Univeristy of California, Santa Barbara
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Discussion: Friday, 16 May 2008
Project Due: Friday, 23 May, 4:00PM

ECE 148 Applications of Signal Analysis and Processing
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Electrical & Computer Engineering

Homework 3

Problem 3.1 Circular Convolution

a) Compute the convolution

$$c[n] = a * b[n] = \sum_{k=-\infty}^{\infty} a[k]b[n-k]$$

for the two signals

$$\begin{aligned} a &= [\ \cdots\ 0\ \boxed{1}\ 2\ -1\ 1\ 3\ 4\ 2\ 1\ 0\ \cdots\ ] \\ b &= [\ \cdots\ 0\ \boxed{4}\ 1\ 1\ 0\ 3\ 4\ 1\ -1\ 0\ \cdots\ ]. \end{aligned}$$

b) Compute the circular convolution

$$c[n] = a \circledast b[n] = \sum_{k=0}^{N-1} a[k]b[n-k],$$

where

$$a[n] = a[n \bmod N], \text{ and } b[n] = b[n \bmod N],$$

of the two N-periodic signals

$$\begin{aligned} a &= [\ \boxed{1}\ 2\ -1\ 1\ 3\ 4\ 2\ 1\ ] \\ b &= [\ \boxed{4}\ 1\ 1\ 0\ 3\ 4\ 1\ -1\ ]. \end{aligned}$$

Problem 3.2: Matrix Operators

The shift operator $\mathscr{S}_1$ has the following impulse response:

$$s_1 = [\ \cdots\ 0\ \boxed{0}\ 1\ 0\ \cdots\ ].$$

The corresponding matrix, when the operating on periodic signals of length $N = 4$ is:

$$S = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

a) For the 4-periodic signal $a = [\ \boxed{1}\ 2\ -1\ 1\ ]^{\mathsf{T}}$, verify that the circular convolution $c = s \circledast a$ and the matrix multiplication $c = Sa$ give the same result.
b) Compute the matrices $S_2 = S_1S_1$, $S_3 = S_2S_1$, and $S_4 = S_3S_1$. What operators do they correspond to?
c) Compute the inverse of the matrix S_1^{-1}.
d) Compute the impulse response of the inverse filter s_1^{-1}.
e) Compute the matrix $\Lambda = F^{-1}S_2F$, where the elements of the matrix F are $F[k,l] = \exp(-j2\pi kl/N)$.
f) Compute the `fft` of the signal s_2 and compare to the result you obtained in e). How are the two related?