# IMAGE COMPRESSION- I

Week VIII



## Reading..

- Chapter 8
  - Section 8.1
  - 8.2.1 Huffman, 8.2.5 run-length
  - 8.2.8 Block transform coding & JPEG
  - 8.2.9 Predictive coding--spatial & temporal,, lossless and lossy
  - 8.2.10 wavelet compression

# Image compression

Objective: To reduce the amount of data required to represent an image.

Important in data storage and transmission
- Progressive transmission of images (internet, www)
- Video coding (HDTV, Teleconferencing, digital cinema)
- Digital libraries and image databases
  - Medical imaging
  - Satellite images



# IMAGE COMPRESSION

- Data redundancy
- Self-information and Entropy
- Error-free and lossy compression
- Huffman coding
- Predictive coding
- Transform coding

# Lossy vs Lossless Compression

**Compression techniques**

Information preserving **(loss-less)**

Images can be compressed and restored without any loss of information.
Application: Medical images, GIS

**Lossy**

Perfect recovery is not possible but provides a large data compression.
Example : TV signals, teleconferencing



# Data Redundancy

- **CODING**: Fewer bits to represent frequent symbols.
- **INTERPIXEL / INTERFRAME**: Neighboring pixels have similar values.
- **PSYCHOVISUAL**: Human visual system can not simultaneously distinguish all colors.

## Coding Redundancy

Fewer number of bits to represent frequently occurring symbols.
Let $p_r(r_k) = n_k / n$, k = 0,1,2, . ., L-1; L # of gray levels.

Let $r_k$ be represented by $l(r_k)$ bits. Therefore average # of bits required to represent each pixel is

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)\ p_r(r_k) \quad \rightarrow (A)$$

Usually $l(r_k) = m$ bits (constant). $\Rightarrow L_{avg} = \sum_k m\, p_r(r_k) = m$



## Coding Redundancy (contd.)

- Consider equation (A): It makes sense to assign fewer bits to those $r_k$ for which $p_r(r_k)$ are large in order to reduce the sum.
- this achieves data compression and results in a variable length code.
- More probable gray levels will have fewer # of bits.

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)\ p_r(r_k) \quad \rightarrow (A)$$

Example of a variable length code

| $r_k$ | $p_r(r_k)$ | Code 1 | $l_1(r_k)$ | Code 2 | $l_2(r_k)$ |
|---|---|---|---|---|---|
| $r_{87} = 87$ | 0.25 | 01010111 | 8 | 01 | 2 |
| $r_{128} = 128$ | 0.47 | 10000000 | 8 | 1 | 1 |
| $r_{186} = 186$ | 0.25 | 11000100 | 8 | 000 | 3 |
| $r_{255} = 255$ | 0.03 | 11111111 | 8 | 001 | 3 |
| $r_k$ for $k \neq 87, 128, 186, 255$ | 0 | — | 8 | — | 0 |

# Coding: Example

Example (From text)

| $r_k$ | $p_r(r_k)$ | Code | $l(r_k)$ |
|---|---|---|---|
| $r_0 = 0$ | 0.19 | 11 | 2 |
| $r_1 = 1/7$ | 0.25 | 01 | 2 |
| $r_2 = 2/7$ | 0.21 | 10 | 2 |
| $r_3 = 3/7$ | 0.16 | 001 | 3 |
| $r_4 = 4/7$ | 0.08 | 0001 | 4 |
| $r_5 = 5/7$ | 0.06 | 00001 | 5 |
| $r_6 = 6/7$ | 0.03 | 000001 | 6 |
| $r_7 = 1$ | 0.02 | 000000 | 6 |

$L_{avg}$

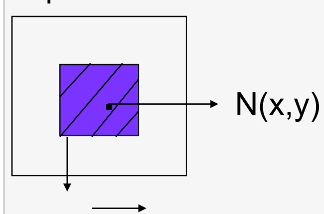
$= \sum \rho(r_k)\, l(r_k)$

$= 2.7$ Bits

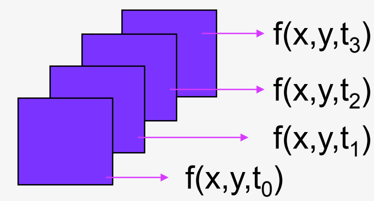
**10% less code**

# Inter-pixel/Inter-frame

spatial




Interframe




f(x,y) Depends on
f (x', y') , (x', y') $\varepsilon$ $N_{xy}$
$N_{xy}$ : Neighbourhood of pixels around (x,y)

$f(x, y, t_i)$ i=1, 2, 3, . . . are related to each other. This can be exploited for video compression.



# Psychovisual

- Human visual system has limitations ; good example is quantization. conveys infromation but requires much less memory/space.
- (Example: Figure 8.4 in text; matlab)

# Image Compression

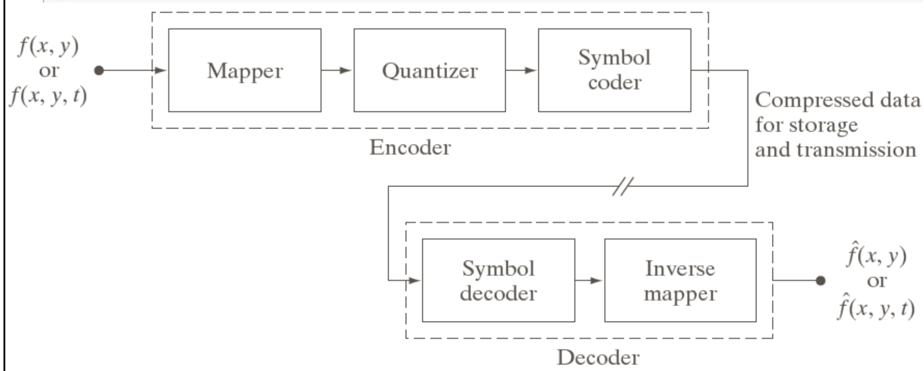





**Image Compression Standards, Formats, and Containers**

**Still Image**

**Binary**

CCITT Group 3
CCITT Group 4
JBIG (or JBIG1)
JBIG2

TIFF

**Continuous Tone**

JPEG
JPEG-LS
JPEG-2000

BMP
GIF
PDF
PNG
TIFF

**Video**

DV
H.261
H.262
H.263
H.264
MPEG-1
MPEG-2
MPEG-4
MPEG-4 AVC

AVS
HDV
M-JPEG
QuickTime
VC-1 (or WMV9)

# Source Encoder

**Mapper:** Designed to reduce interpixel redundancy.

example:

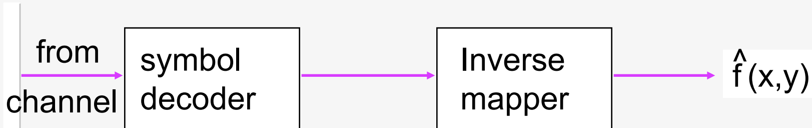
- Run length encoding results in compression.
- Transfrom to another domain where the coefficients are less correlated then the original. Ex: Fourier transfrom.

**Quantizer:** reduces psychovisual redundancies in the image - should be left out if error-free encoding is desired.

**Symbol encoder:** creates a fixed/variable length code - reduces coding redundancies.



# Source Decoder




Note: Quantization is **NOT** reversible

Question(s):

- Is there a minimum amount of data that is sufficient to completely describe an image without any loss of information?
- How do you measure information?

## Self-Information

- Suppose an event E occurs with probability P(E) ; then it is said to contain **I (E) = -log P(E)** units of information.
- P(e) = 1 [always happens] => I(e) = 0 [conveys no information]
- If the base of the logarithm is 2, then the unit of information is called a "bit".
- If P(E) = 1/2 , I(E) = $-\log_2(1/2)$ = 1 bit. Example: Flipping of a coin ; outcome of this experiment requires one bit to convey the information.



## Self-Information (contd.)

Assume an information source which generates the symbols $\{a_0, a_1, a_2, \ldots, a_{L-1}\}$ with

$$prob\ \{a_i\} = p(a_i) \quad ; \quad \sum_{i=0}^{L-1} p(a_i) = 1$$

$$I(a_i) = -\log_2 p(a_i) \quad \text{bits.}$$

# ENTROPY

Average information per source output is

$$H = -\sum_{i=0}^{L-1} p(a_i) \log_2 p(a_i) \quad \text{bits / symbol}$$

H is called the **uncertainity** or the **entropy** of the source.

If all the source symbols are equally probable then the source has a maximum entropy.

*H gives the lower bound on the number of bits required to code a signal.*



# Noiseless coding theorem

(**Shannon**)
It is possible to code, without any loss of information, a source signal with entropy H bits/symbol, using H + ε bits/symbol where ε is an arbitrary small quantity.

ε can be made arbitrarily small by considering increasingly larger blocks of symbols to be coded.

# Error-free coding

**Error-free coding**

**Coding redundancy**

Ex: Huffman coding

**Yeilds smallest possible # of code symbols per source symbol when symbols are coded one at a time.**

**Interpixel redundancy**

Ex: Runlength coding



# Huffman code: example

**Huffman code:** Consider a 6 symbol source

| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ |
|---|---|---|---|---|---|---|
| $p(a_i)$ | 0.1 | 0.4 | 0.06 | 0.1 | 0.04 | 0.3 |

# Huffman coding: example (contd.)

| | | | | | |
|---|---|---|---|---|---|
| $a_2$ | $0.4^{(1)}$ | $0.4^{(1)}$ | $0.4^{(1)}$ | $0.4^{(1)}$ | 0.6(0) |
| $a_6$ | $0.3^{(00)}$ | $0.3^{(00)}$ | $0.3^{(00)}$ | $0.3^{(00)}$ | 0.4(1) |
| $a_1$ | $0.1^{(011)}$ | $0.1^{(011)}$ | $0.2^{(010)}$ | $0.3_{(01)}$ | |
| $a_4$ | $0.1^{(0100)}$ | $0.1_{(0100)}$ | $0.1_{(011)}$ | | |
| $a_3$ | $0.06^{(01010)}$ | $0.1_{(0101)}$ | | | |
| $a_5$ | $0.04_{(01011)}$ | | | | |



# Example (contd.)

**Average length**:
(0.4) (1) + 0.3 (2) + 0.1 X 3 + 0.1 X 4 + (0.06 + 0.04) 5 = 2.2 bits/symbol

$-\Sigma\ p_i \log p_i$ = 2.14 bits/symbol **(Entropy)**

## Huffman code: Steps

- Arrange symbol probabilities $p_i$ in decreasing order
- While there is more than one node
  - Merge the two nodes with the smallest probabilities to form a new node with probabilities equal to their sum.
  - Arbitrarily assign 1 and 0 to each pair of branches
  - merging in to a node.
- Read sequentially from the root node to the leaf node where the symbol is located.



## Huffman code (final slide)

- Lossless code
- Block code
- Uniquely decodable
- Instantaneous (no future referencing is needed)

# Run-length Coding

**Run-length encoding (Binary images)**

0 0 0 0 1 1 1 1 1 1 0 0 0 1 1 1 0 0

4 6 3 3 2

Lengths of 0's and 1's is encoded. Each of the bit planes in a gray scale image can be run length - encoded.

One can combine run-length encoding with variable length coding of the run-lengths to get better compression.