Errata

1) In page 7, the MATLAB® command should read

\[
\text{sys\_ss}=\text{ss}(A,B,C,D,)
\]
\[
\ '\text{InputName}', \{\text{'input1'}, \text{'input2'}, \ldots\},\ldots
\]
\[
\ '\text{OutputName}', \{\text{'output1'}, \text{'output2'}, \ldots\},\ldots
\]
\[
\ '\text{StateName}', \{\text{'state1'}, \text{'state2'}, \ldots\}
\]

2) In page 15 in Figure 2.3, the angle \( \theta_2 \) is incorrectly drawn, it should be drawn as follows:

![Diagram](image)

Moreover, the matrix \( M(q) \) and the vector \( G(q) \) in Example 2.2 should be as follows:

\[
M(q) := \begin{bmatrix}
m_2 \ell_2^2 & 2m_2 \ell_1 \ell_2 \cos \theta_2 + (m_1 + m_2) \ell_1^2 & m_2 \ell_2^2 + m_2 \ell_1 \ell_2 \cos \theta_2 \\
m_2 \ell_1 \ell_2 \cos \theta_2 + m_1 \ell_1^2 & m_2 \ell_2^2 & m_2 \ell_2^2 \\
\end{bmatrix}
\]

\[
G(q) := \begin{bmatrix}
m_2 g \ell_2 \cos(\theta_1 + \theta_2) + (m_1 + m_2) g \ell_1 \cos \theta_1 \\
m_2 g \ell_2 \cos(\theta_1 + \theta_2) \\
\end{bmatrix}
\]

3) In page 17, equation (2.8) should read as follows:

\[
\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} v,
\]

\[
y = \begin{bmatrix} I & 0 \end{bmatrix} x,
\]

\[
x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \in \mathbb{R}^{2k}.
\]

4) In page 18, the first equation in Section 2.4.4 should read as follows:

\[
\dot{x}_1 = x_2
\]

\[
\dot{x}_2 = M^{-1}(x_1) \left( -B(x_1, x_2)x_2 - G(x_1) + u \right).
\]
5) In page 17 in Figure 2.5, the label y at the right should be replaced by \( \begin{bmatrix} q \\ q \end{bmatrix} \).

6) In page 28, the equation just above definition 3.2 should read

\[
\dot{y}(z) = Z[y(t)] := \sum_{t=0}^{\infty} z^{-t} y(t), \quad z \in \mathbb{C}.
\]

7) In page 30, the last equation in Note 4, \( dt \) should be replaced by \( d\tau \), as in:

\[
L[(x \ast y)(t)] = \int_{0}^{\infty} e^{-s \tau} x(\tau) \left( \int_{0}^{\infty} e^{-s \tau} y(t) d\tau \right) d\tau = \lambda(s) \dot{y}(s).
\]

8) In page 34, the sidebar example in Section 4.3.2, the matrix \( N_3 \) should be

\[
N_3 = \begin{bmatrix} -2 & 3 \\ 1 & \frac{3}{2} \end{bmatrix}.
\]

9) In page 35, the proof of Proposition 4.1, the second equation should read as follows:

\[
Z_n = \frac{1}{s} Z_{n-1}, \quad Z_{n-1} = \frac{1}{s} Z_{n-2}, \quad \ldots, \quad Z_2 = \frac{1}{s} Z_1 \quad \Rightarrow \quad Z_k = \frac{1}{s^{k-1}} Z_1.
\]

10) In page 38, the first command in MATLAB® Hint 17 should read

\[
z=0; \quad p=[-1,-3]; \quad k=2;
\]

11) In page 48, the last equation should read

\[
A^{n+1} = -a_1 A^n - a_2 A^{n-1} - \cdots - a_{n-1} A^2 - a_n A \\
= a_1(a_1 A^{n-1} + a_2 A^{n-2} + \cdots + a_{n-1} A + a_n I) - a_2 A^{n-1} - \cdots - a_{n-1} A^2 - a_n A \\
= (a_1^2 - a_2) A^{n-1} + (a_1 a_2 - a_3) A^{n-2} + \cdots + (a_1 a_{n-1} - a_n) A + a_1 a_n I.
\]

12) In page 50, the forth equation should read

\[
\mathcal{L}^{-1} \left[ \frac{a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n}{(s - \lambda_1)^{m_1}(s - \lambda_2)^{m_2} \cdots (s - \lambda_k)^{m_k}} \right] = a_{11} e^{\lambda_1 t} + a_{12} t e^{\lambda_1 t} + \cdots + \frac{a_{1m_1} t^{m_1-1} e^{\lambda_1 t}}{(m_1 - 1)!} \\
+ \cdots + a_{k1} e^{\lambda_k t} + a_{k2} t e^{\lambda_k t} + \cdots + \frac{a_{km_k} t^{m_k-1} e^{\lambda_k t}}{(m_k - 1)!}.
\]

13) In page 56, Note 5, item 2, bullet 3: ". one 2 x 2 block and two 1 x 1 blocks .." should read "one 2 x 2 block and one 1 x 1 block."

14) In page 56, the condition 3 in Theorem 7.2 is incorrect. It should read as follows:

**Theorem 7.2.** For an \( n \times n \) matrix \( A \), the following three conditions are equivalent:

1. \( A \) is semisimple.

2. \( A \) has \( n \) linearly independent eigenvectors.

3. There is a nonzero polynomial without repeated roots that annihilates \( A \); i.e., there is a nonzero polynomial \( p(s) \) without repeated roots for which \( p(A) = 0 \).
Note that condition 3 provides a simple procedure to check for diagonalizability. Since every polynomial that annihilates $A$ must have each eigenvalue of $A$ as a root (perhaps with different multiplicities), one simply needs to compute all the distinct eigenvalues $\lambda_1, \ldots, \lambda_k$ ($k \leq n$) of $A$ and then check if the polynomial $p(s) = (s - \lambda_1) \cdots (s - \lambda_k)$ annihilates $A$.

15) In page 57, equation (7.1) should read

$$A^t = \begin{pmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_k \end{pmatrix} P,$$

16) In page 57, the following clarifying sentence should be added after the expression for $J^1$:

"The formula above presumes that $0! = 1$ and that any entries in $J^1$ that would contain factorials of negative numbers should be set to zero."

17) In page 58, the first equation should read

$$e^{At} := \sum_{k=0}^{\infty} \frac{t^k}{k!} A^k.$$

18) In page 64, item 3: "...this norm corresponds to the usual Euclidean norm $\nu := \ldots$" should read "...this norm corresponds to the usual Euclidean norm $||v||_2 := \ldots$"

19) In page 65 in Definition 8.1, the condition for exponential stability should read

$$||x(t)|| \leq c e^{-\lambda(t-h)} ||x(t_0)||, \quad \forall t \geq 0.$$

20) In page 66, the second inequality in the Attention! box below Theorem 8.1 should read

$$||x(t)|| = ||e^{A(t-t_0)} x_0|| \leq ||e^{A(t-h)}|| ||x_0|| \leq c e^{-\lambda(t-t_0)} ||x_0||, \quad \forall t \in \mathbb{R}.$$

21) In page 69 in the proof of Theorem 8.2, the argument regarding how condition 4 implies condition 5, should consider selecting $Q = I$ (instead of $Q = -I$ as stated).

22) In page 70 at the end of the proof of Theorem 8.2, the definition of $\lambda$ should read

$$\lambda := \frac{\lambda_{\min}[Q]}{\lambda_{\max}[P]}.$$

23) In page 72, the third equation should read

$$v(t+1) = x'(t+1)Px(t+1) = x(t)'P \lambda(t).$$

24) In page 72, in the last equation in Section 8.6 should read

$$v(t+1) = x'(t)(P - Q)x(t) = v(t) - x'(t)Qx(t), \quad \forall t \geq 0.$$

25) In page 72, equation (8.15) should read

$$||x(t) - x_\text{eq}|| \leq c e^{-\lambda(t-t_0)} ||x(t_0) - x_\text{eq}||, \quad \forall t \geq t_0.$$
27) In page 76, the first note should read “This equilibrium point is $x^\text{eq} = [\pi, 0]^T$, $u^\text{eq} = 0$, $y^\text{eq} = \pi$, and therefore
\[
\delta x := x - x^\text{eq} = x - [\pi, 0]^T, \quad \delta u := u - u^\text{eq} = u, \quad \delta y := y - y^\text{eq} = y - \pi.
\]

28) In page 76 and 77, squares are missing in the expression for the eigenvalues of $A$. Specifically, in page 76, it should read as follows: “The eigenvalues of $A$ are given by
\[
\det(\lambda I - A) = \lambda \left( \frac{b}{m \ell^2} \right) + \frac{g}{\ell} = 0 \quad \Leftrightarrow \quad \lambda = -\frac{b}{2m \ell^2} \pm \sqrt{\left( \frac{b}{2m \ell^2} \right)^2 - \frac{g}{\ell}},
\]
and therefore the linearized system is exponentially stable, because
\[
-\frac{b}{2m \ell^2} \pm \sqrt{\left( \frac{b}{2m \ell^2} \right)^2 - \frac{g}{\ell}}
\]
has a negative real part”; and in page 77, it should read as follows: “The eigenvalues of $A$ are given by
\[
\det(\lambda I - A) = \lambda \left( \frac{b}{m \ell^2} \right) - \frac{g}{\ell} = 0 \quad \Leftrightarrow \quad \lambda = -\frac{b}{2m \ell^2} \pm \sqrt{\left( \frac{b}{2m \ell^2} \right)^2 + \frac{g}{\ell}},
\]
and therefore the linearized system is unstable, because
\[
-\frac{b}{2m \ell^2} + \sqrt{\left( \frac{b}{2m \ell^2} \right)^2 + \frac{g}{\ell}} > 0.
\]

29) In page 82, the first equation should read
\[
u_T(\tau) := \begin{cases} 
0 & 0 \leq \tau < T \\
e_i & \tau \geq T
\end{cases}, \quad \forall \tau \geq 0.
\]

30) In page 83, the last equation before Theorem 9.2 should read
\[
\sup_{t \geq 0} \int_0^t |\dot{g}_{ij}(t - \tau)|d\tau = \sup_{t \geq 0} \int_0^t |\dot{g}_{ij}(\rho)|d\rho = \int_0^{\infty} |\dot{g}_{ij}(\rho)|d\rho.
\]

31) In page 84, the last equation should read
\[
g_{ij}(t) = \mathcal{L}^{-1}\left[ \dot{g}_{ij}(s) \right] = a_{11}e^{\lambda_1 t} + a_{12} t e^{\lambda_1 t} + \ldots + \frac{a_{1m_1} t^{m_1 - 1} e^{\lambda_1 t}}{(m_1 - 1)!} \\
+ \ldots + a_{k1}e^{\lambda_k t} + a_{k2} t e^{\lambda_k t} + \ldots + \frac{a_{km} t^{m - 1} e^{\lambda_k t}}{(m_k - 1)!}.
\]

32) In page 85, the last equation should not have the term $\sum \sigma$. 

33) In page 86, the equation in Exercise 9.1 should read
\[
\dot{x} = \begin{bmatrix} -2 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\
0 \\
-1 \end{bmatrix} u, \quad \quad y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x + u.
\]

34) In page 89, the two equations for $J_{LQR}$ should read
\[
J_{LQR} := \int_0^{\infty} x'CQx + u'Ru \, dt
\]
44) In page 120, the third sentence in the proof of Theorem 13.2 should read

\[ \text{Since the number of nonzero rows of } C^0 \text{ and } C^1 \text{ is exactly } \bar{c}, \]
46) In page 123, equation (14.1) should read
\[
\begin{bmatrix}
\dot{x}_c/x^+ \\
\dot{x}_u/x^+
\end{bmatrix}
= \begin{bmatrix}
A_c & A_{12}
0 & A_u
\end{bmatrix}
\begin{bmatrix}
x_c \\
x_u
\end{bmatrix}
+ \begin{bmatrix}
B_c
0
\end{bmatrix} u,
\quad x_c \in \mathbb{R}^n, \quad x_u \in \mathbb{R}^{n-\hat{n}},
\]
\[y = \begin{bmatrix}
C_c & C_u
\end{bmatrix}
\begin{bmatrix}
x_c \\
x_u
\end{bmatrix}
+ Du,
\quad u \in \mathbb{R}^k, \quad y \in \mathbb{R}^m,
\]
where \(\hat{n}\) denotes the dimension of the controllable subspace \(C\) of the original system.

47) In page 126, the second equation in the proof of Theorem 14.3 should read
\[(A'x)^*Px + x^*PA'x = \lambda^x x^*Px + \lambda x^*Px = 2\Re[\lambda]x^*Px.
\]

48) In page 127, the three-line equation in the middle of the page should read
\[
\begin{align*}
\dot{P} + PA' - BA' &= \begin{bmatrix}
A_c & A_{12}
0 & A_u
\end{bmatrix}
\begin{bmatrix}
P_c & 0
0 & \rho P_u
\end{bmatrix} + \begin{bmatrix}
P_c & 0
0 & \rho P_u
\end{bmatrix}
\begin{bmatrix}
A'_c & 0
A'_{12} & A'_u
\end{bmatrix} - \begin{bmatrix}
B_c
0
\end{bmatrix}
\begin{bmatrix}
B'_c & 0
\end{bmatrix}
\end{align*}
\]

49) In page 128, Theorem 14.6 should read as follows:

**Theorem 14.6 (Eigenvalue assignment).** Assume that the system
\[
\dot{x}/x^+ = Ax + Bu,
\quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^k \tag{AB-CLTI}
\]
is controllable. Given any set of \(n\) complex numbers \(\{\lambda_1, \lambda_2, \ldots, \lambda_n\}\) in which complex values appear on conjugate pairs, there exists a state feedback matrix \(K \in \mathbb{R}^{k \times n}\) such that the closed-loop system \(\dot{x}/x^+ = (A - BK)x\) has eigenvalues equal to the \(\lambda_i\).

Note the requirement that the eigenvalues must appear in complex conjugate pairs to get a real-valued matrix \(K\).

50) In page 141, the second equation should read
\[y(t) = C(t)\Phi(t, t_0)x_0 + \sum_{\tau=t_0}^{t-1} C(t)\Phi(t, \tau + 1)B(\tau)u(\tau) + D(t)u(t), \quad \forall t_0 \leq t < t_1.
\]

51) In page 142, the two equations for the state given in Theorem 15.4 should read
\[x(t_0) = W_0(t_0, t_1)^{-1} \sum_{t=t_0}^{t_1} \Phi(t, t_0)^t C(t)^t \dot{g}(t),
\]
and
\[x(t_1) = W_{C_0}(t_0, t_1)^{-1} \sum_{t=t_0}^{t_1} \Phi(t, t_1)^t C(t)^t \dot{g}(t).
\]

52) In pages 142–143, Section 15.8, the word “controllable” should be throughout replaced by “reachable” and vice-versa. Additionally, the controllability gramians \(W_C\) and \(\tilde{W}_C\) should be replaced by the reachability gramians \(W_R\) and \(\tilde{W}_R\).
53) In page 143, equation (15.7) should read
\[
\dot{x} = A'\dot{x} + C'\dot{u}, \quad \dot{y} = B'\dot{x} + D'\dot{u}, \quad \dot{x} \in \mathbb{R}^n, \ u \in \mathbb{R}^m, \ \dot{y} \in \mathbb{R}^k.
\]

54) In page 145, the last equation of Section 15.9 should read
\[
W = \sum_{t=0}^{\infty} (A')^t C A^t = \lim_{t \to \infty} W(t, t_1).
\]

55) In page 145, the system in Exercise 15.1 should be
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{bmatrix} x, \quad y = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} x.
\]
The exercise still makes sense with a \( u \) in the output equation, but it is much less interesting.

56) In page 149, equation (16.2), \([B_o, B_u] = T^{-1}B^*\) should be replaced by \([B_o, B_u] = T^{-1}B^*\).

57) In page 152, equation (16.4a) should read
\[
\begin{bmatrix}
\dot{x}_o/x_o^+ \\
\dot{x}_u/x_u^+
\end{bmatrix} = \begin{bmatrix} A_o & 0 \\
A_{u1} & A_u \end{bmatrix} \begin{bmatrix} x_o \\
x_u \end{bmatrix} + \begin{bmatrix} B_o \\
B_u \end{bmatrix} u, \quad x_o \in \mathbb{R}^{n-h}, \ x_u \in \mathbb{R}^n, \ y = \begin{bmatrix} c_o & 0 \end{bmatrix} \begin{bmatrix} x_o \\
x_u \end{bmatrix} + Du, \quad u \in \mathbb{R}^k, \ y \in \mathbb{R}^m,
\]
where \( \hat{n} \) denotes the dimension of the unobservable subspace \( \Omega \) of the original system.

58) In page 152, the definition for detectability should read

**Definition 16.1 (Detectable system).** The pair \((A, C)\) is **detectable** if it is algebraically equivalent to a system in the standard form for unobservable systems (16.4) with \( \hat{n} = 0 \) (i.e., \( A_u \) nonexistent) or with \( A_u \) a stability matrix.

59) In page 154, Theorem 16.9 should read as follows:

**Theorem 16.9.** Assume that the pair \((A, C)\) is observable. Given any set of \( n \) complex numbers \( \{\lambda_1, \lambda_2, \ldots, \lambda_n\} \) in which complex values appear on conjugate pairs, there exists an output injection matrix \( L \in \mathbb{R}^{n \times m} \) such that \( A - LC \) has eigenvalues equal to the \( \lambda_i \).

Note the requirement that the eigenvalues must appear in complex conjugate pairs to get a real-valued matrix \( L \).

60) In page 155, equation (16.10) should read
\[
\begin{bmatrix}
\dot{x}/x^+ \\
\dot{e}/e^+
\end{bmatrix} = \begin{bmatrix} A - BK & -BK \\
0 & A - LC \end{bmatrix} \begin{bmatrix} x \\
e \end{bmatrix}.
\]

61) In page 156, the system in Exercise 16.9 should be
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{bmatrix} x, \quad y = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} x.
\]
The exercise still makes sense with a \( u \) in the output equation, but it is much less interesting.
62) In page 160, the last sentence of Section 17.2 should read as follows: “But since \( \bar{C} \) has only \( \bar{n} < n \) rows, its rank must be lower than \( n \).”

63) In page 162, the equation below (17.13) should read

\[
Ax = \lambda x \iff \begin{cases} 
- \sum_{i=1}^{n} a_i x_i = \lambda x_1 \\
 x_i = \lambda x_{i+1}, \quad i \in \{1, 2, \ldots, n-1\}.
\end{cases}
\]

64) In page 169, the third sentence after equation (18.1) should read as follows: “We recall that the number of roots of \( d(s) \) (i.e., the number of poles) is equal to the dimension of a minimal realization for \( \hat{q}(s) \).”

65) In page 171, the last sentence in the proof of Lemma 18.1 should read as follows: “However, at the roots of \( D_r(s) \), all \( r \times r \) submatrices are singular and therefore have linearly dependent rows and columns.”

66) In page 184, the second section of Section 19.6 should read as follows: “Since \( L(s) \) and \( R(s) \) are unimodular, \( \hat{G}(s) \) has an inverse if and only if \( SM\hat{G}(s) \) is invertible, which happens only when \( m = r \).”

67) In page 185, the last equation should read

\[
\hat{c}(s) = \hat{g}(s)^{-1} \frac{\hat{q}(s)}{1 - \hat{q}(s)} = \frac{k}{s} \hat{q}(s)^{-1}.
\]

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