability and the attacker between the naive and counter-deception policies. However, in zero-sum games all equilibria yield the same value, so the players have no incentive to choose this equilibrium that is more complex in terms of the decision rules. Finally, it should also be noted that, because of the equilibrium interchangeability property for zero-sum games, the pairs  $\{a^*, \bar{d}^*\}$  and  $\{\bar{a}^*, d^*\}$  are also Nash equilibria [22, p. 28].

We have just seen that the attacker gains nothing from using the measurements available, even though these measurements give precise information about the position of some of the defense units. At an intuitive level, this is because the information available to the attacker is completely controlled by its opponent. And, if the defender chooses to disclose the position of some of its units, this is done solely to get an advantage. This can be seen, for example, in the equilibrium given by (5)–(6). We shall consider next a version of the game where the defender no longer has complete control over the information available to the attacker. For the new game, the attacker may sometimes improve its cost by using the available information.

## 4 Partial Manipulation of Information

In practice, when the defender decides to "show" one of its units it simply does not camouflage it, making it easy to find by the surveillance sensors used by the attacker. In the previous game we assumed that shown units are always detected by the attacker and hidden ones are not. We will deviate now from this ideal situation and assume that (i) shown units may not be detected and, more importantly, (ii) hidden units may sometimes be detected by the attacker. We consider here a generic probabilistic model for the attacker's surveillance, which is characterized by the conditional probability of detecting units next to a particular target, given a specific total number of units next to that target and how many of them are being shown. In particular, denoting by  $\mathcal{D}_A$  the event that defenses are detected next to target A, we have

that

$$P(\mathfrak{D}_A \mid \mathbf{n}_A = n_A, \mathbf{s}_A = s_A) = \chi(n_A, s_A),$$

where  $\chi(\cdot)$  is the *characteristic function of the sensor*,  $\mathbf{n}_A$  denotes the total number of units defending target A, and  $\mathbf{s}_A$  the number of these that are shown. We assume here that  $\mathfrak{D}_A$  is conditionally independent of any other event, given specific values for  $\mathbf{s}_A$  and  $\mathbf{n}_A$ . Since there is no incentive for the defender to show more than one unit  $\mathbf{s}_A \in \{0, 1\}$ , whereas  $\mathbf{n}_A \in \{0, 1, 2, 3\}$ . For simplicity of notation, we assume that the surveillance of target B is identical and independent, with

$$P(\mathfrak{D}_B \mid \mathbf{n}_B = n_B, \mathbf{s}_B = s_B) = \chi(n_B, s_B),$$

where the symbols with the B subscript have the obvious meaning.

For most of the discussion that follows, the characteristic function  $\chi(\cdot, \cdot)$  can be arbitrary, provided that it is monotone non-decreasing with respect to each of its arguments (when the other is held fixed). The monotonicity is quite reasonable since more units (shown or not) should always result in a higher probability of detection. However, it will be sometimes convenient to work with a specific characteristic function  $\chi(\cdot, \cdot)$ . One possible choice is:

$$\chi(n, s) = 1 - (1 - p)^s(1 - q)^{n-s}, \quad (7)$$

$n \in \{0, 1, 2, 3\}$ ,  $s \in \{0, 1\}$ , where  $0 \leq q \leq p \leq 1$ . This model assumes that (i) the surveillance sensors will provide a positive reading (i.e., announce that units were detected next to a particular target) when they are able to detect, at least, one unit, and (ii) the probability of detecting a particular defense unit that is hidden is equal to  $q$ , whereas the probability of detecting a unit that is shown is equal to  $p$ . A few particular cases should be considered:

- When  $\chi(n, s) = 0$ ,  $\forall n, s$  (or when  $p = q = 0$  in (7)) we have the first game considered in this paper, since the defense units are never detected.
- When

$$\chi(n, s) = \begin{cases} 1, & s > 0 \\ 0, & s = 0 \end{cases} \quad \forall n, s$$

(or when  $p = 1$  and  $q = 0$  in (7)) the defense units are detected only when they are shown. This corresponds to the second game considered here, where the defender has full control of the information available to the attacker.

Another interesting situation occurs when placing more units next to a particular target makes that target more likely to be detected by the surveillance sensors *regardless of how many units are shown*. In such a case the attacker's sensors are said to be *reliable*. This can be formally expressed by the condition

$$\chi(n_1, s_1) \geq \chi(n_2, s_2), \quad (8)$$

$\forall n_1 > n_2 : n_1 + n_2 = 3, \forall s_1, s_2$ . Because the characteristic functions of the sensors are monotone non-decreasing with respect to each of its arguments (when the other is held fixed), it is straightforward to show that this reliability condition is equivalent to

$$\chi(2, 0) \geq \chi(1, 1). \quad (9)$$

For the characteristic function in (7), this corresponds to

$$2q - q^2 \geq p.$$

We will see below that the attacker can choose naive policies, when the sensors are reliable. A special case of reliable sensors arises when  $\chi(n, s)$  is independent of  $s$  for all values of  $n$ , (or when  $p = q$  in (7)). In this case we have sensors that cannot be manipulated by the defender since the detection is independent of the number of units "shown" by the defender.

In terms of the policies available, the game considered in this section is very similar to the one in Section 3. The only difference is that, in principle, the attacker may now detect defense units next to both targets. In practice, this means that Table 2(a) should have a fourth column entitled "units detected at A and B," which would result in 16 distinct pure policies. It turns out that not detecting any unit or detecting units next to both targets is essentially the same. Because of this we shall consider for this game only the 8 policies in Table 2(a), with the understanding that when units are detected next to both targets, the attacker acts as if no units were detected. It is not hard to prove that this introduces no loss of generality. The defender's policies are the same as in Section 3, and are given in Table 2(b).

This game can also be represented in extensive form by an  $8 \times 10$  matrix. The reader is referred to [20] on how to construct this matrix. As before the game has Nash equilibria in mixed policies, but now the equilibrium policies depend on the values of  $\chi(n, s)$ .

To make the computation of the Nash Equilibrium simpler, we reduced the size of the matrix of the game

using the intuitive notion that the optimal policies for the attacker and the defender will be symmetric. Then we found a Nash Equilibrium for the reduced game, and construct a solution for the actual game. We finally proved that this solution is in fact a Nash Equilibrium for the original game [20]. The optimal policies that we computed for this game are given in the following theorem:

**Theorem 1.** 1. For  $\chi(2,0) \geq \chi(1,1)$ , one of the Nash Equilibrium solutions for the players is

$$a^* := [0 \ 0 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0 \ 0 \ 0]' ,$$

$$d^* := \begin{cases} [\frac{1}{2} \ \frac{1}{2} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]' & 1 - \chi(3,0) \geq c_1 + c_2 - e_1 \\ [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{2} \ \frac{1}{2}]' & 1 - \chi(3,0) < c_1 + c_2 - e_1, \end{cases}$$

where  $e_1 = (c_2 - c_1)(\chi(2,0) - \chi(1,1))$ .

2. For  $\chi(2,0) < \chi(1,1)$  and  $c_1 + c_2 \geq 1$ , one of the Nash Equilibrium solutions for the players is

$$a^* := [\frac{1}{2} \ \frac{1}{2} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]' ,$$

$$d^* := [0 \ 0 \ \frac{e_2}{2} \ \frac{e_2}{2} \ 0 \ 0 \ 0 \ 0 \ \frac{1-e_2}{2} \ \frac{1-e_2}{2}]' ,$$

where  $e_2 := \frac{\chi(1,1) - \chi(2,0)}{\chi(1,1) - \chi(1,0)}$ .

3. For  $\chi(2,0) < \chi(1,1)$  and  $c_1 + c_2 < 1$ , one of the Nash equilibrium solutions for the players is

$$a^* := [\frac{1-e_3}{2} \ \frac{1-e_3}{2} \ \frac{e_3}{2} \ \frac{e_3}{2} \ 0 \ 0 \ 0 \ 0]' ,$$

$$d^* := [\frac{e_4}{2} \ \frac{e_4}{2} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1-e_4}{2} \ \frac{1-e_4}{2}]' ,$$

where  $e_3 := \frac{1-c_1-c_2}{\chi(3,0)-e_1}$  and  $e_4 := \frac{e_1}{e_1 - \chi(3,0)}$ .

The proof of Theorem 1 can be found in [20].

Having computed the Nash Equilibrium solutions for the matrix, we can conclude the following:

1. When the sensors are reliable (i.e.,  $\chi(2,0) \geq \chi(1,1)$ ), the attacker randomizes among its naive policies and the defender either randomizes among the deception policies or the 3-0 no-information configurations. The latter occurs when the attacker only incurs in significant cost when 3 units are in its path and therefore 2-1 configurations are not acceptable for the defender. The value of the game is

$$a^* G d^* = \frac{\max\{1 - \chi(3,0), c_1 + c_2 - e_1\}}{2} \leq \frac{c_1 + c_2}{2} .$$

This value for the cost is smaller than the one obtained in the previous two games, making it more favorable to the attacker, which is now able to take advantage of the surveillance information.

2. When the sensors are not reliable (i.e.,  $\chi(2,0) < \chi(1,1)$ ) and  $c_1 + c_2 \geq 1$ , the attacker randomizes among its blind policies and the defender randomizes between deception and no-information in 2-1 configurations. The probability distribution used by the defender is a function of the several parameters. However, the value of the game is always

$$a^* G d^* = \frac{c_1 + c_2}{2} .$$

This means that the surveillance sensors of the attacker are effectively rendered useless by the defender's policy. This happens because the sensors are not reliable and therefore the defender can significantly manipulate the information available to the attacker. For the characteristic function of the sensors in (7), this occurs when the probability  $p$  of detecting a unit that is shown is significantly large when compared to the probability  $q$  of detecting a unit that is hidden. It is interesting to note that the region of the  $(p, q)$  parameter space where this happens is actually quite large (cf. Figure 1). This means that such situations are likely to occur in practice.

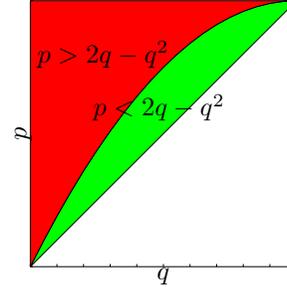


Figure 1:  $(p, q)$  Parameter Space

3. When the sensors are not reliable (i.e.,  $\chi(2,0) < \chi(1,1)$ ) and  $c_1 + c_2 < 1$ , the attacker randomizes between its blind and naive policies, whereas the defender randomizes between deception and no-information in 3-0 configurations. The value of the game is

$$a^* G d^* = \frac{1}{2} - \frac{(1 - c_1 - c_2)\chi(3,0)}{2(\chi(3,0) - e_1)} \leq \frac{1}{2} .$$

Therefore, the attacker can attain a cost smaller than  $\frac{1}{2}$  which would be obtained by only using blind policies.

Note that both in cases 2 and 3 the sensors are not reliable and the defender has sufficient power to manipulate the information available to the attacker so as to make it effectively useless. However, in case 3, the 2-1 configurations required for deception are very costly to the defender and deception is no longer a very attractive alternative.

## 5 Conclusions

We demonstrated that, when one of the players in a competitive game can manipulate the information available to its opponents, deception can be used to increase the player's payoff. We showed that an intelligent player can effectively render the information available to its opponent useless by carefully using deception. This study was carried out for a prototype problem in asset distribution in military operations. The ideas presented here can be applied to devise optimal strategies that use and counteract deception in many other problems. This is the subject of our current research.

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