Averaging

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$\bar{g}(x,y) = \frac{1}{M}\sum_{i=1}^{M} g_i(x,y)$$

$$E(\bar{g}(x,y)) = f(x,y) \text{ and } \sigma^2_{\bar{g}} = \frac{1}{M}\sigma^2_{\eta}(x,y)$$

$\eta(x,y) \rightarrow$ Uncorrelated zero mean

$\sigma^2_{\eta}(x,y) \rightarrow$ Reduces the noise variance

Fig 3.30

Spatial Filtering

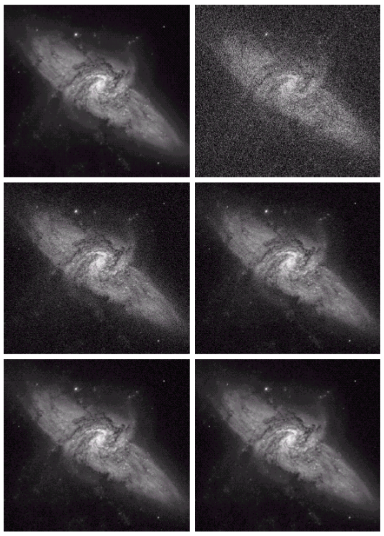
Chapter 3

Required Reading: All sections except 3.8

Image Forensics Project Guidelines

- Process
- Images

Fig 3.30



a b
c d
e f

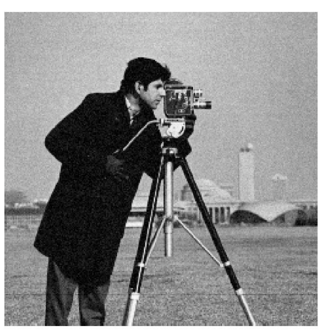
FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

Another example


Images with additive Gausian Noise; Independent Samples.

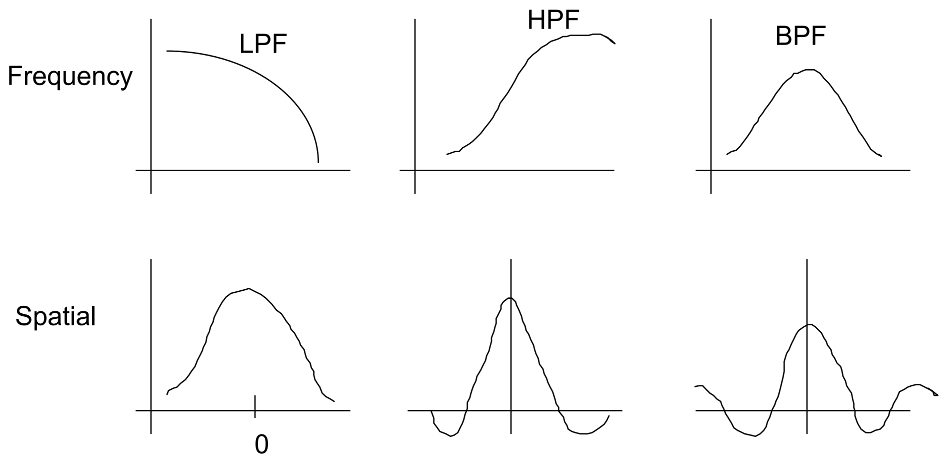
I=imnoise(J,'Gaussian');


Averaged image

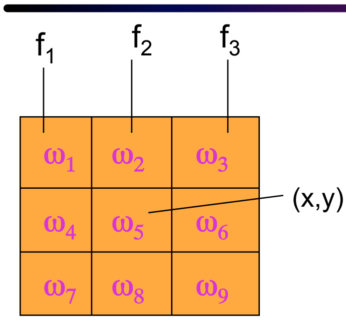



Left: averaged image (10 samples);
Right: original image

Spatial filtering



Smoothing (Low Pass) Filtering



Replace f (x,y) with

$$\hat{f}(x,y) = \sum_i \omega_i f_i$$

Linear filter

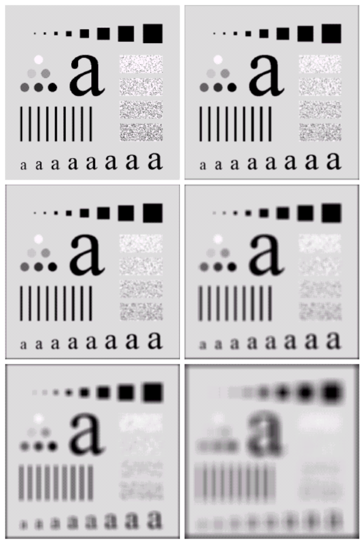
LPF: reduces additive noise → blurs the image

→ sharpness details are lost

(Example: Local averaging)

Fig 3.35

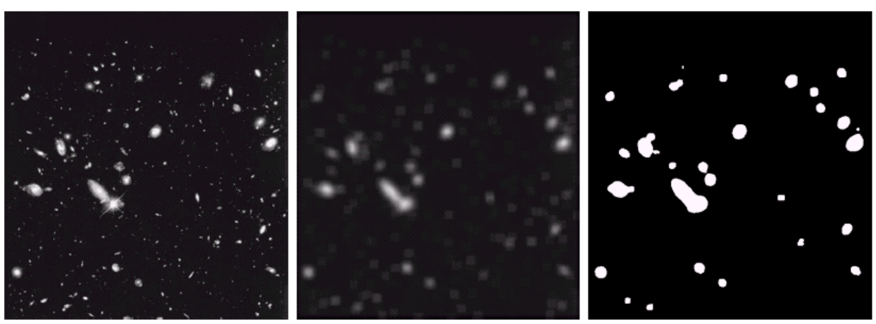
Fig 3.35: smoothing



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Fig 3.36: another example



a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Image Dithering

- Dithering: to produce visually pleasing signals from heavily quantized data.
  - Halftoning: convert a gray scale image to a binary image by thresholding.
  - Dithering to "add" noise so that the resulting image is smoother than just thresholding (but still it is a binary image)
  - Your homework #4 explores this further with a MATLAB exercise.

Median filtering

Replace f (x,y) with median [f (x' , y')]
(x' , y') ε neighbourhood

- Useful in eliminating intensity spikes. (salt & pepper noise)
- Better at preserving edges.

Example:

10	20	20
20	15	20
25	20	100

→ (10,15,20,20,20,20,20,25,100)

Median=20

So replace (15) with (20)

Median Filter: Root Signal

Repeated applications of median filter to a signal results in an invariant signal called the "root signal".
A root signal is invariant to further application of the medina filter.

Example: 1-D signal: Median filter length = 3

0	0	0	1	2	1	2	1	2	1	0	0	0	
0	0	0	1	1	2	1	2	1	1	0	0	0	
0	0	0	1	1	1	2	1	1	1	0	0	0	
0	0	0	1	1	1	1	1	1	1	0	0	0	root signal

Invariant Signals

Invariant signals to a median filter:

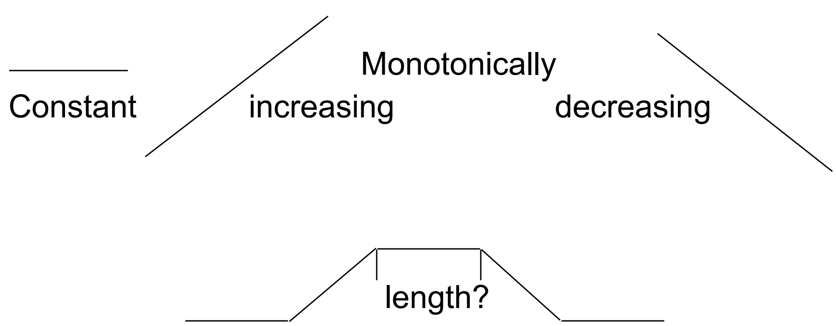
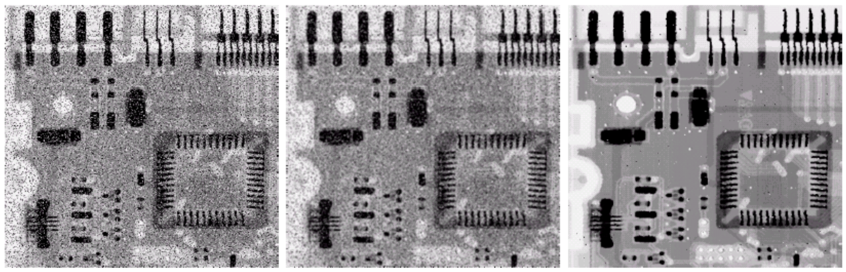


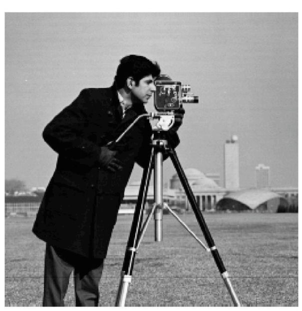
Fig 3.37: Median Filtering example



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Media Filter: another example



Original and with salt & pepper noise
imnoise(image, ‘salt & pepper’);

Donoised images

Local averaging
K=filter2(fspecial('average',3),image)/255.

Median filtered
L=medfil2(image, [3 3]);

Sharpening Filters

- Enhance finer image details (such as edges)
- Detect region /object boundaries.

Example:

−1	−1	−1
−1	8	−1
−1	−1	−1

Unsharp Masking

Subtract Low pass filtered version from the original
emphasizes high frequency information

I' = A (Original) - Low pass
HP = O - LP A > 1
I' = (A - 1) O + HP
A = 1 => I' = HP
A > 1 => LF components added back.

Derivative Filters

1/9

−1	−1	−1
−1	ω	−1
−1	−1	−1

Gradient

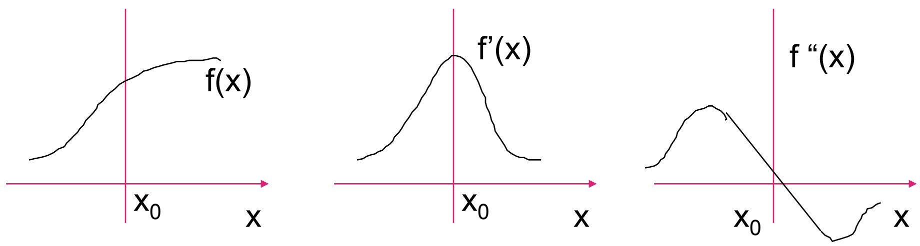
$$\nabla f = \left[\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \right]^T$$

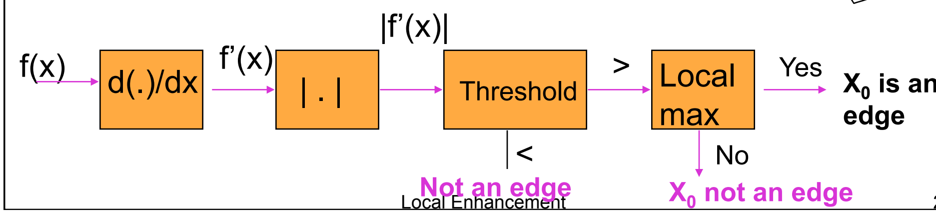
$$\|\nabla f\| = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

Edge Detection

Gradient based methods

$$\nabla f = \left(\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \right)^T$$





Digital edge detectors

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$|\nabla f| \approx \left[(z_5 - z_8)^2 + (z_5 - z_6)^2 \right]^{1/2}$$

$$|\nabla f| \approx |z_5 - z_8| + |z_5 - z_6|$$

Robert's operator

1	0
0	-1

$| z_5 - z_9 |$

0	1
-1	0

$| z_6 - z_8 |$

prewitt

-1	-1	-1
0	0	0
1	1	1

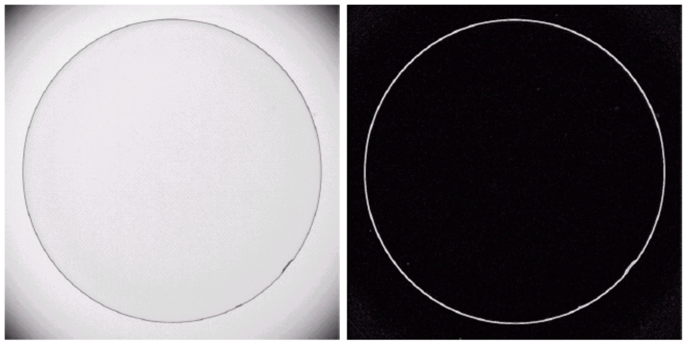
-1	0	1
-1	0	1
-1	0	1

Sobel's

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Fig 3.45: Sobel edge detector



a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

Laplacian based edge detectors

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

	1	
1	-4	1
	1	

- Rotationally symmetric, linear operator
- Check for the zero crossings to detect edges
- Second derivatives => sensitive to noise.