Zero Dynamics and Tracking Performance Limits in Nonlinear Feedback Systems

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Abstract: Among Alberto Isidori's many seminal contributions, his solution of the nonlinear tracking problem and the underlying concept of zero dynamics have had the widest and strongest impact. Here we use these results to investigate and quantify the limit to tracking performance posed by unstable zero dynamics. While some aspects of this limit are nonlinear analogs of Bode's T-integral formula, the dependence on the exosystem dynamics is an added complexity of nonlinear tracking.

1 Introduction

The concept of nonlinear *zero dynamics* is now firmly placed at the foundation of control theory. Its ability to reveal input-output properties and feedback limitations continues to stimulate many researchers to gain deeper insights and develop new design methods.

The foundational and pioneering role of Alberto Isidori in this area is well known. Some quarter of a century ago he captured the attention of most of active researchers in the field by the astonishing novelty of his ideas and brilliant clarity with which he presented them. Any one of his papers and talks was enough to convert his listeners to his way of thinking and motivate them to ask for his preprints and notes, which he most generously shared with his colleagues.

We use this opportunity to express our gratitude to Alberto Isidori and illustrate how his concepts of non-minimum phase nonlinear systems influenced our research. Isidori and coworkers introduced this concept in the 1980's, within the broader framework of input-output linearization theory [11–13, 15–17]

Difficulties with non-minimum phase linear systems have been known in classical feedback theory for many decades, especially in tracking and dis-

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turbance rejection problems. For linear transfer functions Bode has characterized the limitations of feedback connections via his seminal results on integral invariants in frequency domain. Bode showed that integrals of logarithmic sensitivities are constrained by unstable poles and zeros. Over the last several decades Bode-like results have been obtained for wider classes of linear time-invariant systems (see [4, 9, 21–23] and references therein.)

The frequency domain form of Bode integrals makes it unclear whether such constraints apply to nonlinear systems. Occasionally one would even hear conjectures that, by introducing nonlinearities in the controller, Bode's constraints may be avoided.

A few years ago we approached the nonlinear feedback limitations problem using Isidori's work on nonlinear zero dynamics and normal forms. For this we first had to give a state space interpretation of Bode's integrals. We focused on the T-integral and followed a path which started with the 1972 "cheap control" result of Kwakernaak and Sivan [21]. This led us through the singular perturbation analysis [18,26] to the explicit formulas of Qiu and Davison [23]. For a special case a nonlinear analog of the Bode T-integral was obtained by Seron et al. [24] while our general result is presented in [3]. The purpose of this text is to present a brief review on this line of research.

2 Bode T-Integral and Cheap Control

For single input-single output linear time invariant systems, the best attainable tracking performance is constrained by Bode's T-integral

$$\frac{1}{\pi}\int_0^\infty \log|T(j\omega)|\frac{d\omega}{\omega^2} + \frac{1}{2K_v} = \sum_{i=1}^{p}\frac{1}{\alpha_i}$$

with $T = GK(1+GK)^{-1}$ where G is the plant, K is a minimum phase controller, K_v is the velocity constant and $\alpha_1, \ldots, \alpha_p$ are the unstable zeros. Clearly, perfect tracking of a reference input that would result from $T(j\omega) = 1$ for all ω, is impossible in the presence of unstable zeros of plant G.

With a singular perturbation time scale decomposition of the cheap control tracking problem into the slow minimum energy stabilization of the zero dynamics and a rapid output regulation, Seron et al. [24] showed that the Bode T-invariant is, in fact, the minimum amount of output energy needed to stabilize the zero dynamics. This insight is gained from the linear normal form in which the output is the input into the zero dynamics subsystem and must be used for its stabilization. The amount of energy the output needs to stabilize the unstable zeros is therefore not available for tracking and appears as the energy of the tracking error which remains nonzero even when the gain is allowed to tend to infinity. In other words

$$\frac{1}{\pi}\int_0^\infty \log|T(j\omega)|\frac{d\omega}{\omega^2} + \frac{1}{2K_v} = \lim_{\epsilon\to 0}\frac{1}{2}\int_0^\infty e^2(t)\,dt.$$

Seron et al. have shown that this interpretation of Bode T-constraint applies to nonlinear systems having a normal form in which the system output is the sole input into the zero dynamics subsystem and plays the role of the stabilizing control in the corresponding nonlinear minimum energy problem.

The results and expressions summarized in the above discussion are for the tracking of a step input. To prepare for more general nonlinear results discussed in the next section, we briefly review the tracking problem for linear systems

$$\dot{x} = Ax + Bu, \quad y = Cx + Du,$$

$x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^q$, and reference signals $r(t) \in \mathbb{R}^q$ generated by a known *exosystem*

$$\dot{w} = Sw, \quad r = Qw.$$

Davison [7] and Francis [8] have shown that it is possible to design a feedback controller such that the closed-loop system is asymptotically stable and the output $y(t)$ converges to $r(t)$, if and only if (A, B) is stabilizable, (C, A) is detectable, the number of inputs is at least as large as the number of outputs ($m \geq q$), and the zeros of (A, B, C, D) do not coincide with the eigenvalues of S. The *internal model approach*, [8, 10], designs the tracking controller

$$u(t) = Kx(t) + (\mathit{\Gamma} - K\mathit{\Pi})w(t),$$

where $A + BK$ is Hurwitz, and $\mathit{\Pi}$ and $\mathit{\Gamma}$ satisfy

$$\begin{aligned} \mathit{\Pi} S &= A\mathit{\Pi} + B\mathit{\Gamma}, \\ 0 &= C\mathit{\Pi} + D\mathit{\Gamma} - Q. \end{aligned}$$

Then, the *tracking error* $e(t) := y(t) - r(t)$ converges to zero, and the transients

$$\tilde{x} := x - \mathit{\Pi} w, \quad \tilde{u} := u - \mathit{\Gamma} w$$

are governed by $\dot{\tilde{x}} = (A + BK)\tilde{x}$, $\tilde{u} = K\tilde{x}$.

Kwakernaak and Sivan [21] were the first to consider the cheap control problem

$$J_\epsilon := \min_{\tilde{u}} \int_0^\infty \left[\|y(t) - r(t)\|^2 + \epsilon^2 \|\tilde{u}(t)\|^2 \right] dt$$

and to demonstrate that in the presence of non-minimum phase zeros dynamics the limit $J_\epsilon \to J$ as $\epsilon \to 0$ is strictly positive.

Qiu and Davison [23] showed that for $r(t) = \eta_1 \sin \omega t + \eta_2 \cos \omega t$, $\eta = \mathrm{col}(\eta_1, \eta_2)$, the non-minimum phase zeros $z_1, z_2, \ldots, z_p$ determine the limit J as follows:

$$J = \eta' M \eta, \quad \operatorname{trace} M = \sum_{i=1}^{p} \left(\frac{1}{z_i - j\omega} + \frac{1}{z_i + j\omega} \right).$$

For more general reference signals, Su, Qiu, and Chen [25] give explicit formulas which show the dependence of J on the non-minimum phase zeros and their frequency-dependent directional information.

3 Performance Limits in Nonlinear Feedback Systems

The analogous nonlinear tracking problem

$$\dot{x} = f(x,u), \quad y = h(x,u), \tag{1}$$
$$\dot{w} = s(w), \qquad r = q(w), \tag{2}$$

where $f(0,0) = 0$, $s(0) = 0$, $h(0,0) = 0$, has been analyzed by Isidori and Byrnes [14]. They proved that this problem is solvable if and only if there exist smooth maps $\Pi(w)$ and $c(w)$, satisfying

$$\frac{\partial \Pi}{\partial w} s(w) = f(\Pi(w), c(w)), \quad \Pi(0) = 0, \tag{3a}$$
$$h(\Pi(w), c(w)) - q(w) = 0, \quad c(0) = 0. \tag{3b}$$

In [3] we consider the class of nonlinear systems which are locally diffeomorphic to systems in strict-feedback form (see for example [20, Appendix G])[3]:

$$\dot{z} = f_0(z) + g_0(z)\xi_1, \tag{4a}$$
$$\begin{aligned} \dot{\xi}_1 &= f_1(z,\xi_1) + g_1(z,\xi_1)\xi_2, \\ &\vdots \end{aligned}$$
$$\dot{\xi}_{r_d} = f_{r_d}(z,\xi_1,\dots,\xi_{r_d}) + g_{r_d}(z,\xi_1,\dots,\xi_{r_d})u, \tag{4b}$$
$$y = \xi_1, \tag{4c}$$

where $z \in \mathbb{R}^{n_z}$, $\xi := \operatorname{col}(\xi_1,\dots,\xi_{r_d})$, $\xi_i \in \mathbb{R}^m$, $\forall i \in \{1,\dots,r_d\}$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^m$. $f_i(\cdot)$ and $g_i(\cdot)$ are $\mathcal{C}^k$ functions of their arguments (for some large k), $f_i(0,\dots,0) = 0$, and the matrices $g_i(\cdot)$, $\forall i \in \{1,\dots,r_d\}$ are always nonsingular. We assume that initially the system is at rest, $(z,\xi) = (0,0)$.

When the tracking problem is solvable, that is, when it is possible to design a continuous feedback law that drives the tracking error to zero, there exist maps $\Pi = \operatorname{col}(\Pi_0,\dots,\Pi_{r_d})$, $\Pi_0 : \mathbb{R}^p \to \mathbb{R}^{n_z}$, $\Pi_i : \mathbb{R}^p \to \mathbb{R}^m$, $\forall i \in$

[3] When convenient we use the compact form (1) for (4). In that case, $f(\cdot)$ denotes the vector field described by the right-hand-side of (4a)–(4b), $h(\cdot)$ the output map described by (4c), and $x = \operatorname{col}(z,\xi_1,\dots,\xi_{r_d})$.

$\{1,\ldots,r_d\}$, and $c:\mathbb{R}^p\to\mathbb{R}^m$ that satisfy (3). The locally diffeomorphic change of coordinates

$$\tilde{z} = z - \Pi_0(w), \tag{5a}$$
$$\tilde{\xi} := \mathrm{col}(\tilde{\xi}_1,\ldots,\tilde{\xi}_{r_d}), \tag{5b}$$
$$\tilde{\xi}_i = \xi_i - \Pi_i(w), \quad i=1,\ldots,r_d \tag{5c}$$
$$\tilde{u} = u - c(w), \tag{5d}$$

transforms the system (4) into the *error system*

$$\begin{aligned}
\dot{\tilde{z}} &= \tilde{f}_0(\tilde{z},w) + \tilde{g}_0(\tilde{z},w)e,\\
\dot{\tilde{\xi}}_1 &= \tilde{f}_1(\tilde{z},\tilde{\xi}_1,w) + \tilde{g}_1(\tilde{z},\tilde{\xi}_1,w)\tilde{\xi}_2,\\
&\;\vdots\\
\dot{\tilde{\xi}}_{r_d} &= \tilde{f}_{r_d}(\tilde{z},\tilde{\xi}_1,\ldots,\tilde{\xi}_{r_d},w) + \tilde{g}_{r_d}(\tilde{z},\tilde{\xi}_1,\ldots,\tilde{\xi}_{r_d},w)\tilde{u},\\
e &= \tilde{\xi}_1,
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
\tilde{f}_0 &:= f_0(\tilde{z}+\Pi_0(w)) - f_0(\Pi_0(w)) + \Big[g_0(\tilde{z}+\Pi_0(w)) - g_0(\Pi_0(w))\Big]q(w),\\
\tilde{g}_0 &:= g_0(\tilde{z}+\Pi_0(w)),
\end{aligned}$$

$\tilde{f}_0(0,w)=0$, $\tilde{g}_0(\tilde{z},0)=g_0(\tilde{z})$, and $\tilde{f}_i(\cdot)$, $\tilde{g}_i(\cdot)$, $\forall i\in\{1,\ldots,r_d\}$ are appropriately defined functions that satisfy $\tilde{f}_i(0,\ldots,0,w)=0$ and $\tilde{g}_i(\tilde{z},\ldots,\tilde{\xi}_i,0)=g_i(\tilde{z},\ldots,\tilde{\xi}_i)$.

As in the work of Seron et al. [24], the singular perturbation separation of time scales gives rise to the following two optimal control problems:

Cheap control problem: For the system consisting of the error system (6) and the exosystem (2) with initial condition $\big(\tilde{z}(0),\tilde{\xi}(0),w(0)\big)=\big(\tilde{z}_0,\tilde{\xi}_0,w_0\big)$, find the optimal feedback law $\tilde{u}=\alpha^{cc}_{\delta,\epsilon}(\tilde{z},\tilde{\xi},w)$ that minimizes the cost functional

$$\frac{1}{2}\int_0^\infty \left(\|e(t)\|^2 + \delta\|\tilde{z}(t)\|^2 + \epsilon^{2r_d}\|\tilde{u}(t)\|^2\right)dt$$

for $\delta>0$, $\epsilon>0$. We denote by $J^{cc}_{\delta,\epsilon}(\tilde{z}_0,\tilde{\xi}_0,w_0)$ the corresponding optimal value. The best-attainable cheap control performance for tracking is then

$$J := \lim_{(\delta,\epsilon)\to 0} J^{cc}_{\delta,\epsilon}(\tilde{z}_0,\tilde{\xi}_0,w_0).$$

As shown by Krener [19], in some neighborhood of $(0,0,0)$ and for every $\delta>0$, $\epsilon>0$, the value $J^{cc}_{\delta,\epsilon}(\cdot,\cdot,\cdot)$ is $\mathcal{C}^{k-2}$ under the following assumption:

Assumption 1 *The linearization around* $(z, \xi) = (0, 0)$ *of system* (4) *is stabilizable and detectable, and the linearization around* $w = 0$ *of the exosystem* (2) *is stable.*

The fast part of the cheap control problem describes the rapid transient of $e(t)$ to its slow part represented by the minimum energy problem for the stabilization of zero dynamics:

Minimum-energy problem: For the system

$$\dot{\tilde{z}} = \tilde{f}_0(\tilde{z}, w) + \tilde{g}_0(\tilde{z}, w)e, \qquad \tilde{z}(0) = z_0, \tag{7a}$$

$$\dot{w} = s(w), \qquad w(0) = w_0, \tag{7b}$$

with e viewed as the input, find the optimal feedback law $e = \alpha_\delta^{me}(\tilde{z}, w)$ that minimizes the cost

$$\frac{1}{2}\int_0^\infty \left(\delta\|\tilde{z}(t)\|^2 + \|e(t)\|^2\right) dt,$$

for $\delta > 0$. We denote by $J_\delta^{me}(\tilde{z}_0, w_0)$ the corresponding optimal value. Under Assumption 1, $J_\delta^{me}(\cdot,\cdot)$ is $\mathcal{C}^{k-2}$ in some neighborhood of $(0, 0)$.

Our analysis reveals that the best-attainable cheap control performance J is equal to the least control effort (as $\delta \to 0$) needed to stabilize the corresponding zero dynamics system (7) driven by the tracking error e.

Theorem 1. *Suppose that Assumption 1 holds and that* (3) *has a solution in some neighborhood of* $w = 0$*. Then, for any* $(\tilde{z}(0), \tilde{\xi}(0), w(0)) = (\tilde{z}_0, \tilde{\xi}_0, w_0)$ *in some neighborhood of* $(0, 0, 0)$ *there exists a solution to the cheap control problem and the limit to tracking performance is*

$$J = \lim_{\delta \to 0} J_\delta^{me}$$

□

A more detailed analysis leading to this theorem and its proof are soon to appear in [3].

For linear systems we obtain the following:

Corollary 1. *For linear systems with unstable zero-dynamics subsystem described by*

$$\dot{z} = F_0 z + G_0 y,$$

the limit to tracking performance is

$$J = \lim_{\delta \to 0} \frac{1}{2}\omega_0' \Pi_0' P_0(\delta) \Pi_0 \omega_0, \tag{8}$$

where $\omega_0 = \omega(0)$*, and* Π_0 *and* $P_0 > 0$ *satisfy*

$$\Pi_0 S = F_0 \Pi_0 + G_0 Q, \tag{9a}$$

$$F_0' P_0 + P_0 F_0 + \delta I = P_0 G_0 G_0' P_0. \tag{9b}$$

□

Formula (8) follows from the fact that the equations for the minimum-energy problem are

$$\begin{aligned}\dot{\tilde{z}} &= F_0 \tilde{z} + G_0 e,\\ \dot{w} &= Sw\end{aligned}$$

where $\tilde{z} = z - \Pi_0 \omega$, and Π_0 is the solution of (9a). In this case the optimal feedback law for the minimum energy problem is $e = -G_0' P_0 \tilde{z}$ where $P_0 > 0$ is the solution of (9b), and $\frac{1}{2}\tilde{z}_0' P_0(\delta)\tilde{z}_0$ the corresponding optimal value. Note that $\tilde{z}_0 = z(0) - \Pi_0 \omega(0) = -\Pi_0 \omega_0$.

4 Illustrative Example

To illustrate the above results and show how the limits of tracking performance for nonlinear systems depend on the exosystem dynamics, we consider the following system

$$\dot{z} = -z + z^2 + \xi_1, \tag{10a}$$

$$\dot{\xi}_1 = \xi_2, \tag{10b}$$

$$\dot{\xi}_2 = u, \tag{10c}$$

$$y = \xi_1,$$

which is already in normal form. The zero-dynamics subsystem given by (10a) with $\xi_1 \equiv 0$ has an asymptotically stable equilibrium at $z = 0$. Suppose that the tracking task is to asymptotically track any reference $r(t)$ generated by the exosystem

$$\dot{\omega}_1 = a\omega_2, \tag{11a}$$

$$\dot{\omega}_2 = -a\omega_1, \tag{11b}$$

$$r(t) = q(\omega_1, \omega_2), \tag{11c}$$

where $a > 0$ and $q(\omega_1, \omega_2)$ is to be chosen later. When the maps $\Pi(w)$ and $c(w)$ satisfying (3) exist, we apply (5) and obtain the error system

$$\dot{\tilde{z}} = \big(2\Pi_0(\omega) - 1\big)\tilde{z} + \tilde{z}^2 + e, \tag{12a}$$

$$\dot{\tilde{\xi}}_1 = \tilde{\xi}_2, \tag{12b}$$

$$\dot{\tilde{\xi}}_2 = \tilde{u}, \tag{12c}$$

$$e = \tilde{\xi}_1.$$

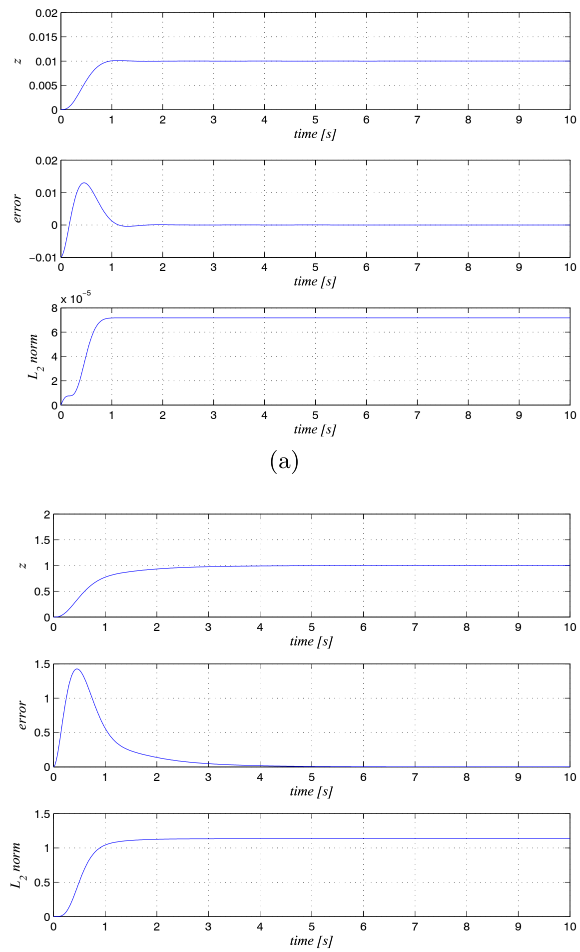

Fig. 1. State z, tracking error e, and $\int_0^t \|e(\tau)\|^2 \, d\tau$ for (a) $\omega(0) = (0.1, 0)'$ and (b) $\omega(0) = (1, 0)'$ (Note the 10^{-5} scale factor !).

The zero-dynamics of the error system are governed by (12a) with $e \equiv 0$ and (11a)–(11b). Clearly, the stability of the zero dynamics and hence the limits of tracking performance depend on the exosystem. In particular, the zero-dynamics are unstable for $2\Pi_0(\omega) > 1$. To illustrate this we let

$$\Pi_0(\omega_1, \omega_2) = \omega_1^2 + \omega_2^2$$

and then evaluate the corresponding $q(\omega_1, \omega_2)$ from

$$\frac{\partial \Pi_0}{\partial \omega_1} a\omega_2 - \frac{\partial \Pi_0}{\partial \omega_2} a\omega_1 = -\Pi_0(\omega_1, \omega_2) + \Pi_0(\omega_1, \omega_2)^2 + q(\omega_1, \omega_2)$$

as dictated by (3). We use this $q(\omega_1, \omega_2)$ in (11c) and perform a series of simulations. To compare the transient errors with different initial conditions $\omega(0) = \omega_0$, we define the normalized transient error $\bar{J} := \frac{J}{\|\omega_0\|^2}$.

Fig. 1 displays the simulation results obtained with $a = 1\, rad/s$ and using a feedback law of the form $u = c(\omega) + k_0(z - \Pi_0(\omega)) + k_1(\xi_1 - \Pi_1(\omega)) + k_2(\xi_2 - \Pi_2(\omega))$. The initial conditions are $(z, \xi_1, \xi_2) = 0$, $\omega(0) = (0.1, 0)'$. In this case $\Pi_0(\omega_1, \omega_2) = 0.01$, so that the subsystem (12a) is locally input-to-state stable and the convergence to the desired reference signal is achieved with a negligibly small transient error $J \simeq 7.2 \times 10^{-5}$ and $\bar{J} \simeq 7.2 \times 10^{-3}$.

In contrast, Fig. 1(b) shows the simulation results obtained with the same controller but with initial condition $\omega(0) = (1, 0)'$ which implies that $\Pi_0(\omega_1, \omega_2) = 1$ and, hence, the error zero-dynamics are not input-to-state stable. As it can be seen, the transient error and its normalized version have increased by several orders of magnitude to $J = \bar{J} \simeq 1.1$.

5 Concluding Remarks

We have shown that, analogous to the tracking problem for linear time-invariant non-minimum phase systems, the tracking performance for nonlinear non-minimum phase systems cannot be improved beyond a limit determined by the least amount of energy required to stabilize the zero dynamics of the tracking error system. In the nonlinear problem, these zero dynamics depend on the dynamics of the exosystem, which may destabilize them for some reference signals, as illustrated on an example.

Since non-minimum phase phenomena create a fundamental limit to the tracking performance that can not be removed by any controller redesign, a direction of practical interest would be to search for reformulations of the tracking problem that would be free of limitations, but still meaningful for applications. One such reformulation, pursued in our work [1, 2, 5, 6] is to replace the tracking problem by a less demanding path following problem, in which the speed along the prescribed geometric path is used as a free design parameter. As shown in [3], for a class of path following problems the limitations of the tracking problems can be avoided.

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