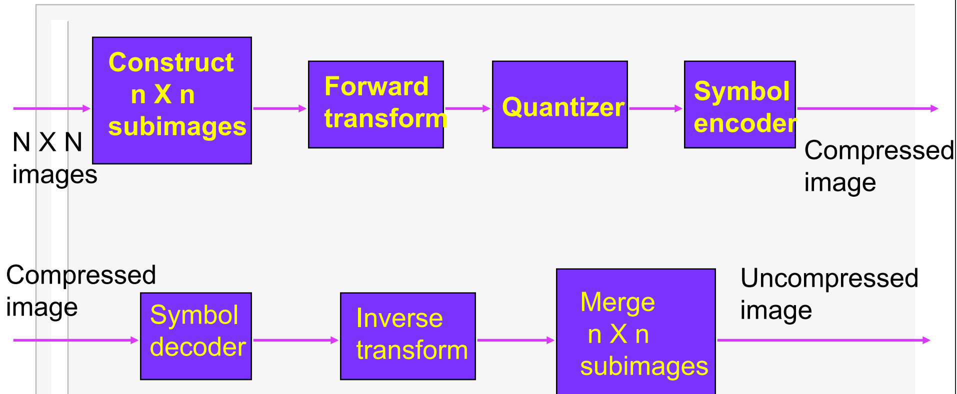
Block Transform Coding

Section 8.2.8

Transform coding



Blocking artifact: boundaries between subimages become visible

Transform Selection

- DFT
- Discrete Cosine Transform (DCT)
- Wavelet transform
- Karhunen-Loeve Transform (KLT)
- ….

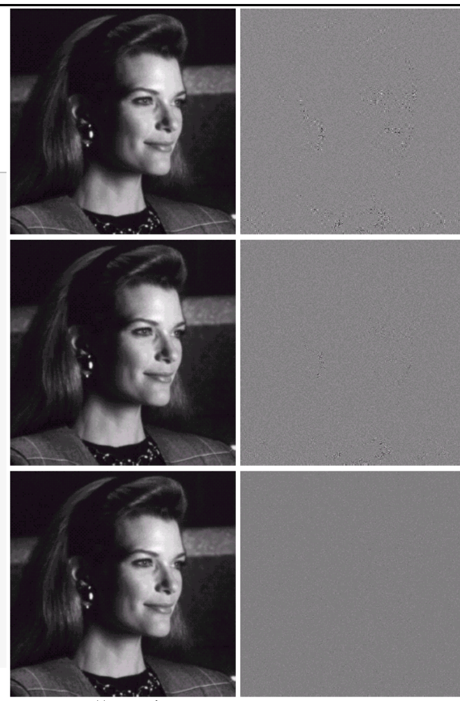
$$T(u,v) = \sum_{x=0}^{n-1}\sum_{y=0}^{n-1} g(x,y)\,r(x,y,u,v) ----(8.2.10)$$

$$g(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} T(u,v)s(x,y,u,v) ----8.2.11$$

Transform Kernels

- Separable if $r(x,y,u,v) = r_1(x,u)r_2(y,v) ----(8.2.12)$
- E.g. DFT: $r(x,y,u,v) = \exp(-j2\pi(ux+vy)/n)$
- E.g. Walsh-Hadamard transform (see page 568, text)
- E.g. DCT

Approximations using DFT, Hadamard and DCT, and the scaled error images



Discrete Cosine Transform

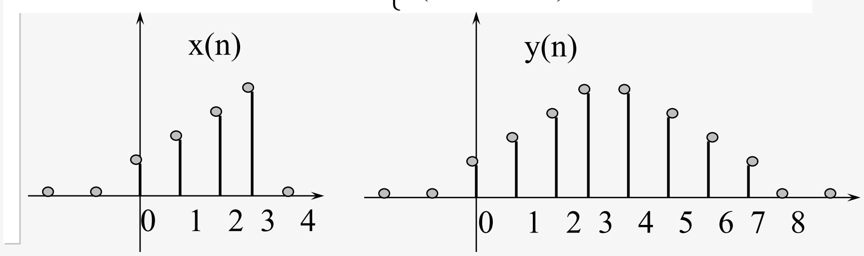
Ahmed, Natarajan, and Rao, IEEE T-Computers, pp. 90-93, 1974.

1-D Case: Extended 2N Point Sequence

Consider 1 - D first; Let x(n) be a N point sequence $0 \le n \le N-1$.

$$x(n) \leftrightarrow \begin{matrix}\text{2 - N point}\\ y(n)\end{matrix} \overset{DFT}{\leftrightarrow} \begin{matrix}2-N\text{ point}\\ Y(u)\end{matrix} \leftrightarrow \begin{matrix}N-\text{point}\\ C(u)\end{matrix}$$

$$y(n) = x(n) + x(2N-1-n) = \begin{cases} x(n), & 0 \le n \le N-1 \\ x(2N-1-n), & N \le n \le 2N-1 \end{cases}$$



DCT & DFT

$$\begin{aligned}
Y(u) &= \sum_{n=0}^{2N-1} y(n)\exp\left(-j\frac{2\pi}{2N}un\right) \\
&= \sum_{n=0}^{N-1} x(n)\exp\left(-j\frac{2\pi}{2N}un\right) + \sum_{n=N}^{2N-1} x(2N-1-n)\exp\left(-j\frac{2\pi}{2N}un\right) \\
&= \sum_{n=0}^{N-1} x(n)\exp\left(-j\frac{2\pi}{2N}un\right) + \sum_{m=0}^{N-1} x(m)\exp\left(-j\frac{2\pi}{2N}u(2N-1-m)\right) \\
&= \exp\left(j\frac{\pi}{2N}u\right)\sum_{n=0}^{N-1} x(n)\exp\left(-j\frac{\pi}{2N}u - j\frac{2\pi}{2N}un\right) \\
&\quad + \exp\left(j\frac{\pi}{2N}u\right)\sum_{n=0}^{N-1} x(n)\exp\left(j\frac{\pi}{2N}u + j\frac{2\pi}{2N}un\right) \\
&= \exp\left(j\frac{\pi}{2N}u\right)\sum_{n=0}^{N-1} 2x(n)\cos\left(\frac{\pi}{2N}u(2n+1)\right).
\end{aligned}$$

DCT

The N-point DCTof x(n), C(u), is given by

$$C(u) = \begin{cases} \exp\left(-j\dfrac{\pi}{2N}u\right)Y(u), & 0 \le \text{u} \le \text{N-1} \\ 0 & \text{otherwise.} \end{cases}$$

The unitary DCT transformations are:

$$F(u) = \alpha(u)\sum_{n=0}^{N-1} f(n)\cos\left(\frac{\pi}{2N}(2n+1)u\right), \quad 0 \le u \le N-1, \text{ where}$$

$$\alpha(0) = \frac{1}{\sqrt{N}}, \quad \alpha(u) = \sqrt{\frac{2}{N}} \text{ for } 1 \le k \le N-1.$$

The inverse transformation is

$$f(n) = \sum_{n=0}^{N-1} \alpha(u)F(u)\cos\left(\frac{\pi}{2N}(2n+1)u\right), \quad 0 \le u \le N-1.$$

Discrete Cosine Transform—in 2-D

$$C(u,v) = \alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\tfrac{(2x+1)u\pi}{2N}\right]\cos\left[\tfrac{(2y+1)v\pi}{2N}\right]$$

for $u, v = 0,1,2,\ldots,N-1$, where

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & \text{for } u = 0 \\ \sqrt{\frac{2}{N}} & \text{for } u = 1,2,..,\text{N-1}. \end{cases} \quad \text{and}$$

$$f(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \alpha(u)\alpha(v)C(u,v)\cos\left[\tfrac{(2x+1)u\pi}{2N}\right]\cos\left[\tfrac{(2y+1)v\pi}{2N}\right]$$

for $x, y = 0,1,2,\ldots,N-1$

DCT Summary

$$T(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)r(x,y,u,v) ----8.2.10$$

$$r(x,y,u,v) = s(x,y,u,v)$$

$$= \alpha(u)\alpha(v)\cos\left[\frac{(2x+1)u\pi}{2N}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right] ....8.2.18$$

DCT Basis functions

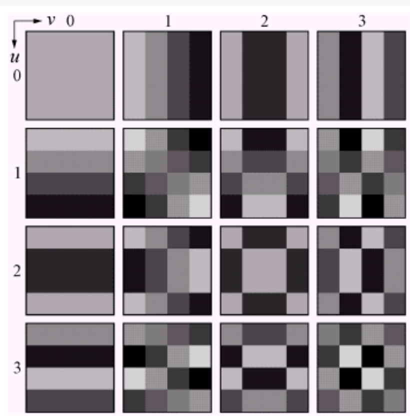


FIGURE 8.30 Discrete-cosine basis functions for $N = 4$. The origin of each block is at its top left.

Implicit Periodicity-DFT vs DCT (Fig 8.32)

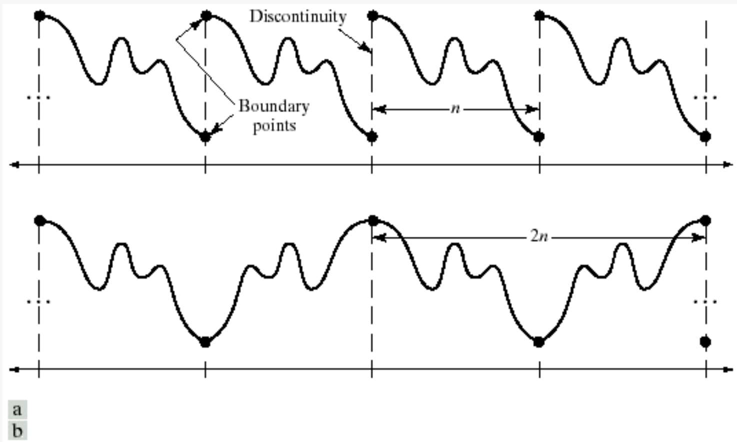


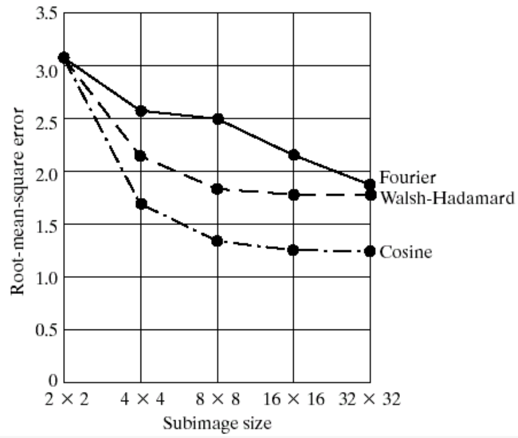
FIGURE 8.32 The periodicity implicit in the 1-D (a) DFT and (b) DCT.

Why DCT?

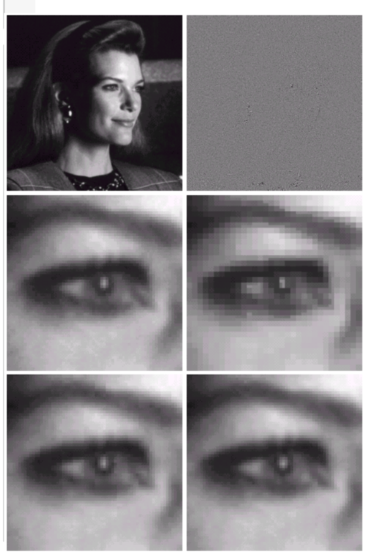
- Blocking artifacts less pronounced in DCT than in DFT.
- Good approximation to the Karhunen-Loeve Transform (KLT) but with basis vectors fixed.
- DCT is used in JPEG image compression standard.

Sub-image size selection (fig 8.26)

FIGURE 8.33 Reconstruction error versus subimage size.



Different sub-image sizes



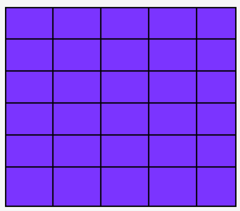
a	b
c	d
e	f

FIGURE 8.34 Approximations of Fig. 8.23 using 25% of the DCT coefficients: (a) and (b) 8 × 8 subimage results; (c) zoomed original; (d) 2 × 2 result; (e) 4 × 4 result; and (f) 8 × 8 result.

Bit Allocation/Threshold Coding

- # of coefficients to keep
- How to quantize them
  - Threshold coding
  - Zonal coding

Threshold coding

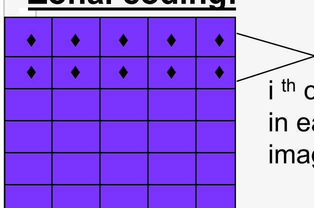


For each subimage i

- Arrange the transform coefficients in decreasing order of magnitude
- Keep only the top X% of the coefficients and discard rest.
- Code the retained coefficient using variable length code.

Zonal Coding

Zonal coding:



i th coefficient in each sub image

(1) Compute the variance of each of the transform coeff; use the subimages to compute this.
(2) Keep X% of their coeff. which have maximum variance.
(3) Variable length coding (proportional to variance)

Bit allocation: In general, let the number of bits allocated be made proportional to the variance of the coefficients. Suppose the total number of bits per block is B. Let the number of retained coefficients be M. Let v(i) be variance of the i-th coefficient. Then

$$b(i) = \frac{B}{M} + \frac{1}{2}\log_2 v(i) - \frac{1}{2M}\sum_{i=1}^{M}\log_2 v(i)$$

Zonal Mask & bit allocation: an example

1	1	1	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8	7	6	4	3	0	0	0
7	6	5	4	0	0	0	0
6	5	4	0	0	0	0	0
4	4	0	0	0	0	0	0
3	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Typical Masks (Fig 8.36)

a b
c d

FIGURE 8.36 A typical (a) zonal mask, (b) zonal bit allocation, (c) threshold mask, and (d) thresholded coefficient ordering sequence. Shading highlights the coefficients that are retained.

(a)

1	1	1	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(b)

8	7	6	4	3	2	1	0
7	6	5	4	3	2	1	0
6	5	4	3	3	1	1	0
4	4	3	3	2	1	0	0
3	3	3	2	1	1	0	0
2	2	1	1	1	0	0	0
1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0

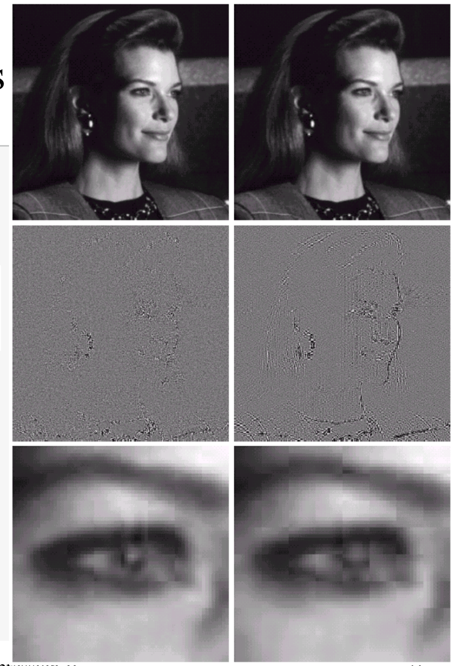
(c)

1	1	0	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(d)

0	1	5	6	14	15	27	28
2	4	7	13	16	26	29	42
3	8	12	17	25	30	41	43
9	11	18	24	31	40	44	53
10	19	23	32	39	45	52	54
20	22	33	38	46	51	55	60
21	34	37	47	50	56	59	61
35	36	48	49	57	58	62	63

Image Approximations



a b
c d
e f

FIGURE 8.35 Approximations of Fig. 8.23 using 12.5% of the 8×8 DCT coefficients: (a), (c), and (e) threshold coding results; (b), (d), and (f) zonal coding results.

The JPEG standard

- The following is mostly from Tekalp's book, Digital Video Processing by M. Tekalp (Prentice Hall).
- For the new JPEG-2000 check out the web site www.jpeg.org.
- See Example 8.17 in the text

JPEG (contd.)

- JPEG is a lossy compression standard using DCT.
- Activities started in 1986 and the ISO in 1992.
- Four modes of operation: Sequential (basline), hierarchical, progressive, and lossless.
- Arbitrary image sizes; DCT mode 8-12 bits/ sample. Luminance and chrominance channels are separately encoded.
- We will only discuss the baseline method.

JPEG-baseline.

- DCT: The image is divided into 8x8 blocks. Each pixel is level shifted by 2^{n-1} where 2^n is the maximum number of gray levels in the image. Thus for 8 bit images, you subtract 128. Then the 2-D DCT of each block is computed. For the baseline system, the input and output data precision is restricted to 8 bits and the DCT values are restricted to 11 bits.
- Quantization: the DCT coefficients are threshold coded using a quantization matrix, and then reordered using zig-zag scanning to form a 1-D sequence.
- The non-zero AC coefficients are Huffman coded. The DC coefficients of each block are DPCM coded relative to the DC coefficient of the previous block.

JPEG -color image

- RGB to Y-Cr-Cb space
  - Y = 0.3R + 0.6G + 0.1B
  - Cr = 0.5 (B-Y) + 0.5
  - Cb = (1/1.6) (R – Y) + 0.5
- Chrominance samples are sub-sampled by 2 in both directions.

Y1	Y2	Y3	Y4
Y5	Y6	Y7	Y8
Y9	Y10	Y11	Y12
Y13	Y14	Y15	Y16

Cr1	Cr2
Cr3	Cr4

Cb1	Cb2
Cb3	Cb4

Non-Interleaved
Scan 1:Y1,Y2,…,Y16
Scan 2: Cr1, Cr2, Cr3, Cr4
Scan 3: Cb1, Cb2, Cb3, Cb4

Interleaved: Y1, Y2, Y3, Y4, Cr1, Cb1,Y5,Y6,Y7,Y8,Cr2,Cb2,…

JPEG – quantization matrices

- Check out the matlab workspace (dctex.mat)
- Quantization table for the luminance channel.
- Quantization table for the chrominance channel.
- JPEG baseline method
  - Consider the 8x8 image (matlab: array s.)
  - Level shifted (s-128=sd).
  - 2d-DCT: dct2(sd)= dcts
  - After dividing by quantiization matrix qmat: dctshat.
  - Zigzag scan as in threshold coding.

  [20, 5, -3, -1, -2,-3, 1, 1, -1, -1, 0, 0, 1, 2, 3, -2, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, EOB].

An 8x8 sub-image (s)

s = (8x8block)

183	160	94	153	194	163	132	165
183	153	116	176	187	166	130	169
179	168	171	182	179	170	131	167
177	177	179	177	179	165	131	167
178	178	179	176	182	164	130	171
179	180	180	179	183	164	130	171
179	179	180	182	183	170	129	173
180	179	181	179	181	170	130	169

sd =(level shifted)

55	32	-34	25	66	35	4	37
55	25	-12	48	59	38	2	41
51	40	43	54	51	42	3	39
49	49	51	49	51	37	3	39
50	50	51	48	54	36	2	43
51	52	52	51	55	36	2	43
51	51	52	54	55	42	1	45
52	51	53	51	53	42	2	41

2D DCT (dcts) and the quantization matrix (qmat)

dcts=

312	56	-27	17	79	-60	26	-26
-38	-28	13	45	31	-1	-24	-10
-20	-18	10	33	21	-6	-16	-9
-11	-7	9	15	10	-11	-13	1
-6	1	6	5	-4	-7	-5	5
3	3	0	-2	-7	-4	1	2
3	5	0	-4	-8	-1	2	4
3	1	-1	-2	-3	-1	4	1

qmat=

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Division by qmat (dctshat)=dcts/qmat

dcthat=

20	5	-3	1	3	-2	1	0
-3	-2	1	2	1	0	0	0
-1	-1	1	1	1	0	0	0
-1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

dcts=

312	56	-27	17	79	-60	26	-26
-38	-28	13	45	31	-1	-24	-10
-20	-18	10	33	21	-6	-16	-9
-11	-7	9	15	10	-11	-13	1
-6	1	6	5	-4	-7	-5	5
3	3	0	-2	-7	-4	1	2
3	5	0	-4	-8	-1	2	4
3	1	-1	-2	-3	-1	4	1

Zig-zag scan of dcthat

dcthat=

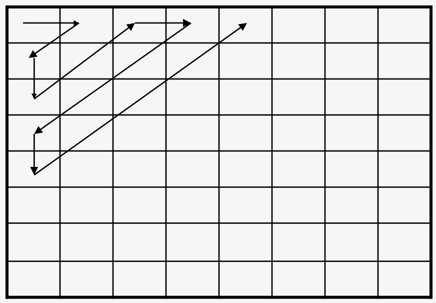
20	5	-3	1	3	-2	1	0
-3	-2	1	2	1	0	0	0
-1	-1	1	1	1	0	0	0
-1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Zigzag scan as in threshold coding.

[20, 5, -3, -1, -2,-3, 1, 1, -1, -1, 0, 0, 1, 2, 3, -2, 1, 1, 0, 0, 0, 0, 0, 0,1, 1, 0, 1, EOB].

Threshold coding -revisited

Zig-zag scanning of the coefficients.



The coefficients along the zig-zag scan lines are mapped into [run,level] where the *level* is the value of non-zero coefficient, and *run* is the number of zero coeff. preceding it. The DC coefficients are usually coded separately from the rest.

JPEG – baseline method example

Zigzag scan as in threshold coding.

[20, 5, -3, -1, -2,-3, 1, 1, -1, -1, 0, 0, 1, 2, 3, -2, 1, 1, 0, 0, 0, 0, 0, 0,1, 1, 0, 1, EOB].

- The DC coefficient is DPCM coded (difference between the DC coefficient of the previous block and the current block.)
- The AC coef. are mapped to run-level pairs.
  (0,5), (0,-3), (0, -1), (0,-2),(0,-3),(0,1),(0,1),(0,-1),(0,-1), (2,1), (0,2), (0,3), (0,-2),(0,1),(0,1),(6,1),(0,1),(1,1),EOB.
- These are then Huffman coded (codes are specified in the JPEG scheme.)
- The decoder follows an inverse sequence of operations. The received coefficients are first multiplied by the same quantization matrix.
  (recddctshat=dctshat.*qmat.)
- Compute the inverse 2-D dct. (recdsd=idct2(recddctshat); add 128 back. (recds=recdsd+128.)

Decoder

Recddcthat=dcthat*qmat

320	55	-30	16	72	-80	51	0
-36	-24	14	38	26	0	0	0
-14	-13	16	24	40	0	0	0
-14	0	0	29	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Recdsd=

67	12	-9	20	69	43	-8	42
58	25	15	30	65	40	-4	47
46	41	44	40	59	38	0	49
41	52	59	43	57	42	3	42
44	54	58	40	58	47	3	33
49	52	53	40	61	47	1	33
53	50	53	46	63	41	0	45
55	50	56	53	64	34	-1	57

Received signal

Reconstructed S=

195	140	119	148	197	171	120	170
186	153	143	158	193	168	124	175
174	169	172	168	187	166	128	177
169	180	187	171	185	170	131	170
172	182	186	168	186	175	131	161
177	180	181	168	189	175	129	161
181	178	181	174	191	169	128	173
183	178	184	181	192	162	127	185

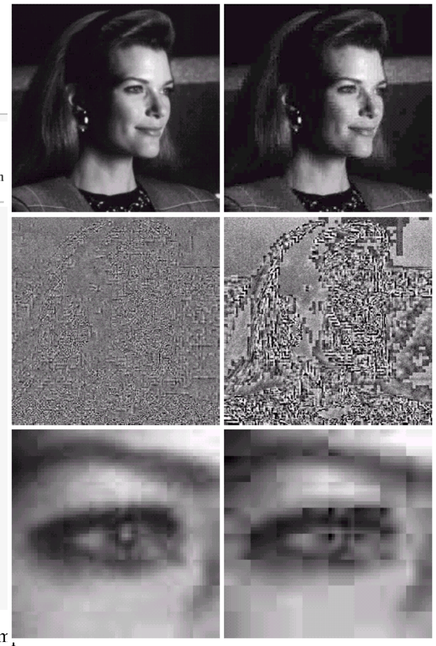
s = (8x8block)

183	160	94	153	194	163	132	165
183	153	116	176	187	166	130	169
179	168	171	182	179	170	131	167
177	177	179	177	179	165	131	167
178	178	179	176	182	164	130	171
179	180	180	179	183	164	130	171
179	179	180	182	183	170	129	173
180	179	181	179	181	170	130	169

Example

a b
c d
e f

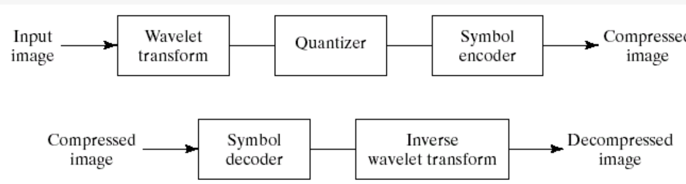
FIGURE 8.38 Left column: Approximations of Fig. 8.23 using the DCT and normalization array of Fig. 8.37(b). Right column: Similar results for 4**Z**.



Wavelet Compression

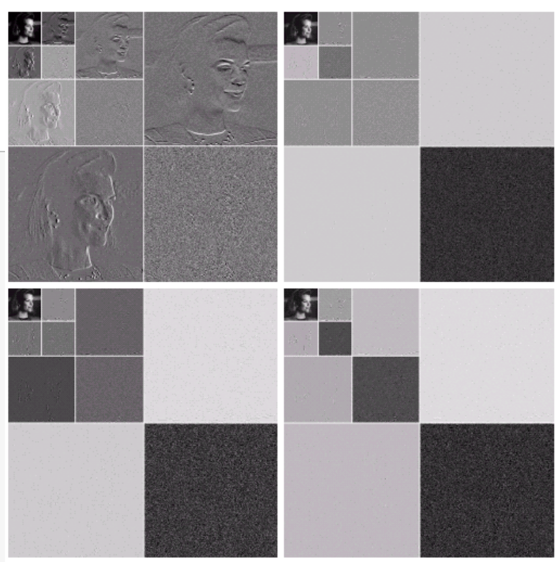
a
b

FIGURE 8.39 A wavelet coding system: (a) encoder; (b) decoder.



a b
c d
e f

FIGURE 8.41 (a), (c), and (e) Wavelet coding results with a compression ratio of 108 to 1; (b), (d), and (f) similar results for a compression of 167 to 1.



a b
c d

FIGURE 8.42 Wavelet transforms of Fig. 8.23 with respect to (a) Haar wavelets, (b) Daubechies wavelets, (c) symlets, and (d) Cohen-Daubechies-Feauveau biorthogonal wavelets.

Image Compression: Summary

- Data redundancy
- Self-information and Entropy
- Error-free compression
  - Huffman coding, Arithmetic coding, LZW coding, Run-length encoding
  - Predictive coding
- Lossy coding techniques
  - Predictive coing (Lossy)
  - Transform coding
    - DCT, DFT, KLT, …
- JPEG image compression standard