In other courses, you have learned to design amplifiers using small signal models for devices. This works reasonably well at lower frequencies, but at high frequencies often the device S.S. model is not accurate enough. Then, measured s-parameters can be used to accurately design the amplifier.

The s-parameter design technique employs relationships between input and output powers, forward and reflected powers that look scary at first but can easily be derived using the signal flow graph method and Mason’s gain rules. (Gonzalez, Sec. 2.6)

Our sequence of topics will include:

1. Signal flow graph method (homework)
2. Power gain definitions
3. Stability of amplifiers
4. Unilateral approximation ($S_{12}=0$)
5. Bilateral design
6. Bias circuits and wideband stability

Goal: Learn to design stable narrowband amplifiers using S parameters
Recall the definition of the $S$ parameters:

\[
\begin{align*}
    b_1 &= S_{11}a_1 + S_{12}a_2 \\
    b_2 &= S_{21}a_1 + S_{22}a_2
\end{align*}
\]

Consider the forward transmission and calculate the transducer power gain:

\[
S_{21} = \frac{2V_{\text{out}}}{V_{\text{gen}}} \bigg|_{a_2=0} = \frac{b_2}{a_1}
\]

In general, for an arbitrary $R_S$ and $R_L$,

\[
P_{AVS} = \frac{V_{\text{gen}}^2}{8R_S} \quad \quad P_L = \frac{V_{\text{out}}^2}{2R_L}
\]

The definition of transducer power gain:

\[
G_T = \frac{P_L}{P_{AVS}}
\]

So, for the special case where $R_S = R_L = Z_O$,

\[
|S_{21}|^2 = \frac{4V_{\text{out}}^2}{V_{\text{gen}}^2} = \frac{V_{\text{out}}^2}{2Z_O} = \frac{8Z_O^2}{V_{\text{gen}}^2} = G_T
\]
But, life is generally not that straightforward because $|S_{21}|^2$ is often much less than the optimum gain that you could obtain from a given transistor. You must add matching networks to transform $Z_0$ to a more suitable $\Gamma_S$ and $\Gamma_L$.

**AMPLIFIER BLOCK DIAGRAM**

How do we calculate gain from s-parameters?

Evaluate the appropriate gain equation:

$$G_T = \text{transducer power gain} = \frac{P_L}{P_{AVS}}$$

$$= \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} \left| S_{21} \right|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{out}\Gamma_L|^2}$$

$G_T$ of device

gain term associated with input match if $\Gamma_S = \Gamma_L = Z_0$

$\Gamma_{out} = S_{22} + \frac{S_{12}S_2\Gamma_S}{1 - S_{11}\Gamma_S}$

So, if you are given the $S$ params and $\Gamma_S, \Gamma_L$ then you can calculate the gain.

Note however that $\Gamma_{out}$ depends on $\Gamma_S$ unless $S_{12} = 0!$
Why does $\Gamma_{\text{OUT}}$ depend on $\Gamma_{S}$?

\[
b_1 = S_{11}a_1 + S_{12}a_2 \\
b_2 = S_{21}a_1 + S_{22}a_2
\]

But, $a_1 = \Gamma_{S}b_1$. Substitute into the equation for $b_1$

\[
b_1 = S_{11}\Gamma_{S}b_1 + S_{12}a_2 \\
b_1(1 - S_{11}\Gamma_{S}) = S_{12}a_2
\]

or

\[
b_1 = \frac{S_{12}a_2}{(1 - S_{11}\Gamma_{S})}
\]

Now, find $\Gamma_{OUT} = \frac{b_2}{a_2}$

\[
b_2 = \left(\frac{S_{21}\Gamma_{S}S_{12}}{(1 - S_{11}\Gamma_{S})} + S_{22}\right)a_2
\]

Thus,

\[
\Gamma_{OUT} = \left(\frac{S_{21}\Gamma_{S}S_{12}}{(1 - S_{11}\Gamma_{S})} + S_{22}\right)
\]

Likewise,

\[
\Gamma_{IN} = \left(\frac{S_{21}\Gamma_{L}S_{12}}{(1 - S_{22}\Gamma_{L})} + S_{11}\right)
\]

So, an amplifier is truly unilateral only when $S_{12} = 0$
Other gain definitions can also be used for specific purposes

**Operating Power Gain**

\[ G_p = \frac{\text{Power delivered to load}}{\text{Power input to network}} \]

\[ G_p = \frac{1}{1 - |\Gamma_{IN}|^2} \left| S_{21} \right|^2 \frac{1 - |\Gamma_L|^2}{1 - S_{22} |\Gamma_L|^2} \]

- This can be useful because it eliminates the dependence of gain on \( \Gamma_S \) - helpful when the device is bilateral – passes signal both ways.

**Available Power Gain**

\[ G_A = \frac{\text{Power available from network}}{\text{Power available from source}} \]

\[ G_A = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \Gamma_S|^2} \left| S_{21} \right|^2 \frac{1}{1 - |\Gamma_{OUT}|^2} \]

- Used in noise calculations – eliminates dependence of gain on \( \Gamma_L \)

See derivations in Sec. 2.6 and equations 3.2.1 – 3.2.4 below

Calculating power gains from \( S \)-param is a mechanical process – or you can use CAD tools such as Agilent/EESOF.
3.2 POWER GAIN EQUATIONS

Several power gain equations appear in the literature and are used in the
design of microwave amplifiers. Figure 3.2.1 illustrates a microwave amplifier
signal flow graph and the different powers used in gain equations. The trans-
ducer power gain $G_T$, the power gain $G_p$ (also called the operating power gain),
and the available power gain $G_A$ are defined as follows:

$$ G_T = \frac{P_L}{P_{AVS}} = \frac{\text{power delivered to the load}}{\text{power available from the source}} $$

$$ G_p = \frac{P_L}{P_{IN}} = \frac{\text{power delivered to the load}}{\text{power input to the network}} $$

and

$$ G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{\text{power available from the network}}{\text{power available from the source}} $$

The expressions for $G_T$, $G_p$, and $G_A$ were already derived in (2.6.14),
(2.6.15), (2.6.18), and (2.6.22)—namely,

$$ G_T = \frac{1 - |\Gamma_s|^2}{1 - \Gamma_{IN}\Gamma_s} \frac{|S_{21}|^2}{|1 - S_{21}\Gamma_s|^2} \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} $$ (3.2.1)

$$ G_T = \frac{1 - |\Gamma_s|^2}{1 - S_{11}\Gamma_s} \frac{|S_{21}|^2}{|1 - S_{21}\Gamma_L|^2} \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} $$ (3.2.2)

$$ G_p = \frac{1}{1 - |\Gamma_s|^2} \frac{|S_{21}|^2}{|1 - S_{21}\Gamma_s|^2} \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} $$ (3.2.3)

$$ G_A = \frac{1}{1 - |\Gamma_s|^2} \frac{|S_{21}|^2}{|1 - S_{21}\Gamma_s|^2} \frac{1}{1 - |\Gamma_L|^2} $$ (3.2.4)

![Figure 3.2.1 Different power definitions.](image)

Voltage Standing Wave Ratio (VSWR)

Often used as part of an amplifier specification. What is it?

Recall that the reflection coefficient on a transmission line varies in phase with position $x$, where $\Gamma(0)$ is the reflection coefficient at the end of the line, usually called the load.

$$\Gamma_{IN}(x) = \Gamma(0)e^{j\beta x}$$

That implies that the voltage on the line also will vary with position as the forward and reflected waves add or subtract. The magnitude of $\Gamma_{IN}$ is constant, $|\Gamma(0)|$.

$$|V(x)| = |V^+| |1 + \Gamma(0)e^{j\beta x}|$$

Thus, the maximum and minimum voltages on the line can be found:

$$|V_{\text{max}}(x)| = |V^+||1 + |\Gamma(0)||$$

$$|V_{\text{min}}(x)| = |V^+||1 - |\Gamma(0)||$$

The VSWR is then defined by:

$$\text{VSWR} = \frac{|V_{\text{max}}|}{|V_{\text{min}}|} = \frac{1 + |\Gamma(0)|}{1 - |\Gamma(0)|}$$

Why is it important? The amount of the available power transferred to the load depends on the reflection coefficient. Recall that

$$P_{\text{Load}} = P_{\text{AVS}}(1 - |\Gamma_L|^2)$$

Thus, a high VSWR means that the load is badly matched to the source, so gain will be lost.

Also, reflections between components within a system can be harmful. Consider the case of two cascaded amplifiers with a finite length transmission line interconnecting them. Suppose that the output of the first amplifier and the input of the second amplifier both are mismatched to the line impedance. There will be reflections at both ends. Standing waves will appear on the line with the position of their maxima and minima varying with frequency. The voltage and current delivered to the input of amp 2 will become frequency dependent because the electrical length of the transmission line depends on frequency. Thus, the gain of the cascaded amplifiers will exhibit ripple.
Design of a microwave amplifier.

Suppose the amplifier specifications are presented in a design sense: given a device, design input and output $MN$ for a particular value of $G_T$.

Now, we find many possible solutions. To determine the best solution we need to first consider the stability of the amplifier – we must guarantee that the amplifier does not oscillate under the expected source and load impedances.

Look at:
- Stability Circles
- Gain Circles

Then, once a stable matching condition region in the $\Gamma_S$ and $\Gamma_L$ planes is identified, gain circles can be plotted to assist in selecting a $\Gamma_S$ and $\Gamma_L$ that is least susceptible to circuit variances.

GOAL:
- Robust design
- Stable and repeatable

Finally, noise performance can also be an important design constraint. We will see later how the design of an amplifier can be optimized for minimum noise using available gain as a design tool.
Stability of amplifiers. First let’s review the concept of negative resistance.

\[ -R \]

\[ \text{Load} \]

- R

V

Load

Note that the negative resistor has the opposite to the passive sign convention. Thus, it delivers power into the load rather than dissipating power as a positive resistor does.

Why does this lead to instability?

If \(|-R| = R_L\), there is no net resistance in the loop. This locates the complex poles right on the \(j\omega\) axis.

\[ j\omega \]

\[ \sigma \]

The transient response then has the form \(e^{j\omega t}\), a sustained sinusoidal oscillation.
Oscillation is possible if either input or output port has a negative resistance. If a net negative real part exists, that is if \( \text{Re}\{Z_s + Z_{\text{in}}\} \) or \( \text{Re}\{Z_{\text{out}} + Z_{\text{L}}\} < 0 \), the transient response will grow and oscillation will occur.

For unconditional stability:

\[
\left\{ \begin{array}{l}
|\Gamma_s| < 1 \\
|\Gamma_L| < 1
\end{array} \right. \text{ for any passive source and load,}
\]

So, to avoid net negative resistance, \( |\Gamma_s \Gamma_{\text{IN}}| < 1 \quad \text{and} \quad |\Gamma_{\text{OUT}} \Gamma_L| < 1 \)

\[
|\Gamma_{\text{in}}| = \left| S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \right| < 1
\]

and

\[
|\Gamma_{\text{out}}| = \left| S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11} \Gamma_s} \right| < 1
\]

This unconditional stability is rather unusual for most microwave or millimeter-wave devices of interest. So, we must establish a method to determine regions in the \( \Gamma_s \) and \( \Gamma_L \) plane that are stable. We can then either avoid the unstable regions or modify the transistor with resistive loading to make it unconditionally stable.
**Conditional Stability:** This is the usual case. Also known as potentially unstable.

\[ \text{Re}\{Z_{in}\} \text{ and } \text{Re}\{Z_{out}\} > 0 \text{ for some } |\Gamma_S| \leq 1 \text{ and } |\Gamma_L| \leq 1 \text{ at some specific frequency.} \]

We can evaluate stability graphically with

**Stability Circles**

First we will consider the \( \Gamma_L \) or “load plane” on the Smith chart. Define load stability circles which locate the boundary (values of \( \Gamma_L \)) between \( |\Gamma_{in}| < 1 \) and \( |\Gamma_{in}| > 1 \).

To do this, set

\[
|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1
\]

and solve for \( \Gamma_L \)

Solution lies on a circle.

radius:

\[
r_L = \left| \frac{S_{12}S_{21}}{S_{22}^2 - |\Delta|^2} \right|
\]

center:

\[
c_L = \left| \frac{(S_{22} - \Delta S_{11}^*)}{S_{22}^2 - |\Delta|^2} \right|
\]

where:

\[
\Delta = S_{11}S_{22} - S_{12}S_{21}
\]

These circles form the boundary between stable and unstable operation. Plot on the \( \Gamma_L \) Smith Chart.

1. If the circle intersects the chart, there is a region of instability.
2. If no intersection, device or amplifier is unconditionally stable.

Fortunately, we can use ADS to plot these for us.
In a similar way, we can set $|\Gamma_{\text{out}}| = 1$ and solve for $\Gamma_S$. The boundary circles which can be plotted on the $\Gamma_S$ or “source plane” are defined by:

$$r_S = \frac{S_{12}S_{21}}{|S_1|^2 - |A|^2}$$

$$c_S = \frac{|(S_{11} - \Delta S_{22})^*|}{|S_{11}|^2 - |A|^2}$$

Plot the source stability circles on the $\Gamma_S$ plane.

---

**Figure 3.3.3** Smith chart illustrating stable and unstable regions in the $\Gamma_L$ plane.

Now, we must determine whether inside or outside of circle is stable. Consider the load plane,

let $\Gamma_L = 0$ (center of chart)

then $|\Gamma_{in}| = |S_{11}|$ (by definition)

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right|$$

if $|S_{11}| < 1$, then this point represents a stable operating condition. So, the region inside the chart excluding the circle is stable.

---

**Figure 3.3.4** Smith chart illustrating stable and unstable regions in the $\Gamma_r$ plane.

if $|S_{11}| > 1$, then opposite case.
Stability circles for all possible values of $\Gamma_S$ must also be considered. Instability can be induced at either port.

**Unconditional Stability**

$|C_L| - \eta_L > 1$
Stability Factor: This is a less specific indicator of stability.

\[ k = \frac{\Delta}{1 + \left| S_{11} S_{22} - S_{12} S_{21} \right|^2 - \left| S_{11} \right|^2 - \left| S_{22} \right|^2} > 1 \]

and \[ |\Delta| = \left| S_{11} S_{22} - S_{12} S_{21} \right| = \det S < 1 \]

will guarantee unconditional stability.

1. If a transistor is potentially unstable, typically \(|\Delta| < 1\) and \(0 < k < 1\)

2. Negative \(k\) values can occur, but result in most of the Smith Chart producing instability.

   So, life can be much easier when you choose a device that will be unconditionally stable. But, this may lead to designs that don’t push the edge of performance.

3. Also, you must check for stability at all frequencies for which the device has a \(k < 1\).
Figure 3.3.7 Input and output stability circles.

\[ S_{11} = 0.65 \angle -94^\circ \]
\[ S_{12} = 0.032 \angle 41.2^\circ \]
\[ S_{21} = 4.62 \angle 116.2^\circ \]
\[ S_{22} = 0.66 \angle -36^\circ \]
Example: see Fig. 3.3.7 (Gonzalez, op.cit.)

- **test:** calculate $k$ and $\Delta$:
  
  \[
  k = 0.547 < 1 \\
  \Delta = 0.504 \angle 250^\circ
  \]
  
  $k < 1$ potentially unstable
  
  $|\Delta| < 1$ ok

Since device is potentially unstable at this frequency, check out stability circles.

**Source Stability Circle:** draw on $\Gamma_s$ Smith Chart

\[
\left| S_{22} \right| < 1, \quad \left| \Gamma_{\text{out}} \right| = \left| S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11} \Gamma_s} \right| < 1
\]

when $\Gamma_s = 0$

center of chart.

**Load Stability Circle:** draw on $\Gamma_l$ Smith Chart

\[
\left| S_{11} \right| < 1, \quad \left| \Gamma_{\text{in}} \right| = \left| S_{11} + \frac{S_{12} S_{21} \Gamma_l}{1 - S_{22} \Gamma_l} \right| < 1
\]

when $\Gamma_l = 0$

So, inside of chart except for area intersected by circles is stable.
OK, so how do we proceed with a design?

1. So, we can either choose $\Gamma_S$ and $\Gamma_S'$ appropriately for stability – or

2. We can resistively stabilize the amplifier so that it cannot oscillate if

$$\text{Re}(Z_S + Z_{in}) > 0 \text{ and } \text{Re}(Z_L + Z_{out}) > 0$$

Let’s illustrate the latter approach first.

**Input Stabilization:**

The input stability circle ($\Gamma_S$ plane) cuts across chart tangent to the $z = 0.18$ constant resistance circle.

$$Z = 0.18 \ (50) = 9\Omega$$

If we add a series resistance of 9 ohms to the input, then the device can never see a source resistance less than 9Ω. Now, $|\Gamma_{OUT}|$ is always less than 1.

Disadvantage:

- Gain reduction
- Increased noise
- Reduced frequency response
Equivalently, a shunt resistance can be added – const. conductance circle \( g = 0.7 \) is tangent to the input stability circle.

\[
\frac{0.7}{50} = 14mS \Rightarrow 71.5\Omega
\]

Output Stabilization: same procedure only on the \( \Gamma_L \) plane.

29\( \Omega \) series \((r = 0.58\) const. resistance\))

or

500\( \Omega \) shunt \((g = 0.1\) const. conductance\))
ADS simulations of the above example:

We see that the circuit is nearly unconditionally stable with the 9 ohm series resistor at the input side of the amplifier. 10 ohms would have been better.
Next topic: Gain Define $G_{\text{max}}$. Use gain circles to identify regions of constant gain on $\Gamma_s$, $\Gamma_L$ planes.
Maximum Available Gain

Picture of amplifier circuit of any type, but no feedback.

overall power gain must be $\leq G_{\text{max}}$. This is called the Maximum Available Gain.

Power gain is equal to $G_{\text{max}}$ if input and output are conjugately matched using lossless matching networks.

Why not lossy networks?

$$G < G_{\text{max}} \text{ because } P_{\text{in device}} = P_{\text{gen}} - P_{\text{resistor}} < P_{\text{generator}}$$
So, amplifiers fail to attain $G_{\text{max}}$ because:

1. They fail to match on both input and output
2. They use lossy elements (resistors) to attain a match or both
3. They are potentially unstable. In that case, $G_{\text{max}}$ is not possible due to oscillation. Then, $G_{\text{MSG}}$, the Maximum Stable Gain, is the upper useful limit for gain.

**Maximum Stable Gain**
For potentially unstable transistor

Define max. stable gain

$$G_{\text{MSG}} = \left| \frac{S_{11}}{S_{22}} \right|$$

which is the $G_{T,\text{max}}$ when $k=1$.

This is used to describe the gain which could possibly be obtained from the device under a stable input and output match selection or after stabilization with resistive loading.
Example: Matched or tuned amplifier

\[ G_{\text{max}} \] is desirable for a tuned system (radio, receiver, etc.) Note that \( G_{\text{max}} \) is a function of frequency. \( f_{\text{max}} \) is the intersection with 0 dB gain – sometimes called the maximum frequency of oscillation. We see that a tuned amplifier (with frequency dependent matching networks) can achieve \( G_{\text{max}} \) at only one frequency. A distributed amp, as discussed earlier, can achieve \( G_{\text{max}} \) (at its highest frequency of operation) over a wide range of frequencies.
Example: resistively-terminated amplifier and feedback amplifier

The gain-frequency curve clearly has to lie under the $G_{\text{max}}$ curve. In fact, the gain-frequency curve may be constrained well below this by the $(f_T/f)^2$ line, or even lower.

⇒ Flat gain, but at the expense of performance below the fundamental limit ($G_{\text{max}}$) of the device.
POWER GAINS FOR LUMPED-ELEMENT, and RESONANT AMPLIFIER TYPES

- Transistor short-circuit current gain
- Transistor maximum available power gain
- Lumped-Element* (resistively terminated)

*Bandwidth limited by device input capacitance.
**Amplifier Gain**

At a given frequency, the maximum gain that an amplifier can deliver is limited by either its $G_{\text{max}} = G_{T,\text{max}}$ or by stability $G_{\text{MSG}}$.
**Constant Gain Circles: Unilateral Case**

Now that we have determined a method to find the stable regions of $\Gamma_S$ and $\Gamma_L$ and if necessary to add resistance to guarantee stability, we can explore other considerations for setting the gain.

1. Avoid instability – then
2. Choose $\Gamma_S, \Gamma_L$ for simple $MN$ manipulation
3. $Q$ selection in narrowband design
4. Max. unilateral gain or Max. stable gain.

Unilateral: $S_{12}=0$

This is never really true, but it can be a useful approximation in some cases.

Why? if $S_{12}=0$, then

$\Gamma_{in} = S_{11}$ and $\Gamma_{out} = S_{22}$

no interaction between input and output.

Can we really consider device to be unilateral?

$S_{12} \neq 0$ ever

But we can estimate maximum gain error:

$$\frac{1}{(1+u)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-u)^2}$$

$$u = \frac{|S_{12}| |S_{21}| |S_{11}| |S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

unilateral figure of merit.

If this is small (it might be at low enough frequency) the unilateral approximation is justified.

**Can a unilateral device still be unstable?**  Yes.

It is possible that $|S_{11}| > 1$ and/or $|S_{22}| > 1$. 
Unilateral Transducer Power Gain

\[ G_{TU} = \frac{1 - |\Gamma_S|^2}{1 - S_{11} \Gamma_S} \left| S_{21} \right|^2 \frac{1 - |\Gamma_L|^2}{1 - S_{22} \Gamma_L} \]

\[ = G_S \cdot G_0 \cdot G_L \]

fraction of gain/loss due to input match

fraction of gain/loss due to output match

If unilateral, \( G_T = G_{TU} \) = unilateral transducer power gain

How do we obtain the maximum \( G_{TU} \) (sometimes called maximum available gain)?

We want simultaneous input/output conjugate match, \( \Gamma_S = S_{11}^* \) and \( \Gamma_L = S_{22}^* \).

Then,

\[ G_{TU,max} = \frac{1}{1 - |S_{11}|^2} \left| S_{21} \right|^2 \frac{1}{1 - |S_{22}|^2} \]

if \( S_{12} = 0 \)

The unity gain frequency for \( G_{max} \) or \( MAG \) or \( G_{TU,max} \) is called \( f_{max} \). This represents the upper limit – the highest frequency that the device could ever have a power gain of 1.
Now describe Gain circles. (when unilateral assumption is valid)

We wish to describe the variation in $G_T$ with $\Gamma_S$ and $\Gamma_L$ in a graphical form. Let’s assume that $|S_{11}| < 1$ and $|S_{22}| < 1$.

* Values of $\Gamma_S$ and $\Gamma_L$ that produce constant gain lie on circles in the $\Gamma$ plane.

* Maximum gain occurs when $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$. These are points on $\Gamma_S$ and $\Gamma_L$ planes respectively. The centers of the circles lie on the line connecting these points with the origin.

* By necessity, 0 dB circle will always pass through origin ($\Gamma_S = 0$ or $\Gamma_L = 0$). This comes about because $G_S = 1$ and $G_L = 1$ when $\Gamma_S = \Gamma_L = 0$, i.e. matched to $Z_0$.

- Circles of constant $G_L$ can be similarly drawn on the $\Gamma_L$ plane.

In ADS, input and output gain vs $\Gamma_S$ or $\Gamma_L$ can be plotted using Gscir or GLcir functions.
1. Draw line from origin to $S_{11}^*$ (for $\Gamma_s$ plane) or $S_{22}^*$ (for $\Gamma_L$ plane)

2. Determine gain steps of interest and calculate normalized gain factor $g_i = \frac{G_i}{G_{i,\text{max}}}$ where $0 \leq g_i \leq 1$. $i = S$ or $L$.

   example: Suppose $G_{S,\text{max}} = 3.3\text{dB}$ and you want to draw gain circle for $2\text{ dB}$.

   \[
   \begin{align*}
   3.3\text{dB} & \Rightarrow 2.14 \\
   2\text{dB} & \Rightarrow 1.58 \\
   g_S & = \frac{1.58}{2.14} = 0.743
   \end{align*}
   \]

3. Calculate $C_g = \frac{g_s S_{11}^*}{1 - |S_{11}|^2 (1 - g_s)}$

4. Calculate $r_{gs} = \frac{\sqrt{1 - g_s} (1 - |S_{11}|)}{1 - |S_{11}|^2 (1 - g_s)}$ (see Fig. 3.4.4)

or, use ADS to plot the circles.

$G_S\text{ Cir}$, $G_L\text{ Cir}$ icons found in Simulation-$S_{\text{param}}$ palette.
Figure 3.4.4 (a) Constant-gain circles for $G_r = 2, 1, 0, \text{ and } -1 \text{ dB}; (b) calculations of constant-gain circles.

Gain circles show you where $\Gamma_S$ or $\Gamma_L$ must be to achieve certain gain from the device,

$$ G_{TU} = G_S G_0 G_L $$

$G_0$ remains constant

$$ = |S_{21}|^2 $$

$G_S, G_L$ depend on $\Gamma_S, \Gamma_L$ respectively.

Since this unilateral case was defined to be unconditionally stable, ($|S_{11}| < 1$ and $|S_{22}| < 1$), we do not need to base our selection of $\Gamma_S$ and $\Gamma_L$ on stability but rather on design convenience, or other factors such as VSWR or bandwidth or reproducibility.

Where should you choose $\Gamma_S$ for say 2dB gain?  
A vs B?
What about stability?

If $|S_{11}| < 1$ and $|S_{22}| < 1$, then

$|\Gamma_{in}| < 1 \quad |\Gamma_{out}| < 1$

unconditionally stable

Can unilateral devices be unstable?

yes if $|S_{11}| > 1$ or $|S_{22}| > 1$.

$$G_i = \frac{1 - |\Gamma_i|^2}{|1 - S_{ii}\Gamma_i|^2}$$

Since $|S_{ii}| > 1$, $S_{ii}\Gamma_i = 1$ when

$$\Gamma_i = \frac{1}{S_{ii}} \quad \text{(and } |\Gamma_i| < 1 \text{ in this case)}$$

infinite gain
Summarize Amplifier Design Methodology

1. **Unilateral Case #1** (Unconditionally stable)
   
   A. Check for stability: IF $K>1; |\Delta|<1$, then
   
   **Unconditionally Stable**
   
   B. Check $U$ to determine the maximum gain error for unilateral approximation.
   
   C. If this is satisfactory, then the solution is EASY:
      
      => For $G_{TU,max}$: $\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$
      
      => Or, plot gain circles on $\Gamma_s$ and $\Gamma_L$

2. **Unilateral Case #2** (Potentially Unstable)
   
   A. Check for stability: If $K<1; |\Delta|<1$, then
      
      **Potentially Unstable**
      
      B. Check $U$ to determine the maximum gain error for unilateral approximation.
      
      C. Plot Stability and $G_T$ Gain circles. $G_{TU,max}$ isn't available - oscillator!
      
      Design for stability and low sensitivity to $\Gamma_s$ and $\Gamma_L$ at the desired gain.
Review last lecture on Stability of amplifiers

1. Oscillations possible if either input or output port can produce negative resistance

\[ \text{Re} \{ Z_{\text{in}} \} \text{ and Re} \{ Z_{\text{out}} \} \text{ must be positive for any } Z_S, Z_L \text{ in order to have unconditional stability.} \]

a. \( Z_i < 0 \) gives \( |\Gamma_i| > 1 \) since

\[ \Gamma_i = \frac{Z_i - Z_0}{Z_i + Z_0} \]

2. Stability circles represent the boundary where \( |\Gamma_i| = 1 \).

\[ |\Gamma_{\text{in}}| = \left| S_{11} + \frac{S_{12}S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \right| < 1 \]

\[ |\Gamma_{\text{out}}| = \left| S_{22} + \frac{S_{12}S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} \right| < 1 \]

If the circle intersects the load or source Smith Charts, potentially unstable.
3. Unconditional stability can be proven by

\[ k > 1 \]
\[ |\Delta| < 1 \]

where

\[ k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} \]

\[ \Delta = S_{11}S_{22} - S_{12}S_{21} \]
Bilateral Case  (GONZALEZ 3.6, 3.7)  $S_{12} \neq 0$

This is nearly always the case for any device with high performance.

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$\Gamma_{out} = S_{11} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{22} \Gamma_S}$$

Clearly we have a more difficult case. The choice of $\Gamma_L$ affects $\Gamma_{in}$ and $\Gamma_S$ affects $\Gamma_{out}$. We can no longer design matching networks independently.

1. If you have an unconditionally stable device $k > 1$ and $|\Delta| < 1$, you can solve for the maximum transducer power gain simultaneous conjugate match conditions:

$$\Gamma_{MS} \text{ and } \Gamma_{ML} \quad \quad \Gamma_S^* = \Gamma_{in} \text{ and } \Gamma_L^* = \Gamma_{out}$$

using eq. (3.6.5) – (3.6.8)

$$\Gamma_{MS} = \frac{B_1 \pm \sqrt{B_1^2 - 4 |C_1|^2}}{2C_1}$$

$$\Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4 |C_2|^2}}{2C_2}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^*$$

or use the SmGamma 1 and SmGamma 2 icons in ADS to determine $\Gamma_S$ and $\Gamma_L$ for $G_{r,\text{max}}$.

$$G_{r,\text{max}} = \frac{|S_{21}| \left| k - \sqrt{k^2 - 1} \right|}{|S_{12}|} = \text{Max available gain. } \quad (= \ G_{MSG} \text{ when } k = 1)$$

This is the max. transducer gain for a bilateral device with $k > 1$. We see here that overstabilizing the amplifier will cost us some gain.
$k = \text{stability factor}$

2. When the $k < 1$, device will be unstable at simultaneous conjugate match condition. Note that $k$ varies with frequency, so there may be some frequencies where $G_{T,\text{max}}$ (MAG) can be obtained and some where $G_{\text{MSG}}$ is possible.

![Graph showing MAG, MSG, and $|S_{21}|^2$ vs. frequency](image)

Figure 3.6.2 Typical MAG (i.e., $G_{A,\text{max}}$), MSG (i.e., $G_{\text{MSG}}$), and $|S_{21}|^2$ versus frequency at $V_{CE} = 18$ V and $I_C = 30$ mA for the HXTR-5103. (From HP Microwave and RF Designer’s Catalog 1990–1991: courtesy of Hewlett Packard.)

3. If $k > 1$ but $|\Delta| > 1$, a simultaneous conjugate match is possible even though the device is potentially unstable. This match condition produces a minimum gain,
not the maximum gain (infinity if unstable) and uses eq. (3.6.5) – (3.6.8) as described on p. 242.

\[ G_{T,\text{min}} = \frac{|S_{21}|}{|S_{12}|} \left( k + \sqrt{k^2 - 1} \right) \]

Okay, but now suppose you need a solution with less than the \( G_{T,\text{max}} \). Suppose \(|k| > 1\) and \(|\Delta| < 1\) so the device is unconditionally stable, but bilateral.

The \( G_T \) gain circle approach that worked well for the unilateral device doesn’t work now. \( G_S \) depends on \( G_L \) and \( \Gamma_L \) depends on \( \Gamma_S \).

Possibly frustrating iterative process --------

We can instead plot circles of operating power gain, \( G_P \).

\[ G_P = \frac{P_L}{P_{in}} = \frac{\text{power delivered to load}}{\text{power delivered to input}} \]

\( G_P \) is a function of \( \Gamma_L \). \( \Gamma_S \) is automatically set to the conjugate input match.
Operating Power Gain Circles

\[ G_P = \frac{1}{1 - |\Gamma_{in}|^2} \left| S_{21} \right|^2 \left( 1 - \frac{|\Gamma_L|^2}{\left| 1 - S_{22} \Gamma_L \right|^2} \right) \]

Note independent of \( \Gamma_S \).

\( G_P \) is independent of the source match, whereas

\[ G_T = \frac{P_L}{P_{AVS}} \]

includes the gain term between \( P_{AVS} \) and \( P_m \) which depends on \( \Gamma_S \).

Power gain circles can be plotted in the \( \Gamma_L \) plane. They will be independent of \( \Gamma_S \), so the iterative design problem is cured.

If unconditionally stable, choose \( \Gamma_L \),

calculate \( \Gamma_{in} \)

set \( \Gamma_S = \Gamma_{in}^* \)

\( G_P = G_T \) under this condition since input is conjugately matched \( (VSWR_{in} = 1) \)

output may have significant mismatch.

Procedure:

1. For required \( G_P < G_{P,\text{max}} \), find center and radius of operating power gain circle.

\[ C_P = \frac{g_P C_0^*}{1 + g_P \left( \left| S_{22} \right|^2 - |\Delta|^2 \right)} \]

\[ r_P = \frac{\left| 1 - 2 \Delta S_{12} S_{21} g_P + \left| S_{12} S_{21} \right|^2 g_P^2 \right|^2}{\left| 1 + g_P \left( \left| S_{22} \right|^2 - |\Delta|^2 \right) \right|^2} \]

\[ g_P = \frac{G_P}{\left| S_{21} \right|^2} \]

2. Select \( \Gamma_L \) on the circle.
3. Determine $\Gamma_{in}$. 

$$\Gamma_{S} = \Gamma_{in}^*$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_{L}}{1 - S_{22}\Gamma_{L}}$$

Then $VSWR_{in} = 1$

$VSWR_{out}$ can be large. If necessary, you can iteratively test different $\Gamma_{L}$ values to get better $VSWR_{out}$.

ADS can also be used. The $G_p$ Cir icon can be placed on the schematic or better yet, the Gpcir equation can be written on the display. A gain must be specified for each circle. Several icons (schematic) or several equations (display) can be used to plot multiple circles with different $G_p$ levels.

Format for power gain circle equation on ADS: $x = gp\_cir(S, \text{gain}, \# \text{points})$
Available Gain Circles

Device data sheets often plot $G_T$ with the output matched on the $\Gamma_s$ plane. This is the available power gain $= G_A$.

$$G_A = \frac{P_{AVN}}{P_{AVS}}$$

Since output is always matched, $G_A$ is independent of $\Gamma_L$.

$$G_A = \frac{1-|\Gamma_{31}|^2}{|1-S_{11}\Gamma_s|^2} \left| S_{21} \right|^2 \frac{1}{1-|\Gamma_{out}|^2}$$

(since $\Gamma_L = \Gamma_{out}^*$)

Depends upon input match because actual power absorbed in the input is not necessarily the same as $P_{AVS}$ (unless conj. match at input).

If input is also conjugately matched, then

$$G_A = G_{A,max} = MAG = G_{T,max}$$

$$= \frac{1}{\left(1-|S_{11}|^2\right)} \left| S_{21} \right|^2 \frac{1}{1-|\Gamma_{out}|^2}$$

$$\frac{1}{1-|S_{22}|^2}$$ (if unilateral)

We will use these circles for the design of low noise amplifiers because the noise figure depends primarily on the input match. The input is often mismatched to obtain the best noise figure at the expense of gain. Then, the output is normally conjugately matched for maximum gain under the mismatched input conditions.
3. **Bilateral Case #1:** Unconditionally Stable

A. Check K, |Δ|

B. \[ G_{T,\text{max}} = \left| \frac{S_{21}}{S_{12}} \right| \left( K - \sqrt{K^2 - 1} \right) \]

C. Calculate conjugate match for \( \Gamma_S \) and \( \Gamma_L \) from (3.6.5) - (3.6.8) (or use Smgm1 and Smgm2 function in ADS)

D. Design matching networks for \( \Gamma_L \) and \( \Gamma_S \). Consider biasing.

4. **Bilateral Case #2:** Potentially Unstable

A. \[ G_{MSG} = \left| \frac{S_{21}}{S_{12}} \right| \]

B. Plot stability circles. Use resistive stabilization or avoid unstable regions of Smith chart.

C. Plot \( G_P \) constant gain circles to select \( \Gamma_L \). Calculate \( \Gamma_S = \Gamma_{IN}^* \)

D. Verify that \( \Gamma_S \) is stable.


In all cases, you must also verify that the amplifier is stable over a wide frequency range.