NONCOOPERATIVE GAME THEORY:

AN INTRODUCTION FOR ENGINEERS AND COMPUTER SCIENTISTS

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September 22, 2021

Errata

1) In page 11, the **P1.8** bullet should read:

P1.8 Once player P2 picks the attack policy, P1's routing policy is the best response from this player's perspective.

2) In page 12, in the left image in Figure 1.5 the link from node 3 to node 6 should be labeled with 0% (instead of 100%).

3) In page 44, the left-most "sup" in equation (4.10) should actually by an "inf":

$$\underbrace{\inf_{\gamma\in\Gamma_1} J(\gamma,\sigma^*) = \underline{V}_{\Gamma_1,\Gamma_2}(G)}_{\sigma^* \text{ achieves the maximum}} \coloneqq \sup_{\sigma\in\Gamma_2}\inf_{\gamma\in\Gamma_1} J(\gamma,\sigma). \tag{4.10}$$

4) In page 53, the equation just above Figure 5.1 should be:

$$x_1,x_2 \in \mathrm{co}(v_1,v_2,\ldots,v_k) \quad \Rightarrow \quad \lambda x_1 + (1-\lambda)x_2 \in \mathrm{co}(v_1,v_2,\ldots,v_k), \quad \forall \lambda \in [0,1],$$

(without the division by two that appears in the book).

5) In page 63, the "minimum" at the top of the page should actually be a "maximum":

$$V_m(A) = \underbrace{\begin{array}{ll} \text{maximum} & v \\ \text{subject to} & \left.\begin{array}{c} z \dot{\geqslant} 0 \\ \mathbf{1}z = 1 \end{array}\right\} \quad z \in \mathscr{Z}. \\ & Az \dot{\geqslant} v\mathbf{1} \end{array}}_{\text{optimization over } n+1 \text{ parameters } (v,z_1,z_2,\ldots,z_n)}$$

6) In page 85, the equation for the Bell number should be

$$b_{k+1} = \sum_{i=0}^{k} \binom{k}{i} b_i$$

and the equation just below this one should be

$$b_m + b_n - 1$$

7) In page 90, the Example 8.2 equations to the right of the tables representing the player's policies should be:

$$z_1^\alpha,\, z_2^\alpha,\, z_3^\alpha \geqslant 0,\; z_1^\alpha + z_2^\alpha + z_3^\alpha = 1;$$

and

$$y_1^\beta,\, y_2^\beta \geqslant 0,\; y_1^\beta + y_2^\beta = 1$$
$$y_1^\xi,\, y_2^\xi \geqslant 0,\; y_1^\xi + y_2^\xi = 1.$$

8) In page 91, the expression "$J(y,z)$" that appears in bullet 2, should be "$J(\gamma,\sigma)$."

9) In page 112, the 1st sentence of the proof of Proposition 9.1 should read: "This proposition is a straightforward consequence of the facts that (i) zero-sum games enjoy the order interchangeability property that we found in Proposition 4.2 (item P4.8) and (ii) ..."

10) In page 124, in the 2nd set of equations "**2**" should be replaced by "**1**":

$$\begin{cases} Az \geqslant p\mathbf{1} \\ B'y \geqslant q\mathbf{1} \end{cases} \quad \Rightarrow \quad \begin{cases} y'Az \geqslant p \\ z'B'y \geqslant q \end{cases} \quad \Rightarrow \quad y'(A+B)z - p - q \geqslant 0.$$

11) In page 124, in the 2nd set of equations "**2**" should be replaced by "**1**":

$$\begin{cases} Az \geqslant p\mathbf{1} \\ B'y \geqslant q\mathbf{1} \end{cases} \quad \Rightarrow \quad \begin{cases} y'Az \geqslant p \\ z'B'y \geqslant q \end{cases} \quad \Rightarrow \quad y'(A+B)z - p - q \geqslant 0.$$

12) In page 130, in Definition 11.3 the expression in the first sentence should be $(y^{1*}, y^{2*}, \ldots, y^{N*}) \in \mathscr{Y}^1 \times \mathscr{Y}^2 \times \cdots \times \mathscr{Y}^N$.

13) In page 148, equation (13.11) should read

$$\phi(\gamma) = \sum_{k=1}^{N} \Big(H_k(\gamma_k) + \sum_{j=1}^{k-1} W_{kj}(\gamma_k, \gamma_j) \Big), \quad \forall \gamma \in \Gamma. \tag{13.11}$$

14) In page 157, Theorem 13.1 refers to zero-sum games and therefore to the specific case $N = 2$. The equation in this theorem should thus be:

$$\{\hat{y}^1, \hat{y}^2, \ldots\}, \qquad \hat{y}^k = (\hat{y}_1^k, \hat{y}_2^k) \in \mathscr{Y} := \mathscr{Y}^1 \times \mathscr{Y}^2$$

15) In page 167, the equation numbers are reversed in Exercise 13.10, which should read: "Verify that (13.13) is a potential function for the congestion game with outcomes given by (13.12)."

16) In page 187, the inequality (15.22) should be strict:

$$R + B'P_{k+1}B > 0, \quad \forall k \in \{1, 2, \ldots, K\}. \tag{15.22}$$

17) In page 193, the first sentence of Note 14 should be: "Since $\frac{\partial V(\tau,x)}{\partial \tau}$ in (16.5) does not depend of u,..."

18) In page 197, the superscript in equation (16.15) should by "FB":

$$\gamma^{\mathrm{FB}}(t, x(t)) := \arg\min_{u \in \mathscr{U}} g(t, x(t), u) + \frac{\partial V(t, x(t))}{\partial x} f\big(t, x(t), u\big), \quad \forall t \in [0, T_{\mathrm{end}}]. \tag{16.15}$$

19) In page 202, superscripts are missing for the functions V_{k+1} in equations (17.4) and (17.6):

$$\gamma_k^*(x) := \arg\min_{u_k \in \mathscr{U}_k} \Big(g_k(x, u_k, \sigma_k^*(x)) + V_{k+1}^1\big(f_k(x, u_k, \sigma_k^*(x))\big) \Big) \quad \forall k \in \{1, 2, \ldots, K\}. \tag{17.4}$$

$$\sigma_k^*(x) := \arg\max_{d_k \in \mathscr{D}_k} \Big(g_k(x, \gamma_k^*(x), d_k) + V_{k+1}^2\big(f_k(x, \gamma_k^*(x), d_k)\big) \Big), \quad \forall k \in \{1, 2, \ldots, K\}. \tag{17.6}$$

20) In page 203, the equation for $\sigma_K^*(x)$ should have "arg max" rather than 'arg min":

$$V_K^2(x) = \sup_{d_K \in \mathscr{D}_K} g_K(x, \gamma_K^*(x), d_K), \qquad \sigma_K^*(x) := \arg\max_{d_K \in \mathscr{D}_K} g_K(x, \gamma_K^*(x), d_K),$$

Acknowledgments

I would like to thank Prof. Rodolfo García, Prof. Shaunak Bopardikar, and Guosong Yang for helping me to find and correct several typos in the book.