Image Enhancement: Histogram Processing

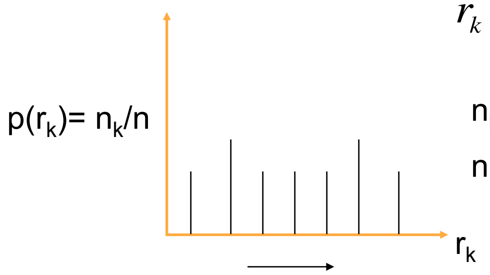
Reading:
Chapter 3 (Spatial domain)

Histogram Processing

- Histogram Equalization
- Histogram Specification/Matching

Histogram

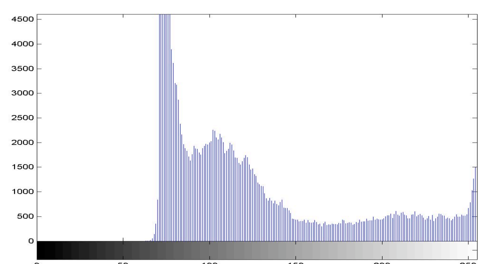
$$r_k \in \{0,1,2,\ldots,L-1\}$$



n_k : # pixels with gray level r_k

n : # pixels in the image

Histogram



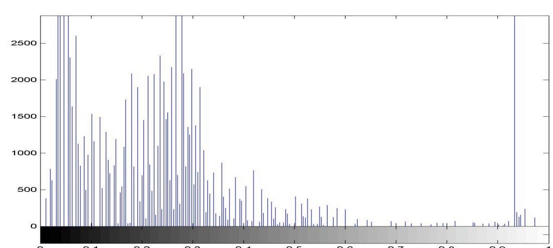
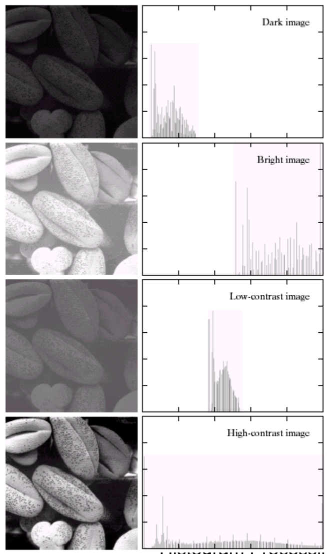


Figure 3.15: histograms



a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Histogram Modification

r : Input gray level $\in$ [0, 1]
s : Transformed gray level $\in$ [0, 1]

s = T (r) T : Transformation function

Histogram Equalization

(i) $T(r)$ is single valued valued and monotonically increasing in
$0 \le r \le 1$

(ii) $0 \le T(r) \le 1$ for $0 \le r \le 1$

$$[0, 1] \xrightarrow{T} [0, 1]$$

Inverse transformation : $T^{-1}(s) = r \quad 0 \le s \le 1$

$T^{-1}(s)$ also satisfies (i) and (ii)

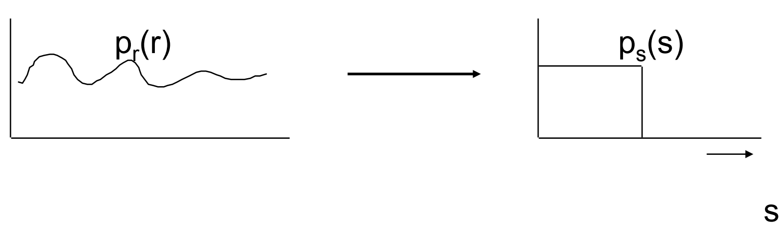
The gray levels in the image can be viewed as random variables taking values in the range [0,1].

Let $p_r(r)$: p.d.f. of input level r and let $p_s(s)$: p.d.f. of s

$s = T(r) \quad ; \quad \therefore p_s(s) = p_r(r) \left. \frac{dr}{ds} \right|_{r=T^{-1}(s)}$ (from ECE 140)

Equalization (contd.)

We are interested in obtaining a transformation function T( ) which transforms an arbitrary p.d.f. to an uniform distribution



Consider $s = T(r) = \int_0^r p_r(w)\, dw \quad 0 \le r \le 1$

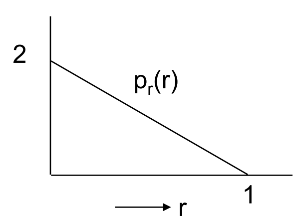
(Cumulative distribution function of r)

$$p_s(s) = p_r(r) \, {dr}/{ds} \Big|_{r=T^{-1}(s)} \; ;$$

$$\frac{ds}{dr} = \frac{d}{dr}\left[\int_0^r p_r(w) dw\right] = p_r(r)$$

$$\therefore\; p_s(s) = p_r(r) \frac{1}{p_r(r)}\Bigg|_{r=T^{-1}(s)} \equiv 1 \qquad 0 \le s \le 1$$

Equalization: Example



$$p_r(r) = \begin{cases} -2r + 2 & 0 \le r \le 1 \\ 0 & \text{Else} \end{cases}$$

$$s = T(r) = \int_0^r (2 - 2w)\, dw = (2w - w^2)\Big|_0^r = 2r - r^2$$

$$\therefore r^2 - 2r + s = 0$$

Equalization (example: contd.)

$$r = \frac{+2 \pm \sqrt{4 - 4s}}{2} = 1 \pm \sqrt{1 - s}$$

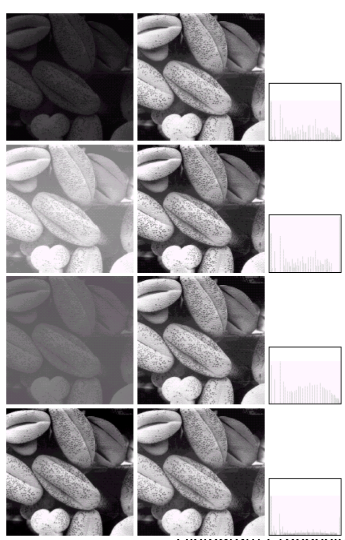
$$r = T^{-1}(s) = 1 - \sqrt{1 - s} \qquad \text{as } r \in [0,1]$$

$$p_s(s) = p_r(r)\frac{dr}{ds} = (-2r + 2)\ \frac{d}{ds}(1 - \sqrt{1 - s})\Big|_{r = 1 - \sqrt{1 - s}}$$

$$= (-2r + 2)\left(\frac{-1}{2}(1 - s)^{-1/2}(-1)\right)$$

$$= (-2 + 2\sqrt{1 - s} + 2)\frac{+1}{2\sqrt{1 - s}} = \ 1 \qquad 0 \le r \le 1$$

Equalized Histograms

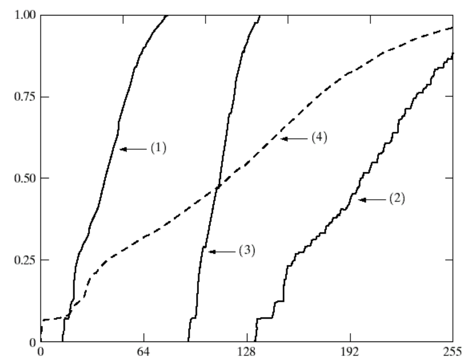


a b c

FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

Fig 3.18: Transformation curves

FIGURE 3.18 Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).



Equalization: Discrete Case

$$p_r(r_k) = \frac{n_k}{n} \qquad 0 \le r_k \le 1 \quad ; \quad k = 0,1,\ldots,L-1$$

$L \rightarrow$ Number of levels

$$s_k = T(r_k) \quad = \sum_{j=0}^{k} p_r(r_j) \quad = \quad \sum_{j=0}^{k} {n_j}/{n}$$

Discrete Case: Example

64x64 image; 8 gray levels.

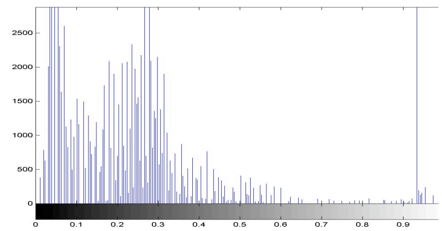
Notice that equalized histogram is not perfectly flat!

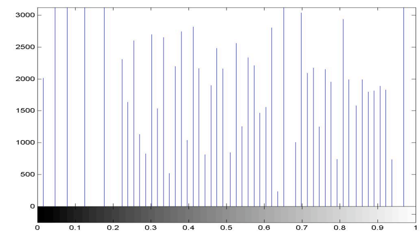
k	r_k	n_k	n_k/n	$S_k = \sum_{j=0}^{k} n_j/n$	$p_s(s_k)$
0	0	790	0.19	$0.19 \to 1/7 \to s_0$	0.19
1	$1/7$	1023	0.25	$0.44 \to 3/7 \to s_1$	0.25
2	$2/7$	850	0.21	$0.65 \to 5/7 \to s_2$	0.21
3	$3/7$	656	0.16	$0.81 \to 6/7 \to s_3$	0.24
4	$4/7$	329	0.08	$0.89 \to 6/7 \to s_3$	0.24
5	$5/7$	245	0.06	$0.95 \to 1 \to s_4$	0.11
6	$6/7$	122	0.03	$0.98 \to 1 \to s_4$	0.11
7	$7/7$	81	0.02	$1.0 \to 1 \to s_4$	0.11

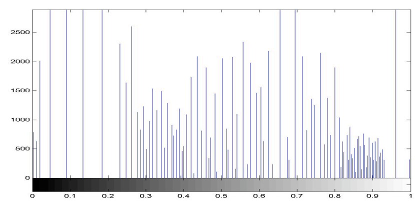
Equalization: Image Examples

Original, Equalized (64)
Equalized (256)

..and their histograms







Original, equalized (64 levels)
Equalized (256 levels)

Histogram specification

Suppose $s = T(r) = \int_0^r p_r(w)\, dw$

$p_r(r) \rightarrow$ Original histogram ; $p_z(z) \rightarrow$ Desired histogram

Let $v = G(z) = \int_0^z p_z(w)\, dw$ and $z = G^{-1}(v)$

But s and v are identical p.d.f.

$\therefore z = G^{-1}(v) = G^{-1}(s) = G^{-1}(T(r))$

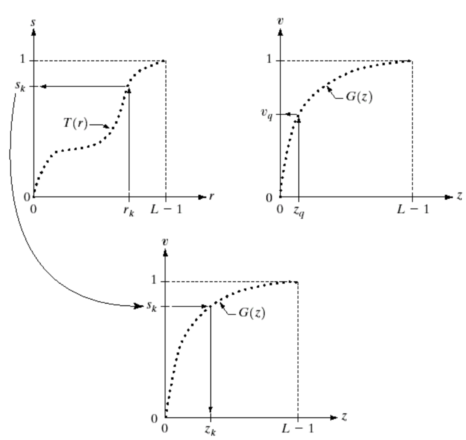
Question

- What happens when you apply equalization to an already equalized histogram
  - In the continuous case?
  - In the discrete case?

Fig 3.19: Matching

a b
c

FIGURE 3.19 (a) Graphical interpretation of mapping from r_k to s_k via $T(r)$. (b) Mapping of z_q to its corresponding value v_q via $G(z)$. (c) Inverse mapping from s_k to its corresponding value of z_k.



Matching: Summary

Steps:

(1) Equalize the levels of original image

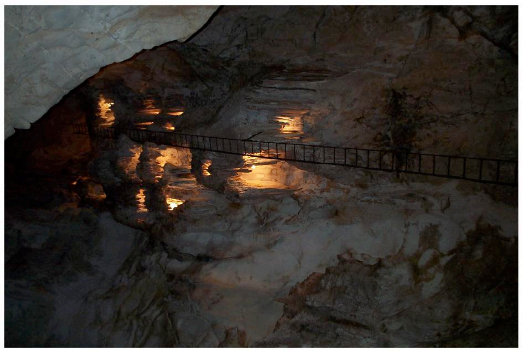
(2) Specify the desired $p_z(z)$ and obtain $G(z)$

(3) Apply $z=G^{-1}(s)$ to the levels s obtained in step 1

Matching: an example

z_k	$p_z(z_k)$	$v_k = G(r_k)$	n_k	$p_z(z_k)$
$z_0 = 0$	0	0	0	0
$z_1 = 1/7$	0	0	0	0
$z_2 = 2/7$	0	0	0	0
$z_3 = 3/7$	0.15	$0.15 \leftrightarrow s_0 = 1/7$	790	0.19
$z_4 = 4/7$	0.2	$0.35 \leftrightarrow s_1 = 3/7$	1023	0.25
$z_5 = 5/7$	0.3	$0.65 \leftrightarrow s_2 = 5/7$	850	0.21
$z_6 = 6/7$	0.2	$0.85 \leftrightarrow s_3 = 6/7$	985	0.24
$z_7 = 1$	0.15	$1.0 \leftrightarrow s_4 = 1$	448	0.11

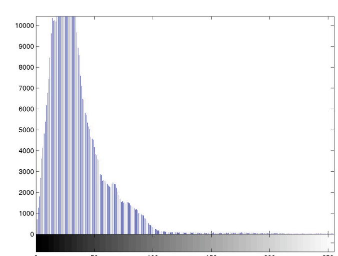
Histogram Matching: example



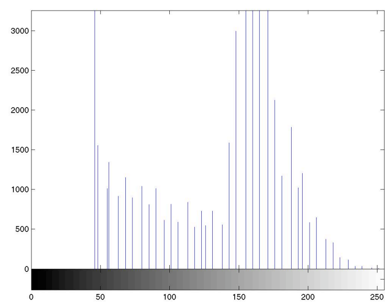
Original image
(Jenolan caves,
blue mountain,
Sydney, Ausralia)

Color to grayscale

```
I=imread('sydney1.jpg);
I1=rgb2gray(I);
I1=imresize(I1,0.5);
Imhist(I1);
```

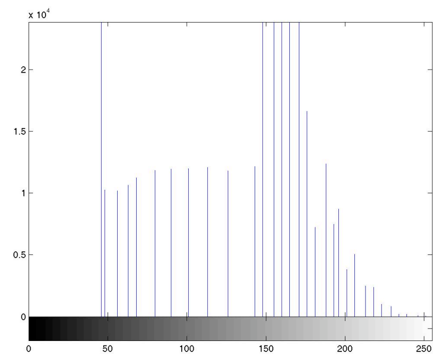


Desired & modified histograms



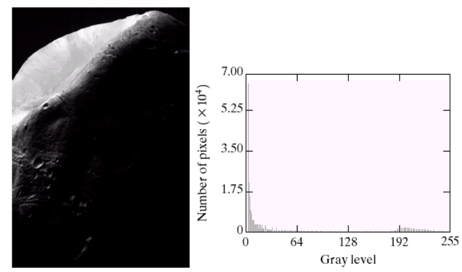
Imhist(J)
J=some image

I2=histeq(I1,imhist(J));
Imhist(I2);



Histogram modified image

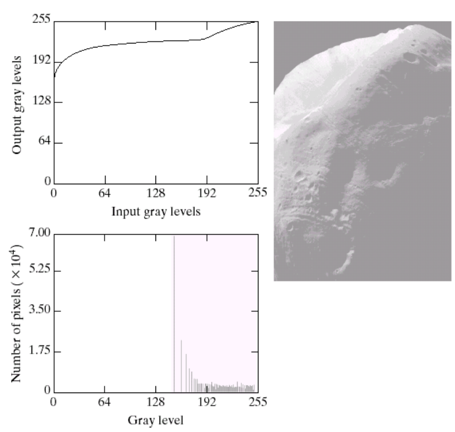
Fig 3.20: Another example



a b

FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's ***Mars Global Surveyor.*** (b) Histogram. (Original image courtesy of NASA.)

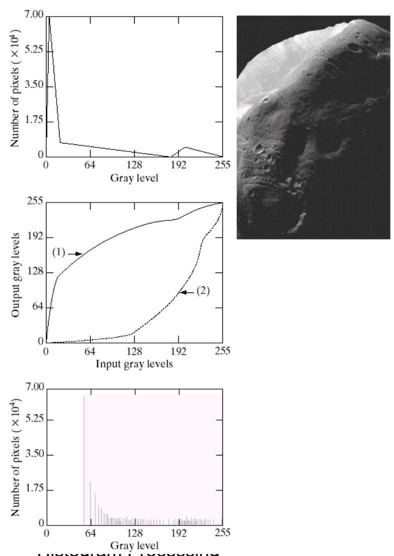
Fig 3.21



a b
c

FIGURE 3.21 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

Fig 3.22



a c
b
d

FIGURE 3.22
(a) Specified histogram.
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).