Supervisory Control of Families of Noise Suppressing Controllers*

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Abstract

This paper describes a high-level "supervisor" capable of switching into feedback with a single-input/single-output (SISO), linear time-invariant (LTI) process a sequence of linear controllers, so as to suppress from the output of the process the effect of a persistent disturbance input consisting of the sum of a finite number of sinusoids of unknown amplitude, phase, and frequency. Each of the controllers being supervised is designed so as to solve the disturbance rejection problem for a specific set of disturbance frequencies. The stability results stated in this paper are valid when the frequencies of the disturbance sinusoids are known to belong to a finite set.

1 Introduction

It is not uncommon for the input to a process to be corrupted by a persistent signal which cannot be measured. This paper deals with this situation. Our goal is to asymptotically suppress from the output y of a SISO, LTI process the effect of a noise signal or disturbance $\mathbf{d}$ which is added to its input u. We assume the disturbance is equal to the sum a finite number of sinusoids of unknown amplitude, phase, and frequency.

Control algorithms to solve the above problem have a wide range of applications in active noise and vibration control in which the disturbance $\mathbf{d}$ is typically due to rotating machinery. Typical applications include noise cancellation in automobile engines [1], turboprop aircraft engines [2], and ventilation systems [3]. In such applications, the frequency and amplitude of the disturbance may vary due to changes in the operating conditions of the systems that generate it. We are particularly interested in situations in which practical considerations prevents the addition of sensors capable of accurately measuring the disturbance.

Were the frequencies of the sinusoids in $\mathbf{d}$ known, we could design a linear time invariant controller based on the internal model principle to asymptotically reject the effect of the disturbance $\mathbf{d}$ from the output y of the process [4]. Since the frequencies are not presumed known, we propose a supervisory control system which orchestrates the switching among a family of candidate controllers, each designed for a specific set of disturbance frequencies, so as to reject the effect of $\mathbf{d}$ from y. Following certainty equivalence, controller selection is based on the current estimate of what the frequencies of $\mathbf{d}$. The algorithm proposed is inspired by [5, 6, 7].

The problem of suppressing from the output of a process the effect of a disturbance equal to the sum a finite number of sinusoids of unknown amplitude, phase, and frequency, is addressed in [8, 9, 10, 11]. In [8] a model reference adaptive algorithm is proposed to solve this problem with an unknown process model. To this effect the family of model reference controllers is over-parameterized to allow for disturbance cancellation. Although in [8] the problem is solved only for one-dimensional processes, the approach seems to be generalizable to higher dimensional systems which are minimum phase. In [9] an adaptive control law based on the Youla parameterization [12] is proposed to solve the problem considered here. The resulting closed loop system is shown to be locally exponentially stable. In [10] two adaptive algorithms based on exact cancellation of a disturbance consisting of a single sinusoid are considered. These algorithms make use of online estimation of disturbance's amplitude, phase, and frequency. The analysis given is approximate and only valid when the initial estimates are close to their true values.

The remaining of this paper is organized as follows. The problem addressed here is formalized in Section 2

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and Section 3 describes the supervisory control system proposed. The main result of the paper—namely, that the proposed control law is capable of asymptotically suppressing from the output of the process the effect of the disturbance—is stated and proved in Section 4. Finally Section 5 contains some concluding remarks and direction for future research.

2 Problem

The problem of interest is to construct a control system capable of asymptotically suppressing from the output y of a process Σ_P the effect of a persistent disturbance input $\mathbf{d}$ consisting of the sum of a finite number of sinusoids of unknown amplitude, phase, and frequency. The process is presumed to admit a SISO, LTI model with strictly proper transfer function $\tau_P(s)$ and realization

$$\dot{x}_P = A_P x_P + b_P(u + \mathbf{d}), \qquad y = c_P x_P \qquad (1)$$

where y denotes the measured output and u the control signal. It is assumed that $\mathbf{d}$ consists of the sum of m sinusoids whose frequencies $\omega^{(1)}, \omega^{(2)}, \ldots, \omega^{(m)}$ all lie within a pre-specified closed, bounded subset Ω on the positive real line. Without loss of generality, we assume that the elements of the frequency list $\omega \triangleq \{\omega^{(1)}, \omega^{(2)}, \ldots, \omega^{(m)}\}$ are ordered so that $\omega^{(1)} \leq \omega^{(2)} \leq \cdots \leq \omega^{(m)}$. We write $\mathcal{P}$ for the set of all such lists.

Presumed given is an m-parameter family of "off-the-shelf" feedback controller transfer functions $\mathcal{K} \triangleq \{\kappa_p : p \in \mathcal{P}\}$, with at least the following properties:

Pole Content Property: *For each element $p^{(i)}$ of each list $p = \{p^{(1)}, p^{(2)}, \ldots, p^{(m)}\} \in \mathcal{P}$, $s = jp^{(i)}$ is either a transmission zero of $\tau_P(s)$ or a pole of $\kappa_p(s)$, but not both.*

Stability Margin Property: *There is a positive constant λ such that for each $p \in \mathcal{P}$, $-\lambda$ is greater than the real parts of all of the closed-loop poles*[1] *of the feedback interconnection*

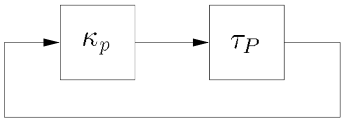

Figure 1: Feedback Interconnection

Also presumed given is an integer $n_C \geq 0$ and a family of n_C-dimensional realizations $\{A_p, b_p, f_p, g_p\}$, one for each $\kappa_p \in \mathcal{K}$. These realizations are required to be chosen so that for each $p \in \mathcal{P}$, $(c_p, \lambda I + A_p)$ is detectable and $(\lambda I + A_p, b_p)$ is stabilizable. As noted in [13], there are many different ways to construct such realizations, once one has in hand an upper bound n_κ on the McMillan Degrees of the κ_p. Given such a family of realizations, the sub-system to be supervised is thus of the form shown in Figure 2, where $\Sigma_C(\sigma)$ is the

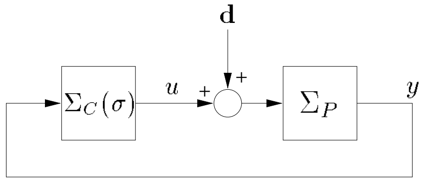

Figure 2: Supervised Sub-System

n_C-dimensional "state-shared" dynamical system

$$\dot{x}_C = A_\sigma x_C + b_\sigma y, \qquad u = f_\sigma x_C + g_\sigma y, \qquad (2)$$

called a *multi-controller* and σ is a piecewise constant *switching signal* taking values in $\mathcal{P}$.

As a consequence of the Pole Content Property, if we were to set

$$\sigma(t) = \omega, \qquad t \geq 0 \qquad (3)$$

then this would cause y converge to zero at $t \to \infty$ [4]. Since the list ω is not presumed to be known, implementation of (3) is not possible. What we shall do instead is to construct a provably correct "supervisor" which is capable of generating σ so as to achieve 1. *global boundedness* {of all system signals} and 2. *asymptotic regulation* {i.e., $y \to 0$}.

3 Estimator-Based Supervision

The supervisor to be considered can be explained informally in terms of the "multi-estimator" architecture shown in Figure 3.

Here each y_p is a suitably defined estimate of y which would be asymptotically correct if the disturbance $\mathbf{d}$ consisted of a sum of m sinusoids whose frequencies were exactly the components of the m-vector p. For each $p \in \mathcal{P}$, $e_p \triangleq y_p - y$ denotes the pth output estimation error and π_p is a "normed" value of e_p or a "performance signal" which is used by the supervisor to assess the potential performance of controller p. Σ_H is a switching logic whose function is to determine σ on the basis of the current values of the π_p.

The underlying decision making strategy used by an estimator-based supervisor is basically this: From time to time select for σ, that candidate control index

[1]By the closed-loop poles are meant the zeros of the polynomial $\rho_p\beta - \mu_p\alpha$, where $\frac{\alpha}{\beta}$ and $\frac{\mu_p}{\rho_p}$ are the reduced transfer functions τ_P and κ_p respectively.

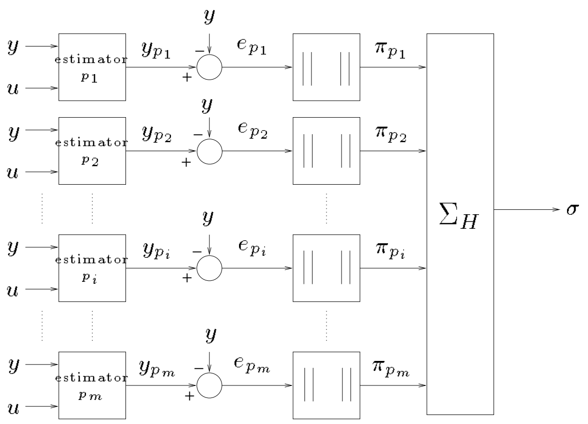

Figure 3: Estimator-Based Supervisor

q whose corresponding performance signal π_q is the smallest among the π_p, $p \in \mathcal{P}$. Motivation for this idea is straightforward: the set of disturbance frequencies whose associated performance signal is the smallest, "best" approximates what the true set of frequencies is, and thus the candidate controller designed on the basis of that set of disturbance frequencies ought to be able to do the best job of rejecting $\mathbf{d}$.

The specific supervisor of interest—shown in Figure 4—, while input-output equivalent to the supervisor just described, admits a slightly different realization than that shown in Figure 3. Internally the supervisor we want to discuss consists of three subsystems: a *multi-estimator dynamic* Σ_E, a *performance-weight generator* Σ_W, and a *scale-independent hysteresis switching logic* Σ_H.

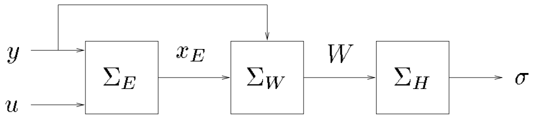

Figure 4: Estimator-Based Supervisor

Σ_E is a n_E-dimensional linear dynamical system of the form

$$\dot{x}_E = \begin{bmatrix} A_E & 0 \\ 0 & A_E \end{bmatrix} x_E + \begin{bmatrix} b_E \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ b_E \end{bmatrix} u \qquad (4)$$

where $n_E \triangleq 2(n_\tau + 2m)$ and (A_E, b_E) is a parameter-independent, $(n_\tau + 2m)$-dimensional SISO, controllable pair with $\lambda I + A_E$ asymptotically stable. Here n_τ is the McMillan degree of the process transfer function $\tau_P(s)$. In [5] it is explained how to construct a continuous function $p \longmapsto c_p$ so that for each $p \in \mathcal{P}$,

$$1 - c_p \begin{bmatrix} \phi_E \\ 0 \end{bmatrix} = \frac{\gamma_p \beta}{\omega_E}, \qquad c_p \begin{bmatrix} 0 \\ \phi_E \end{bmatrix} = \frac{\gamma_p \alpha}{\omega_E} \qquad (5)$$

where ω_E is the characteristic polynomial of A_E; $\phi_E \triangleq (sI - A_E)^{-1} b_E$; α and β are coprime polynomials with β monic such that $\tau_P = \alpha/\beta$; and, for each $p \in \mathcal{P}$, $\gamma_p(s) = \Pi_{i=1}^m \left(s^2 + \left(p^{(i)}\right)^2\right)$, where $p^{(i)}$ is the ith component of p. The c_p are used in the definition of Σ_W which will be given in a moment. The c_p also enable us to define *output estimates*

$$y_p \triangleq c_p x_E, \qquad p \in \mathcal{P} \qquad (6)$$

and the corresponding *output estimation errors*

$$e_p \triangleq y_p - y, \qquad p \in \mathcal{P} \qquad (7)$$

Since $\mathbf{d}$ is the sum of m sinusoids whose frequencies are the components of ω, one has[2]

$$\gamma_\omega \mathbf{d} = 0 \qquad (8)$$

From this and (5), one can deduce that e_ω must go to zero as fast as $e^{-\lambda t}$ {cf. Appendix}. While the error signals defined by (7) are not actually generated by the supervisor, they play an important role in explaining how the supervisor functions.

The supervisor's second subsystem, Σ_W, is a causal dynamical system whose inputs are x_E and y and whose state and output W is a "weighting matrix" which takes values in a linear space $\mathcal{W}$. W together with a suitably defined *performance function* $\Pi : \mathcal{P} \times \mathcal{W} \to \mathbb{R}$ determine a family of scalar-valued *performance signals* of the form

$$\pi_p \triangleq \Pi(p, W), \qquad p \in \mathcal{P} \qquad (9)$$

Each π_p is viewed by the supervisor as a measure of the expected performance of controller $\Sigma_C(p)$. Σ_W and Π are defined by

$$\dot{W} = -2\lambda W + \begin{bmatrix} x_E \\ y \end{bmatrix} \begin{bmatrix} x_E \\ y \end{bmatrix}', \qquad W(0) > 0 \qquad (10)$$

and

$$\Pi(p, W) = \begin{bmatrix} c_p & -1 \end{bmatrix} W \begin{bmatrix} c_p & -1 \end{bmatrix}' \qquad (11)$$

respectively. The definitions of Σ_W and Π are prompted by the observation that if π_p are given by (9), then

$$\dot{\pi}_p = -2\lambda \pi_p + e_p^2, \qquad \pi_p(0) > 0, \qquad p \in \mathcal{P} \qquad (12)$$

because of (7), (10) and (11).

The supervisor's third subsystem, called a *scale-independent hysteresis switching logic* Σ_H, is a hybrid dynamical system whose input is W and whose state and output is σ. To specify Σ_H it is necessary to first pick a positive number $h > 0$ called a *hysteresis constant*. Σ_H's internal logic is then defined by the computer diagram shown in Figure 5 where the π_p are defined by (9) and, at each time t, q denotes the element

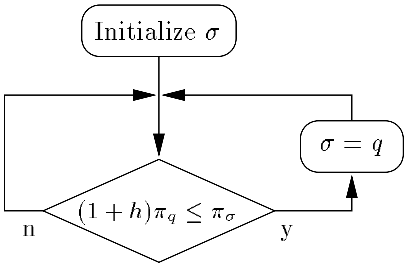

Figure 5: Computer Diagram of Σ_H.

of $\mathcal{P}$ that minimizes $\Pi(q, W)$. In interpreting this diagram it is to be understood that σ's value at each of its switching times $\bar{t}$ is equal to its limit from above as $t \downarrow \bar{t}$. Thus if $\bar{t}_i$ and $\bar{t}_{i+1}$ are any two successive switching times, then σ is constant on $[\bar{t}_i, \bar{t}_{i+1})$. The functioning of Σ_H is roughly as follows. Suppose that at some time t_0, Σ_H has just changed the value of σ to p. σ is then held fixed at this value unless and until there is a time $t_1 > t_0$ at which $(1+h)\pi_q \leq \pi_p$ for some $q \in \mathcal{P}$. If this occurs, σ is set equal to q and so on.

4 Main Result

The overall system just described, admits the block diagram in Figure 6. The following theorem is the main

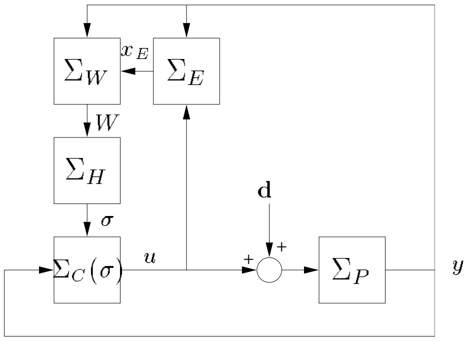

Figure 6: Supervisory Control System

result of this paper.

Theorem 1 *Assume that the set Ω of admissible frequencies is finite. For any initialization of the closed-loop system with $W(0) > 0$ and any disturbance signal $\mathbf{d}$ consisting of the sum of any m sinusoids whose frequencies are the components of some m-vector in $\mathcal{P}$, the signals x_C, x_E, and W are bounded. Moreover, y converges to 0 faster than $e^{-\lambda t}$.*

Proof of Theorem 1. It is shown in the Appendix that for any piecewise-constant signal $s : [0, \infty) \to \mathcal{P}$ and any initialization of the system consisting of (1), (2), (4), and (10) with $\sigma = s$, e_ω converges to zero faster than $e^{-\lambda t}$. From this and

$$e^{2\lambda t}\pi_\omega(t) = \int_0^t e^{2\lambda\tau}e_\omega^2 d\tau + \pi_\omega(0)$$

{cf. equation (12)} we further conclude that $e^{2\lambda t}\pi_\omega$ is bounded on $[0, \infty)$. It follows that if we define the *scaled performance signals*

$$\bar{\pi}_p(t) = e^{2\lambda t}\pi_p(t), \qquad t \geq 0, \quad p \in \mathcal{P} \qquad (13)$$

then (i) $\bar{\pi}_\omega$ will be bounded on $[0, \infty)$. Moreover, because of (12),

$$\dot{\bar{\pi}}_p = e^{2\lambda t}e_p^2, \qquad \bar{\pi}_p(0) > 0 \qquad p \in \mathcal{P}$$

thus we also conclude that (ii) each $\bar{\pi}_p$ is positive and has a limit as $t \to \infty$. This is because each $\bar{\pi}_p$ is a nondecreasing functions of t that starts positive. In the sequel we make use of the above properties (i) and (ii).

The interconnected system defined by (1), (2), (4), and (10) is a dynamical system of the form

$$\dot{z} = f_\sigma(z), \qquad \pi_p = g_p(z), \qquad p \in \mathcal{P} \qquad (14)$$

where $z \triangleq \{x_P, x_C, x_E, W\}$ and, for each $p \in \mathcal{P}$, f_p and g_p are locally Lipschitz. Because of the hysteresis constant h, for each initial state $\{z(0); \sigma(0)\}$ for with $W(0) > 0$, there must be an interval $[0, T)$ of maximal length on which there is a unique pair $\{z; \sigma\}$ with z continuous and σ piecewise constant, which satisfies (14) assuming σ is generated by the scale-independent hysteresis switching logic [14]. Moreover, on each proper subinterval $[0, \tau) \subset [0, T)$, σ can switch at most a finite number of times. Suppose that T was finite. Since $\mathbf{d}$ is bounded and the system defined by (1), (2), and (4) is linear for every piecewise constant σ, one would conclude that x_P, x_C, x_E, u, and y were bounded on $[0, T)$. Moreover, W would also be bounded, because of (10). In this case, the solution to (1), (2), (4), and (10) could be continued onto at least an open half interval of the form $[T, T_1)$ thereby contradicting the hypothesis that $[0, T)$ is the system's interval of maximal existence. By contradiction we can therefore conclude that $T = \infty$.

The term "scale-independence" is prompted by the fact that if χ is any piecewise continuous signal which is positive on $[0, \infty)$, the state σ of the switching logic remains unchanged if each performance signal π_p, $p \in \mathcal{P}$ is replaced $\chi\pi_p$. Thus, because of (13), and as far as the signal σ is concerned, we can think of the switching logic as being driven by the scaled performance signals $\bar{\pi}_p \triangleq \chi\pi_p$ with $\chi(t) \triangleq e^{2\lambda t}$, $t \geq 0$. The facts that $\mathcal{P}$ is a finite set and that the $\bar{\pi}_p$ possess properties (i) and (ii) noted above, enable us to exploit the Scale-independent Hysteresis Switching Lemma [14, 15] and consequently to draw the following conclusion.

[2] For any signal ψ and polynomial ξ we use the notation $\xi(s)\psi$ to denote the action of the differential operator polynomial $\xi(s)|_{s=d/dt}$ on ψ.

Lemma 1 *For any initialization of the close-loop system with $W(0) > 0$, let $\{x_P, x_C, x_E, W, \sigma\}$ denote the unique solution to (1), (2), (4) and (10) with σ generated by Σ_H. There is a time $T^* < \infty$ beyond which σ is constant and $e^{2\lambda t}\pi_{\sigma(T^*)}(t)$ is bounded on $[0, \infty)$.*

Let x_P, x_C, x_E, W, σ, and T^* be as in Lemma 1 and set $q \triangleq \sigma(T^*)$. For $t \geq T^*$, $\sigma(t) = q$ and therefore $\Sigma_C(\sigma)$ is a linear time-invariant system with the transfer function κ_q. Because of the stability margin property and the boundedness of $\mathbf{d}$, the signals x_P, x_C, y, and u are bounded on $[T^*, \infty)$. Boundedness of x_E and W on $[0, \infty)$ follows because of (4), (10), and the asymptotic stability of A_E. Therefore, z is bounded on $[0, \infty)$. Because of (12) and the observation that $e^{2\lambda t}\pi_{\sigma(T^*)}(t)$ is bounded on $[0, \infty)$, we further conclude that

$$\int_0^\infty e_q^2 d\tau \leq \int_0^\infty (e^{\lambda\tau} e_q)^2 d\tau < e^{2\lambda T}\pi_q < \infty$$

Thus, $e_q \in \mathcal{L}_2$. Moreover $\dot{e}_q$ is bounded on $[0, \infty)$, because of (7), (14) and boundedness of z. Therefore, e_q converges to 0 as t tends to ∞ [16, Lemma 1, p. 58].

Because of equations (4), (5), and (6), we conclude that

$$\omega_E y_q = (\omega_E - \gamma_q \beta) y + \gamma_q \alpha u \qquad (15)$$

Since the transfer function of the process is $\frac{\alpha}{\beta}$,

$$\beta y = \alpha(u + \mathbf{d}) \qquad (16)$$

because of (1). Using (16) to eliminate αu from (15) yields

$$\omega_E (y_q - y) = -\alpha\gamma_q \mathbf{d}$$

This and equation (7) further imply that

$$\omega_E e_q = -\alpha\gamma_q \mathbf{d} \qquad (17)$$

Since $\mathbf{d}$ is a sum of m sinusoids, $\alpha\gamma_q\mathbf{d}$ is also a sum of m sinusoids. Hence one can write $\alpha\gamma_q\mathbf{d}$ as

$$\alpha\gamma_q \mathbf{d} = \sum_{i=1}^m \left(A^{(i)} e^{j\omega^{(i)} t} + B^{(i)} e^{-j\omega^{(i)} t}\right) \qquad (18)$$

where each $A^{(i)}$ and $B^{(i)}$ is a fixed complex number. From equation (17) and (18), we get

$$\omega_E e_q = -\sum_{i=1}^m \left(A^{(i)} e^{j\omega^{(i)} t} + B^{(i)} e^{-j\omega^{(i)} t}\right) \qquad (19)$$

Since (19) is a stable differential equation forced by a sinusoidal term and its solution e_q converges to zero, we must have

$$\sum_{i=1}^m \left(A^{(i)} e^{j\omega^{(i)} t} + B^{(i)} e^{-j\omega^{(i)} t}\right) = 0, \qquad t \geq 0$$

Because all the $\omega^{(i)}$ are distinct, the exponentials in the above equation are linearly independent functions of time and therefore we must have

$$A^{(i)} = B^{(i)} = 0, \qquad \forall i \in \{1, 2, \ldots, m\}$$

which means that

$$\alpha\gamma_q \mathbf{d} = 0 \qquad (20)$$

For $\sigma = q$ the reduced transfer function of the multi-controller is $\frac{\mu_q}{\rho_q}$, thus

$$\rho_q u = \mu_q y$$

From this and (16) we conclude that

$$(\rho_q \beta - \mu_q \alpha) y = \alpha\rho_q \mathbf{d} \qquad (21)$$

But, because of pole content property, any root of $\alpha\gamma_q$ is also a root of $\alpha\rho_q$ and therefore, in view of (20), $\alpha\rho_q\mathbf{d} = 0$. Thus (21) implies that

$$(\rho_q \beta - \mu_q \alpha) y = 0$$

The stability margin property then guarantees that the roots of $\rho_q\beta - \mu_q\alpha$ have real part smaller then $-\lambda$ and therefore y converges to 0 faster than $e^{-\lambda t}$. ■

5 Conclusions

In this paper we proposed a supervisory control approach to suppress from the output of a SISO, LTI process the effect of a persistent disturbance input consisting of the sum of a finite number of sinusoids of unknown amplitude, phase, and frequency. The approach proposed in inspired by the literature on supervisory control.

The main shortcoming of the present paper is that the stability analysis in Section 4 is only valid when the set of frequencies Ω is finite and when there is no measurement noise and unmodeled dynamics in the process model. However, simulation results suggest that the overall closed loop system may, in fact, be stable even when Ω has infinitely many elements (e.g. is an interval) and may also be robust with respect to unmodeled dynamics and measurement noise.

Acknowledgments

The second author wishes to thank António Pascoal for useful discussions on the control of autonomous underwater vehicles close to the surface of a wavy sea. These discussions partially motivated this work.

Appendix

In the sequel we show that for any piecewise-constant signal $s : [0, \infty) \to \mathcal{P}$ and any initialization of the system consisting of (1), (2), (4), and (10) with $\sigma = s$, e_ω converges to zero faster than $e^{-\lambda t}$. Since the transfer function of the process is $\frac{\alpha}{\beta}$,

$$\beta y = \alpha(u + \mathbf{d}) \tag{22}$$

because of (1). From equations (8) and (22), we conclude that

$$\gamma_\omega \beta y = \gamma_\omega \alpha u \tag{23}$$

Equations (4), (5), and (6) imply that

$$\omega_E y_\omega = (\omega_E - \gamma_\omega \beta) y + \gamma_\omega \alpha u \tag{24}$$

Subtracting (23) from (24) and using the fact that $e_\omega = y_\omega - y$, yields

$$\omega_E e_\omega = 0$$

Since ω_E is the characteristic polynomial of A_E and $\lambda I + A_E$ is asymptotically stable, e_ω converges to zero faster than $e^{-\lambda t}$. ∎

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