Errata

1) In page 7, the MATLAB® command should read

\[
\text{sys}\_\text{ss}=\text{ss}(A,B,C,D,\ldots, 'InputName', \{\text{input1'}, \text{input2'}, \ldots\}, \ldots, 'OutputName', \{\text{output1'}, \text{output2'}, \ldots\}, \ldots, 'StateName', \{\text{state1'}, \text{state2'}, \ldots\});
\]

2) In page 15 in Figure 2.3, the angle \(\theta_2\) is incorrectly drawn, it should be drawn as follows:

Moreover, the matrix \(M(q)\) and the vector \(G(q)\) in Example 2.2 should be as follows:

\[
M(q) := \begin{bmatrix}
m_2 \ell_2^2 + 2m_2 \ell_1 \ell_2 \cos \theta_2 + (m_1 + m_2) \ell_1^2 & m_2 \ell_2^2 + m_2 \ell_1 \ell_2 \cos \theta_2 \\
m_2 \ell_1 \ell_2 \cos \theta_2 + m_2 \ell_2^2 & m_2 \ell_2^2 
\end{bmatrix}
\]

\[
G(q) := \begin{bmatrix}
m_2 g \ell_2 \cos(\theta_1 + \theta_2) + (m_1 + m_2) g \ell_1 \cos \theta_1 \\
m_2 g \ell_2 \cos(\theta_1 + \theta_2)
\end{bmatrix}.
\]

3) In page 17, equation (2.8) should read as follows:

\[
\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} v, \quad y = \begin{bmatrix} I & 0 \end{bmatrix} x, \quad x := \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \in \mathbb{R}^{2k}.
\]

4) In page 18, the first equation in Section 2.4.4 should read as follows:

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = M^{-1}(x_1) \left( -B(x_1, x_2) x_2 - G(x_1) + u \right).
\]

5) In page 17 in Figure 2.5, the label \(y\) at the right should be replaced by \(\begin{bmatrix} q \\ \dot{q} \end{bmatrix}\).

6) In page 28, the equation just above definition 3.2 should read

\[
\dot{y}(z) = \mathcal{Z}[y(t)] := \sum_{i=0}^{\infty} z^{-i} y(t), \quad z \in \mathbb{C}.
\]

7) In page 30 in the last equation in Note 4, \(dt\) should be replaced by \(d\hat{t}\), as in:

\[
\mathcal{L}[x * y(t)] = \int_{0}^{\infty} e^{-st} x(t) \left( \int_{0}^{\infty} e^{-st} y(t)d\hat{t} \right) d\tau = \hat{x}(s) \hat{y}(s).
\]
8) In page 34 in the sidebar example in Section 4.3.2, the matrix \( N_3 \) should be
\[
N_3 = \begin{bmatrix} -24 & 3 \\ 1 & 2 \end{bmatrix}
\]

9) In page 35 in the proof of Proposition 4.1, the second equation should read as follows:
\[
Z_n = \frac{1}{s}Z_{n-1}, \quad Z_{n-1} = \frac{1}{s}Z_{n-2}, \quad \ldots, \quad Z_2 = \frac{1}{s}Z_1 \quad \Rightarrow \quad Z_k = \frac{1}{s^{k-1}}Z_1.
\]

10) In page 38, the first command in MATLAB\textsuperscript{®} Hint 17 should read
\[
z=0; p=[-1,-3]; k=2;
\]

11) In page 48, the last equation should read
\[
A^{n+1} = -a_1 A^n - a_2 A^{n-1} - \cdots - a_{n-1} A^2 - a_n A
\]
\[
= a_1 (a_1 A^{n-1} + a_2 A^{n-2} + \cdots + a_{n-1} A + a_n I) - a_2 A^{n-2} - \cdots - a_{n-1} A^2 - a_n A
\]
\[
= (a_1^2 - a_2) A^{n-1} + (a_1 a_2 - a_3) A^{n-2} + \cdots + (a_1 a_{n-1} - a_n) A + a_1 a_n I.
\]

12) In page 50, the forth equation should read
\[
\mathcal{L}^{-1} \left[ \frac{a_1 s^n + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n}{(s - \lambda_1)^m (s - \lambda_2)^{m_2} \cdots (s - \lambda_k)^{m_k}} \right] = a_{11} e^{\lambda_1 t} + a_{12} t e^{\lambda_1 t} + \cdots + \frac{a_{1m_1} t^{m_1-1} e^{\lambda_1 t}}{(m_1 - 1)!}
\]
\[
+ \cdots + a_{k1} t^{m_1} + a_{k2} t^{m_2} + \cdots + \frac{a_{km_k} t^{m_k-1} e^{\lambda_k t}}{(m_k - 1)!}.
\]

13) In page 56, the condition 3 in Theorem 7.2 is incorrect. It should read as follows:

**Theorem 7.2.** For an \( n \times n \) matrix \( A \), the following three conditions are equivalent:

1. \( A \) is semisimple.
2. \( A \) has \( n \) linearly independent eigenvectors.
3. There is a nonzero polynomial without repeated roots that annihilates \( A \); i.e., there is a nonzero polynomial \( p(s) \) without repeated roots for which \( p(A) = 0 \).

Note that condition 3 provides a simple procedure to check for diagonalizability. Since every polynomial that annihilates \( A \) must have each eigenvalue of \( A \) as a root (perhaps with different multiplicities), one simply needs to compute all the distinct eigenvalues \( \lambda_1, \ldots, \lambda_k \) (\( k \leq n \)) of \( A \) and then check if the polynomial \( p(s) = (s - \lambda_1) \cdots (s - \lambda_k) \) annihilates \( A \).

14) In page 57, equation (7.1) should read
\[
A^t = P^{-1} J P P^{-1} J P \cdots P^{-1} J P = P^{-1} J^t P = P^{-1}
\]
\[
\begin{bmatrix}
    J_{11} & 0 & \cdots & 0 \\
    0 & J_{22} & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & J_{ll}
\end{bmatrix}
\]

15) In page 57, the following clarifying sentence should be added after the expression for \( J_i^t \): "The formula above presumes that \( 0! = 1 \) and that any entries in \( J_i^t \) that would contain factorials of negative numbers should be set to zero."
16) In page 58, the first equation should read
\[ e^{At} := \sum_{k=0}^{\infty} \frac{t^k}{k!} A^k. \]

17) In page 65 in Definition 8.1, the condition for exponential stability should read
\[ \|x(t)\| \leq c e^{-\lambda(t-t_0)} \|x(t_0)\|, \quad \forall t \geq 0. \]

18) In page 66, the second inequality in the Attention! box below Theorem 8.1 should read
\[ \|x(t)\| = \|e^{A(t-t_0)} x_0\| \leq \|e^{A(t-t_0)}\| \|x_0\| \leq c e^{-\lambda(t-t_0)} \|x_0\|, \quad \forall t \in \mathbb{R}. \]

19) In page 69 in the proof of Theorem 8.2, the argument regarding how condition 4 implies condition 5, should consider selecting \( Q = I \) (instead of \( Q = -I \) as stated).

20) In page 70 at the end of the proof of Theorem 8.2, the definition of \( \lambda \) should read
\[ \lambda := \frac{\lambda_{\min}[Q]}{\lambda_{\max}[P]}. \]

21) In page 72, the third equation should read
\[ v(t+1) = x'(t+1)Px(t+1) = x'(t)PAx(t). \]

22) In page 72, in the last equation in Section 8.6 should read
\[ v(t+1) = x'(t)(P-Q)x(t) = v(t) - x'(t)Qx(t), \quad \forall t \geq 0. \]

23) In page 72, equation (8.15) should read
\[ \|x(t) - x^{eq}\| \leq c e^{-\lambda(t-t_0)} \|x(t_0) - x^{eq}\|, \quad \forall t \geq t_0. \]

24) In page 76, the first note should read “This equilibrium point is \( x^{eq} = [\pi \ 0]' \), \( u^{eq} = 0 \), \( y^{eq} = \pi \), and therefore \( \delta x := x - x^{eq} = x - [\pi \ 0]' \), \( \delta u := u - u^{eq} = u \), \( \delta y := y - y^{eq} = y - \pi \).”

25) In page 76 and 77, squares are missing in the expression for the eigenvalues of \( A \). Specifically, in page 76, it should read as follows: “The eigenvalues of \( A \) are given by
\[ \det(\lambda I - A) = \lambda \left(\lambda + \frac{b}{2m} \right) + \frac{g}{\ell} = 0 \quad \Leftrightarrow \quad \lambda = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{g}{\ell}}, \]
and therefore the linearized system is exponentially stable, because
\[ -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{g}{\ell}} \]
has a negative real part”; and in page 77, it should read as follows: “The eigenvalues of \( A \) are given by
\[ \det(\lambda I - A) = \lambda \left(\lambda + \frac{b}{2m} \right) - \frac{g}{\ell} = 0 \quad \Leftrightarrow \quad \lambda = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 + \frac{g}{\ell}}, \]
and therefore the linearized system is unstable, because
\[ -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 + \frac{g}{\ell}} > 0. \]
26) In page 82, the first equation should read
\[ u_T(\tau) := \begin{cases} 
0 & 0 \leq \tau < T \\
\pi & \tau \geq T 
\end{cases} \quad \forall \tau \geq 0. \]

27) In page 83, the last equation before Theorem 9.2 should read
\[
\sup_{i \geq 0} \int_{0}^{t} |\dot{g}_{ij}(t-\tau)|d\tau = \sup_{i \geq 0} \int_{0}^{t} |\dot{g}_{ij}(\rho)|d\rho = \int_{0}^{\infty} |\dot{g}_{ij}(\rho)|d\rho.
\]

28) In page 84, the last equation should read
\[
g_{ij}(t) = \mathcal{L}^{-1}[\dot{g}_{ij}(s)] = a_{11}e^{ht} + a_{12}te^{ht} + \ldots + \frac{a_{1m_1}t^{m_1-1}e^{ht}}{(m_1-1)!} \\
+ \ldots + a_{k1}e^{ht} + a_{k2}te^{ht} + \ldots + \frac{a_{km}t^{m_k-1}e^{ht}}{(m_k-1)!}.
\]

29) In page 86, the equation in Exercise 9.1 should read
\[
\dot{x} = \begin{bmatrix} -2 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\
0 \\
-1 \end{bmatrix} u, \\
y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x + u.
\]

30) In page 89, the two equations for \( I_{LQR} \) should read
\[
I_{LQR} := \int_{0}^{\infty} x'Qx + u'Ru \, dt \\
= H(x(\cdot);u(\cdot)) + \int_{0}^{\infty} x'Qx + u'Ru + (Ax + Bu)'Px + x'P(Ax + Bu) \, dt \\
= H(x(\cdot);u(\cdot)) + \int_{0}^{\infty} x'(A'P + PA + C'QC)x + u'Ru + 2u'B'Px \, dt,
\]
and
\[
I_{LQR} = H(x(\cdot);u(\cdot)) + \int_{0}^{\infty} x'(A'P + PA + C'QC - PBR^{-1}B'P)x + (u' + x'K')R(u + Kx) \, dt.
\]

31) In page 90, the closed-loop equation below (10.3) should read
\[
\dot{x} = Ax - BKx = (A - BR^{-1}B'P)x.
\]

32) In page 96, the sentence just above equation (11.2) should read "Similarly, determining the controllable subspace amounts to finding for which vectors \( x_0 \in \mathbb{R}^n \) the equation ..."

33) In page 102, the equation after (11.9) should read
\[
\int_{t_0}^{t_1} \|u(\tau)\|^2 \, d\tau = \int_{t_0}^{t_1} \|\bar{B}(\tau)\Phi(t_1, \tau)\eta_1 + v(\tau)\|^2 \, d\tau \\
= \eta_1' \bar{W}_{p}(t_0, t_1) \eta_1 + \int_{t_0}^{t_1} \|v(\tau)\|^2 \, d\tau + 2\eta_1' \int_{t_0}^{t_1} \Phi(t_1, \tau)B(\tau)v(\tau) \, d\tau.
\]
34) In page 112, the second to last equation should read

\[ B'x, B'A'x, \ldots, B'(A')^{n-1}x, \]

35) In page 114, equation (12.8) should read

\[(A'x)^*Wx + x^*WA'x = \lambda x^*Wx + \lambda x^*Wx = 2\Re[\lambda]x^*Wx.\]

36) In page 114, the last sentence of the paragraph that includes equation (12.8) should read "We conclude that every eigenvector of \( A' \) is not in the kernel of \( B' \), which implies controllability by the eigenvector test."

37) In page 115, the last equation should read

\[(A'x)^*P_{x} x^*PA'x = \lambda x^*P_{x} x + \lambda x^*P_{x} = 2\Re[\lambda]x^*P_{x} = -\|B'x\|^2.\]

38) In page 120, the third sentence in the proof of Theorem 13.2 should read "Since the number of nonzero rows of \( C \) is exactly \( \hat{n} \), all these rows must be linearly independent."

39) In page 121, the last equation should read

\[ T(s) = [C_c \ C_u] \left[ (sl - A_c)^{-1} \times \begin{bmatrix} B_c \\ 0 \end{bmatrix} \right] = C_c(sl - A_c)^{-1}B_c + D_c. \]

40) In page 123, equation (14.1) should read

\[
\begin{bmatrix}
\dot{x}_c/\dot{x}_u^+ \\
\dot{x}_u/\dot{x}_u^+
\end{bmatrix} =
\begin{bmatrix}
A_c & A_{12} \\
0 & A_u
\end{bmatrix}
\begin{bmatrix}
x_c \\
x_u
\end{bmatrix}
+ \begin{bmatrix}
B_c \\
0
\end{bmatrix} u,
\quad x_c \in \mathbb{R}^n, x_u \in \mathbb{R}^{n - \hat{n}},
\]
\[
y = [C_c \ C_u] \begin{bmatrix} x_c \\ x_u \end{bmatrix} + Du,
\quad u \in \mathbb{R}^k, y \in \mathbb{R}^m,
\]

where \( \hat{n} \) denotes the dimension of the controllable subspace \( C \) of the original system.

41) In page 126, the second equation in the proof of Theorem 14.3 should read

\[(A'x)^*P_{x} x^*PA'x = \lambda x^*P_{x} x + \lambda x^*P_{x} = 2\Re[\lambda]x^*P_{x}.\]

42) In page 127, the three-line equation in the middle of the page should read

\[
\begin{align*}
\bar{A}P + PA' - BB' &= \begin{bmatrix} A_c & A_{12} \\ 0 & A_u \end{bmatrix} \begin{bmatrix} P_c & 0 \\ 0 & \rho P_u \end{bmatrix} + \begin{bmatrix} P_c & 0 \\ 0 & \rho P_u \end{bmatrix} \begin{bmatrix} A'_c & 0 \\ A'_{12} & A'_u \end{bmatrix} - \begin{bmatrix} B_c \\ 0 \end{bmatrix} \begin{bmatrix} B'_c \\ 0 \end{bmatrix} \\
&= - \begin{bmatrix} Q_c & -\rho A_{12} P_u \\ -\rho P_u A'_{12} & Q_u \end{bmatrix}
\end{align*}
\]

43) In page 128, Theorem 14.6 should read as follows:

**Theorem 14.6 (Eigenvalue assignment).** Assume that the system

\[
\dot{x}/x^+ = Ax + Bu,
\]

\[ x \in \mathbb{R}^n, \ u \in \mathbb{R}^k \quad (\text{AB-CLTI})\]

is controllable. Given any set of \( n \) complex numbers \( \{\lambda_1, \lambda_2, \ldots, \lambda_n\} \) in which complex values appear on conjugate pairs, there exists a state feedback matrix \( K \in \mathbb{R}^{k \times n} \) such that the closed-loop system \( \dot{x}/x^+ = (A - BK)x \) has eigenvalues equal to the \( \lambda_i \). □
Note the requirement that the eigenvalues must appear in complex conjugate pairs to get a real-valued matrix $K$.

44) In page 141, the second equation should read

$$y(t) = C(t)\Phi(t, t_0)x_0 + \sum_{\tau = t_0}^{t-1} C(t)\Phi(t, \tau + 1)B(\tau)u(\tau) + D(t)u(t), \quad \forall t_0 \leq t < t_1.$$  

45) In page 142, the two equations for the state given in Theorem 15.4 should read

$$x(t_0) = W_O(t_0, t_1)^{-1} \sum_{\tau = t_0}^{t-1} \Phi(t, t_0)^T C(t)^T \hat{y}(t),$$

and

$$x(t_1) = W_C(t_0, t_1)^{-1} \sum_{\tau = t_0}^{t-1} \Phi(t, t_1)^T C(t)^T \hat{y}(t).$$

46) In page 143, equation (15.7) should read

$$\dot{x} = A'x + C'\bar{a}, \quad \hat{y} = B'x + D'\bar{a}, \quad \bar{x} \in \mathbb{R}^n, \ \bar{a} \in \mathbb{R}^m, \ \bar{y} \in \mathbb{R}^k.$$  

47) In page 145, the last equation of Section 15.9 should read

$$W = \sum_{\tau = 0}^{\infty} (A')^T C' CA' = \lim_{t_1-0 \to \infty} W_O(t_0, t_1).$$

48) In page 145, the system in Exercise 15.1 should be

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} x, \quad y = [c_1 \ c_2 \ c_3]^T x.$$  

The exercise still makes sense with $u$ in the output equation, but it is much less interesting.

49) In page 152, equation (16.4a) should read

$$\begin{bmatrix} \dot{x}_u \\ \dot{x}_u^T \end{bmatrix} = \begin{bmatrix} A_0 & 0 \\ A_21 & A_0 \end{bmatrix} \begin{bmatrix} x_u \\ x_u^T \end{bmatrix} + \begin{bmatrix} B_0 \\ B_u \end{bmatrix} u, \quad x_0 \in \mathbb{R}^{n-n}, \ x_u \in \mathbb{R}^{n},$$

$$y = \begin{bmatrix} C_0 & 0 \end{bmatrix} \begin{bmatrix} x_u \\ x_u \end{bmatrix} + Du, \quad u \in \mathbb{R}^k, y \in \mathbb{R}^m,$$

where $\hat{n}$ denotes the dimension of the unobservable subspace $\mathcal{U}O$ of the original system.

50) In page 152, the definition for detectability should read

**Definition 16.1 (Detectable system).** The pair $(A, C)$ is detectable if it is algebraically equivalent to a system in the standard form for unobservable systems (16.4) with $\hat{n} = 0$ (i.e., $A_u$ nonexistent) or with $A_u$ a stability matrix.

51) In page 154, Theorem 16.9 should read as follows:

**Theorem 16.9.** Assume that the pair $(A, C)$ is observable. Given any set of $n$ complex numbers $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ in which complex values appear on conjugate pairs, there exists an output injection matrix $L \in \mathbb{R}^{n \times m}$ such that $A - LC$ has eigenvalues equal to the $\lambda_i$.  

$\square$
Note the requirement that the eigenvalues must appear in complex conjugate pairs to get a real-valued matrix $L$.

52) In page 155, equation (16.10) should read
\[
\begin{bmatrix}
\frac{x}{x^+} \\
\frac{e}{e^+}
\end{bmatrix} = \begin{bmatrix} A - BK & -BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}.
\]

53) In page 156, the system in Exercise 16.9 should be
\[
\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} x, \quad y = [c_1 \ c_2 \ c_3] x.
\]
The exercise still makes sense with a $u$ in the output equation, but it is much less interesting.

54) In page 160, the last sentence of Section 17.2 should read as follows: “But since $\hat{C}$ has only $\hat{n} < n$ rows, its rank must be lower than $n$.”

55) In page 162, the equation below (17.13) should read
\[
\dot{x} = \lambda x \quad \Leftrightarrow \quad \begin{cases} -\sum_{i=1}^{n} q_i x_i = \lambda x_1 \\ x_i = \lambda x_{i+1}, & i \in \{1, 2, \ldots, n-1\}. \end{cases}
\]

56) In page 169, the third sentence after equation (18.1) should read as follows: “We recall that the number of roots of $d(s)$ (i.e., the number of poles) is equal to the dimension of a minimal realization for $\hat{g}(s)$.”

57) In page 171, the last sentence in the proof of Lemma 18.1 should read as follows: “However, at the roots of $D_r(s)$, all $r \times r$ submatrices are singular and therefore have linearly dependent rows and columns.”

58) In page 184, the second section of Section 19.6 should read as follows: “Since $L(s)$ and $R(s)$ are unimodular, $\hat{G}(s)$ has an inverse if and only if $SM_\epsilon(s)$ is invertible, which happens only when $m = r$.”

59) In page 185, the last equation should read
\[
\dot{\epsilon}(s) = \hat{g}(s)^{-1} \frac{\hat{q}(s)}{1 - \hat{q}(s)} = \frac{k}{s} \hat{g}(s)^{-1}.
\]

Acknowledgements

I would like to thank several UCSB students (including Cenk Oguz Saglam, John Simpson-Porco, and Justin Pearson), Prof. Maurice Heemels, Prof. Valeriu Prepelita, Prof. Emre Tuna, and Prof. Bogdan Udrea, for helping me to find and correct several typos in the book.