ECE 178 Digital Image Processing Discussion Session #2

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Linearity and Time Invariance:

Consider a system $L\{.\}$ that relates the input $x[m,n]$ to $y[m,n]$,

$$y_1[m,n] = L\{x_1[m,n]\}$$

$$y_2[m,n] = L\{x_2[m,n]\}$$

The system $L\{.\}$ is called linear if

$$Ay_1[m,n] + By_2[m,n] = L\{Ax_1[m,n] + Bx_2[m,n]\}$$

The system $L\{.\}$ is called time invariant if

$$y[m-m_0, n-n_0] = L\{x[m-m_0, n-n_0]\}$$

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Importance of Linearity and Time Invariance:

Consider a system $L\{.\}$ that relates the input $x[m,n]$ to $y[m,n]$.

$$y[m,n] = L\{x[m,n]\} \tag{1}$$

We can write down $x[m,n]$ as

$$x[m,n] = \sum_{k,l} x[k,l]\delta[m-k,n-l]$$

Then the Equation 1 becomes:

$$y[m,n] = L\{\sum_{k,l} x[k,l]\delta[m-k,n-l]\}$$

If $L\{.\}$ is linear:

$$y[m,n] = \sum_{k,l} x[k,l]L\{\delta[m-k,n-l]\}$$

If $L\{.\}$ is time invariant and the response of $L\{.\}$ to the $\delta[m,n]$ is $h[m,n]$, i.e.

$$h[m,n] = L\{\delta[m,n]\}$$

The Equation 1 boils down to

$$y[m,n] = \sum_{k,l} x[k,l]h[m-k,n-l]$$

Which is the convolution of $x[m,n]$ and $h[m,n]$.

$$y[m,n] = x[m,n] * h[m,n] = \sum_{k,l} x[k,l]h[m-k,n-l]$$

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Exercise: 1

Determine, for each of the following systems defined by the input/output relations, if the system is (a) linear, and (b) shift invariant. In the following, $y[n]$ denotes the output of a system for input $x[n]$.

a)

$$y[n] = x[n] - x[n-1]$$

$$y_1[n] = x_1[n] - x_1[n-1]$$
$$y_2[n] = x_2[n] - x_2[n-1]$$

$$Ay_1[n] + By_2[n] = Ax_1[n] + BAx_2[n] - Ax_1[n-1] - Bx_2[n-1]$$

$\Rightarrow$ Linear.

$$y[n-n_0] = x[n-n_0] - x[n-n_0-1]$$

$\Rightarrow$ Time-invariant.

b)

$$y[m,n] = mx[m,n-1] + nx[m-1,n]$$

$$y_1[m,n] = mx_1[m,n-1] + nx_1[m-1,n]$$
$$y_2[m,n] = mx_2[m,n-1] + nx_2[m-1,n]$$

$$Ay_1[m,n] + By_2[m,n] = m(Ax_1[m,n-1] + Bx_2[m,n-1]) + n(Ax_1[m-1,n] + Bx_2[m-1,n])$$

$\Rightarrow$ Linear.

$$y[m,n] = mx[m-m_0,n-n_0-1] + nx[m-m_0-1,n-n_0]$$

$$\neq (m-m_0)x[m-m_0,n-n_0-1] + (n-n_0)x[m-m_0-1,n-n_0] = y[m-m_0,n-n_0]$$

$\Rightarrow$ Time-variant.

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Exercise: 2 (2.15)

a)

4-connected: As shown in Figure 1-a, there is no 4-path between p and q since one cannot reach q from p by traveling along points that are 4-connected and have values in V.

8-connected: As shown in Figure 1-b, the shortest 8-path has length 4 and is unique.

m-connected: As shown in Figure 1-c, the shortest m-path has length 5 and is unique.

b)

4-connected: As shown in Figure 1-d, q can be reached from p along the shortest path of length. Note that this path is non-unique as the dashed line shows another path of the same length.

8-connected: As shown in Figure 1-e, the shortest 8-path has length 4 and is non-unique. Check the dashed line for an alternative.

m-connected: The shortest m-path is non-unique and has length 6. The paths coincide with the shortest 4-paths.

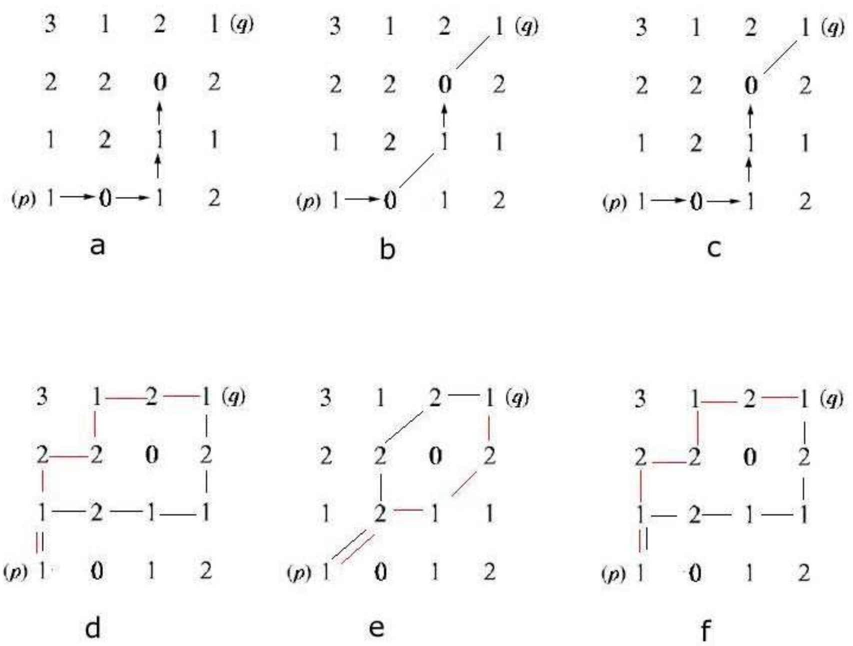

Figure 1: