

# Image Enhancement: Histogram Processing



Reading:  
Chapter 3 (Spatial domain)

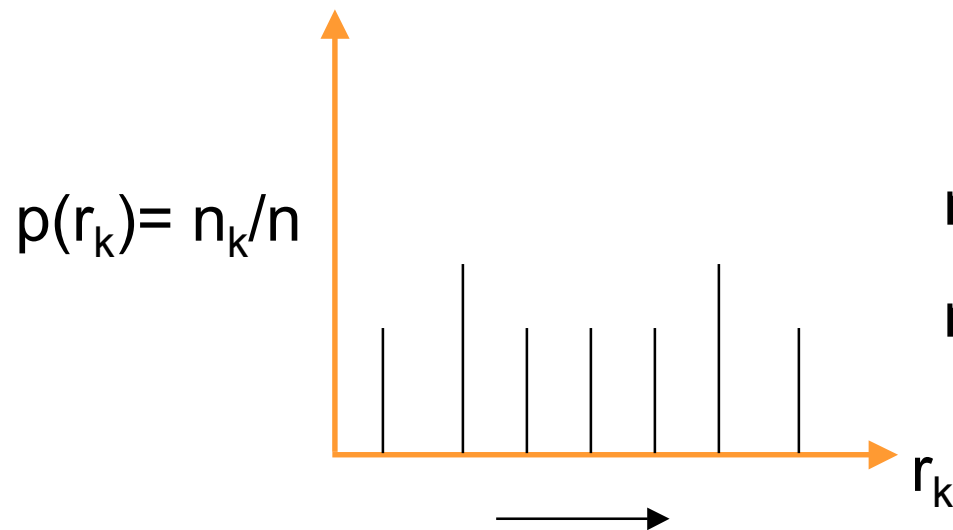
# Histogram Processing



- Histogram Equalization
- Histogram Specification/Matching

# Histogram

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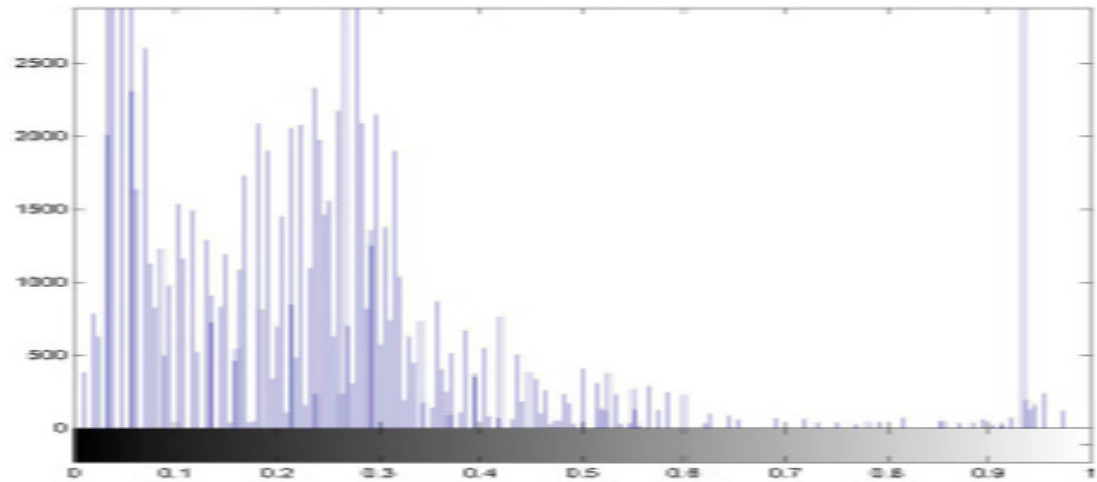
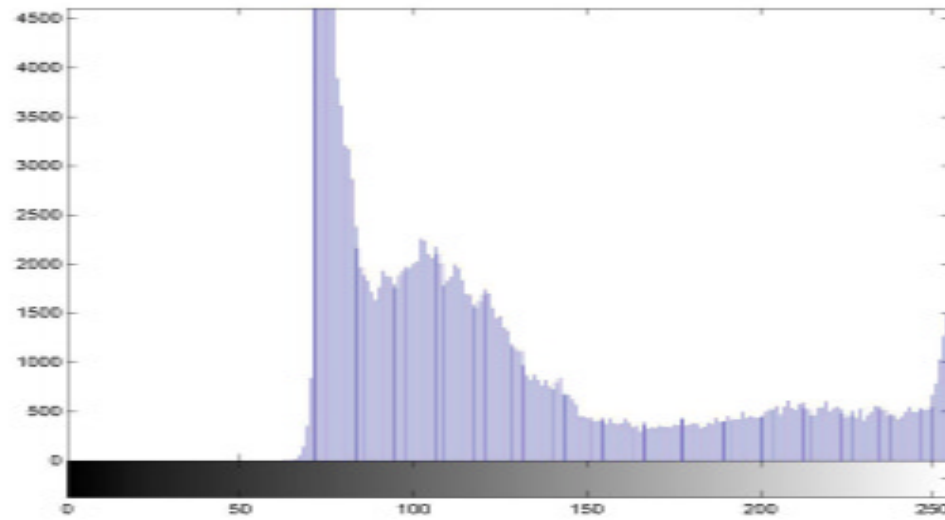


$$r_k \in \{0, 1, 2, 3, \dots, L-1\}$$

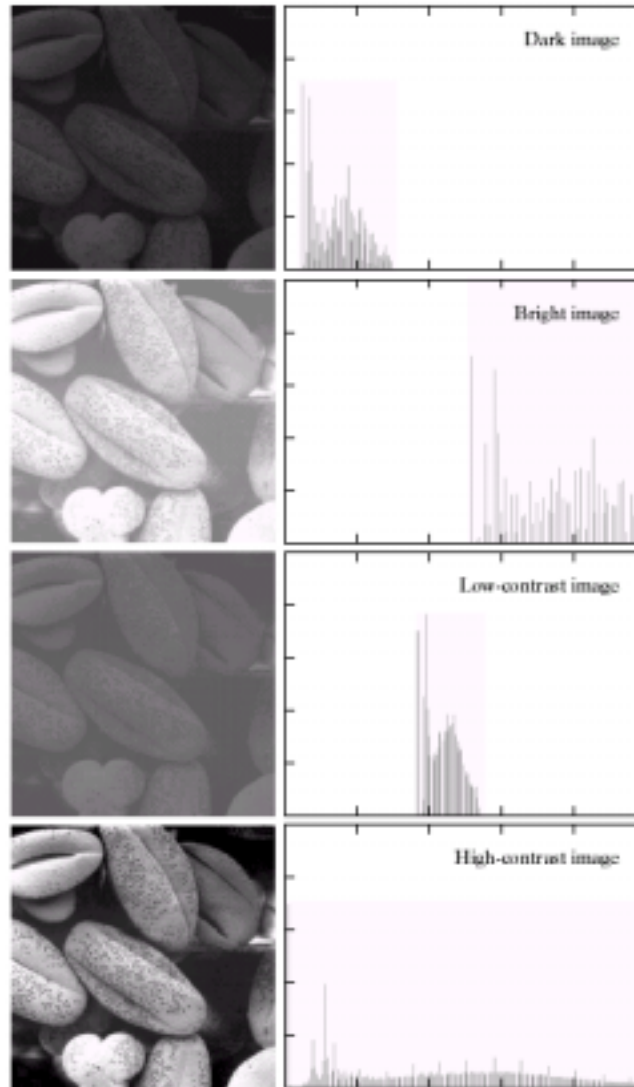
$n_k$  : # pixels with gray level  $r_k$

$n$  : # pixels in the image

# Histogram



# Figure 3.15: histograms



a b

**FIGURE 3.15** Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

# Histogram Modification

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$r$  : Input gray level  $\in [0, 1]$

$s$  : Transformed gray level  $\in [0, 1]$

$s = T(r)$        $T$  : Transformation function

# Histogram Equalization

( i )  $T(r)$  is single valued and monotonically increasing in  
 $0 \leq r \leq 1$

( ii )  $0 \leq T(r) \leq 1$  for  $0 \leq r \leq 1$

$$[0, 1] \xrightarrow{T} [0, 1]$$

Inverse transformation :  $T^{-1}(s) = r$   $0 \leq s \leq 1$

$T^{-1}(s)$  also satisfies ( i ) and ( ii )

The gray levels in the image can be viewed as random variables taking values in the range  $[0,1]$ .

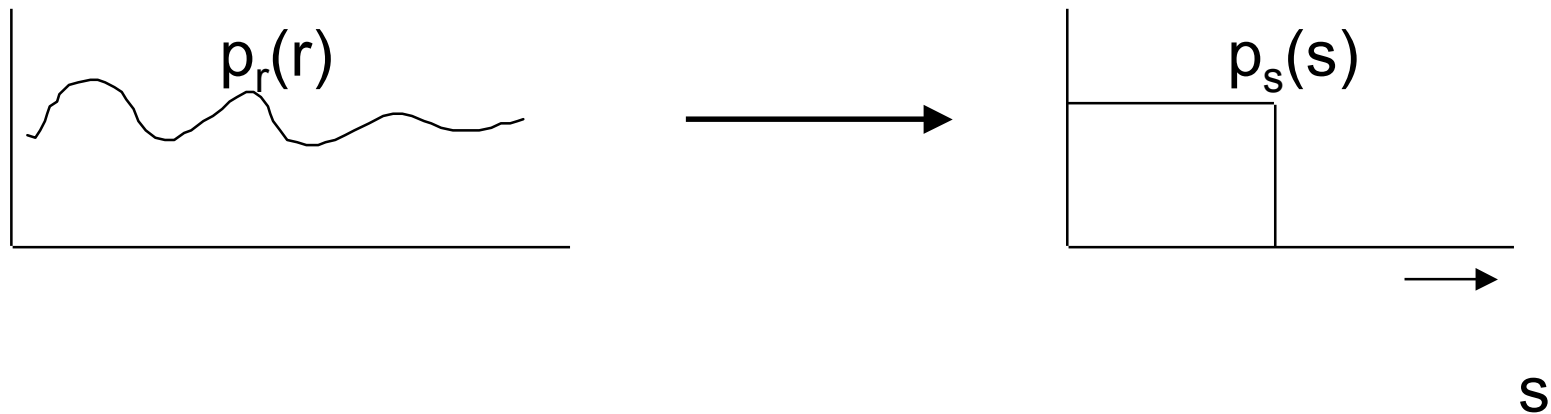
Let  $p_r(r)$  : p.d.f. of input level  $r$  and let  $p_s(s)$  : p.d.f. of  $s$

$$s = T(r) \quad ; \quad \therefore p_s(s) = p_r(r) \left. \frac{dr}{ds} \right|_{r=T^{-1}(s)} \quad (\text{from ECE 140})$$

# Equalization (contd.)

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We are interested in obtaining a transformation function  $T(\cdot)$  which transforms an arbitrary p.d.f. to an uniform distribution





— Consider  $s = T(r) = \int_0^r p_r(w) dw \quad 0 \leq r \leq 1$

(Cumulative distribution function of  $r$ )

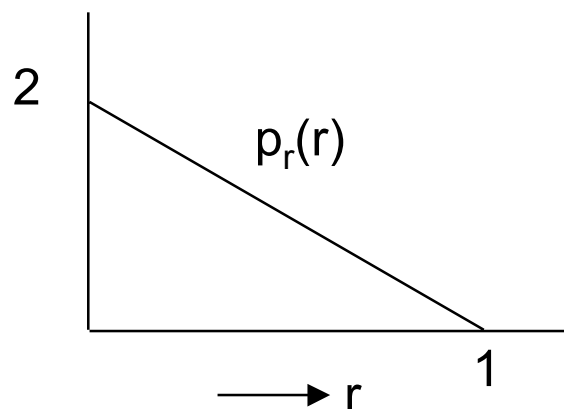
$$p_s(s) = p_r(r) \left. \frac{dr}{ds} \right|_{r=T^{-1}(s)} ;$$

$$\frac{ds}{dr} = \frac{d}{dr} \left[ \int_r^0 p_r(w) dw \right] = p_r(r)$$

$$\therefore p_s(s) = p_r(r) \left. \frac{1}{p_r(r)} \right|_{r=T^{-1}(s)} \equiv 1 \quad 0 \leq s \leq 1$$

# Equalization: Example

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$$p_r(r) = \begin{cases} -2r + 2 & 0 \leq r \leq 1 \\ 0 & \text{Else} \end{cases}$$

$$s = T(r) = \int_0^r (2 - 2w) dw = (2w - w^2) \Big|_0^r = 2r - r^2$$

$$\therefore r^2 - 2r + s = 0$$

# Equalization (example: contd.)

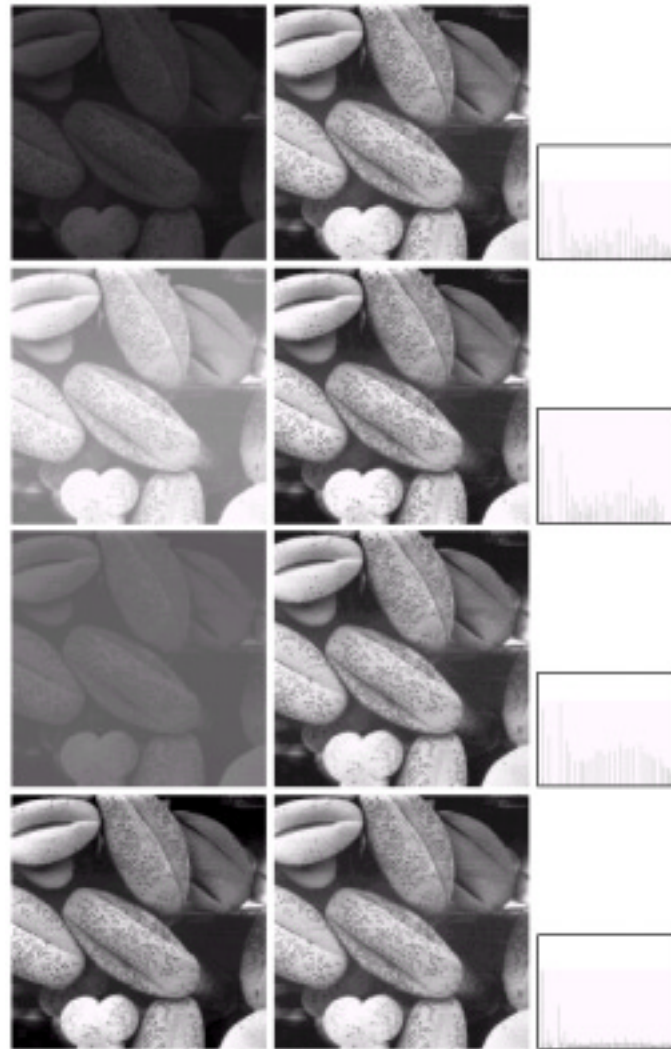
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$$r = \frac{+2 \pm \sqrt{4 - 4s}}{2} = 1 \pm \sqrt{1 - s}$$

$$r = T^{-1}(s) = 1 - \sqrt{1 - s} \quad \text{as } r \in [0,1]$$

$$\begin{aligned} p_s(s) &= p_r(r) \frac{dr}{ds} = (-2r + 2) \left. \frac{d}{ds} (1 - \sqrt{1 - s}) \right|_{r=1-\sqrt{1-s}} \\ &= (-2r + 2) \left( \frac{-1}{2} (1 - s)^{-1/2} (-1) \right) \\ &= (-2 + 2\sqrt{1 - s} + 2) \frac{+1}{2\sqrt{1 - s}} = 1 \quad 0 \leq r \leq 1 \end{aligned}$$

# Equalized Histograms

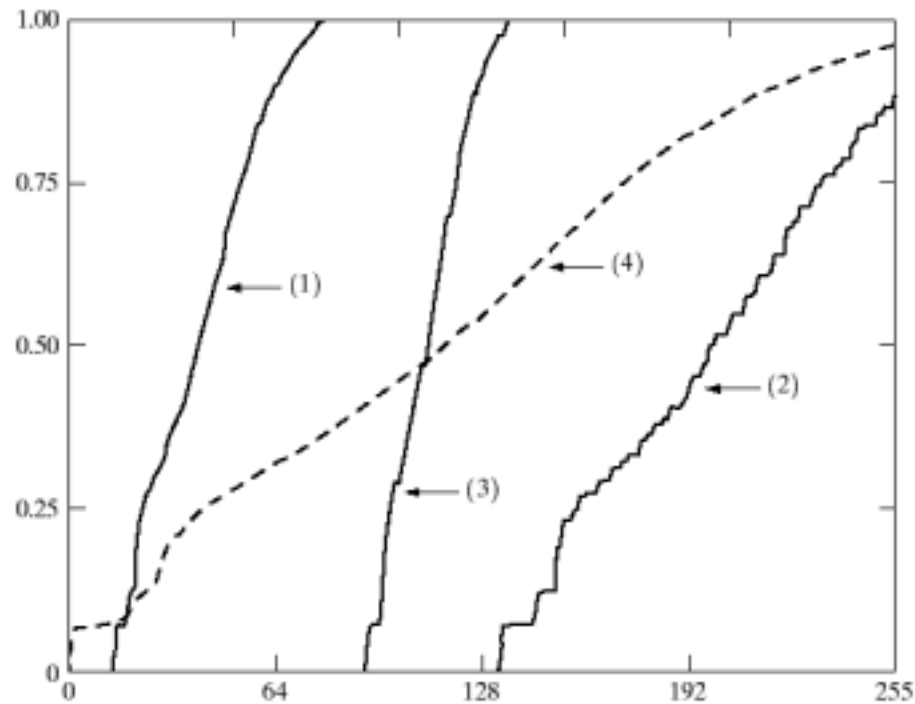


a b c

FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

# Fig 3.18: Transformation curves

**FIGURE 3.18**  
Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).



# Equalization: Discrete Case

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$$p_r(r_k) = \frac{n_k}{n} \quad 0 \leq r_k \leq 1 \quad ; \quad k = 0, 1, \dots, L-1$$

$L \rightarrow$  Number of levels

$$s_k = T(r_k) = \sum_{j=0}^k p_r(r_j) = \sum_{j=0}^k \frac{n_j}{n}$$

# Discrete Case: Example

**64x64 image;  
8 gray levels.**

**Notice that  
equalized  
histogram is  
not  
perfectly flat!**

$k$	$r_k$	$n_k$	$n_k/n$	$S_k = \sum_{j=0}^k n_j/n$	$p_s(s_k)$
0	0	790	0.19	0.19 $\rightarrow 1/7 \rightarrow s_0$	0.19
1	$1/7$	1023	0.25	0.44 $\rightarrow 3/7 \rightarrow s_1$	0.25
2	$2/7$	850	0.21	0.65 $\rightarrow 5/7 \rightarrow s_2$	0.21
3	$3/7$	656	0.16	0.81 $\rightarrow 6/7 \rightarrow s_3$	0.24
4	$4/7$	329	0.08	0.89 $\rightarrow 6/7 \rightarrow s_3$	0.24
5	$5/7$	245	0.06	0.95 $\rightarrow 1 \rightarrow s_4$	0.11
6	$6/7$	122	0.03	0.98 $\rightarrow 1 \rightarrow s_4$	0.11
7	$7/7$	81	0.02	1.0 $\rightarrow 1 \rightarrow s_4$	0.11

# Equalization: Image Examples

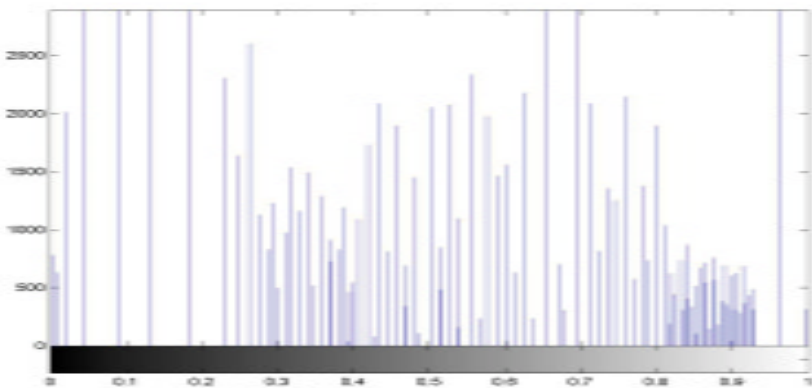
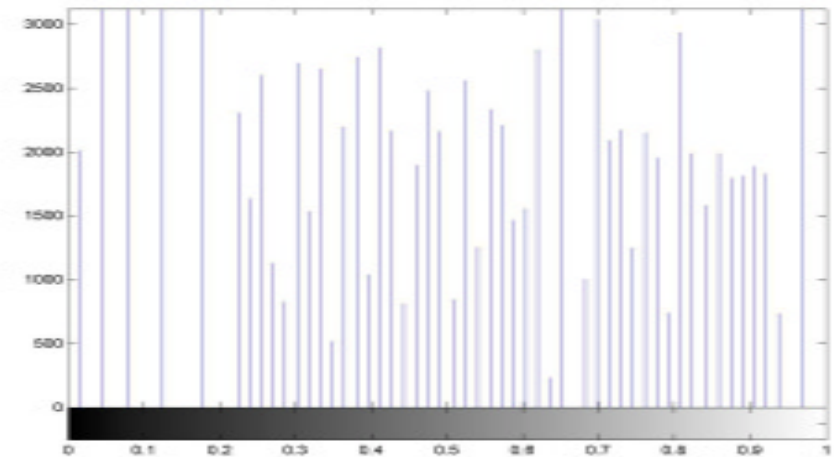
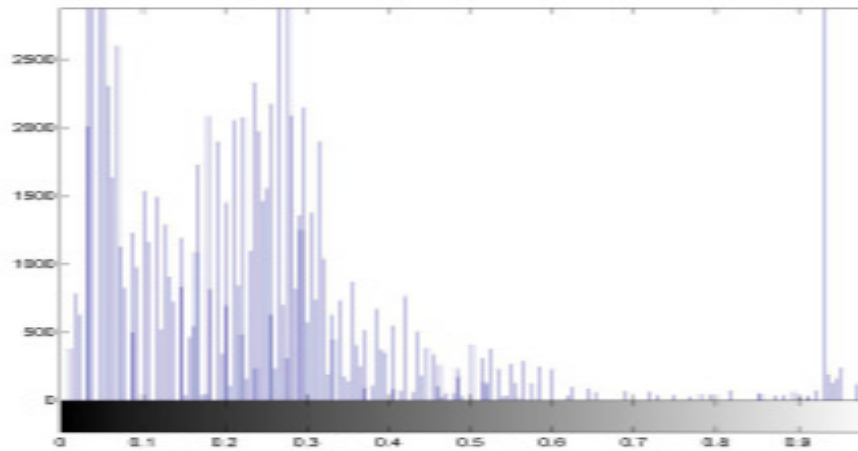
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Original, Equalized (64)  
Equalized (256)



# ..and their histograms



Original, equalized (64 levels)  
Equalized (256 levels)

# Histogram specification

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Suppose  $s = T(r) = \int_0^r p_r(w) dw$

$p_r(r) \rightarrow$  Original histogram ;  $p_z(z) \rightarrow$  Desired histogram

Let  $v = G(z) = \int_0^z p_z(w) dw$  and  $z = G^{-1}(v)$

But  $s$  and  $v$  are identical p.d.f.

$\therefore z = G^{-1}(v) = G^{-1}(s) = G^{-1}(T(r))$

# Question

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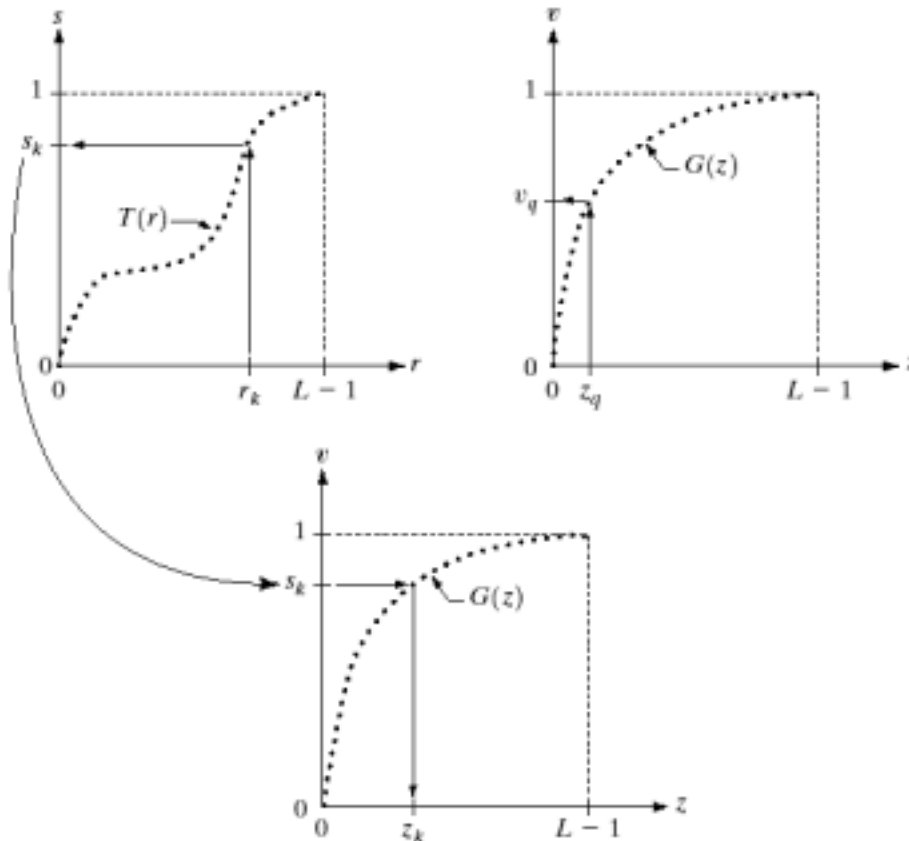
- What happens when you apply equalization to an already equalized histogram
  - In the continuous case?
  - In the discrete case?

# Fig 3.19: Matching

a b  
c

**FIGURE 3.19**

(a) Graphical interpretation of mapping from  $r_k$  to  $s_k$  via  $T(r)$ .  
(b) Mapping of  $z_q$  to its corresponding value  $v_q$  via  $G(z)$ .  
(c) Inverse mapping from  $s_k$  to its corresponding value of  $z_k$ .



# Matching: Summary

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Steps:

- (1) Equalize the levels of original image
- (2) Specify the desired  $p_z(z)$  and obtain  $G(z)$
- (3) Apply  $z=G^{-1}(s)$  to the levels  $s$  obtained in step 1

# Matching: an example

$z_k$	$p_z(z_k)$	$v_k = G(r_k)$	$n_k$	$p_z(z_k)$
$z_0 = 0$	0	0	0	0
$z_1 = 1/7$	0	0	0	0
$z_2 = 2/7$	0	0	0	0
$z_3 = 3/7$	0.15	$0.15 \leftrightarrow s_0 = 1/7$	790	0.19
$z_4 = 4/7$	0.2	$0.35 \leftrightarrow s_1 = 3/7$	1023	0.25
$z_5 = 5/7$	0.3	$0.65 \leftrightarrow s_2 = 5/7$	850	0.21
$z_6 = 6/7$	0.2	$0.85 \leftrightarrow s_3 = 6/7$	985	0.24
$z_7 = 1$	0.15	$1.0 \leftrightarrow s_4 = 1$	448	0.11

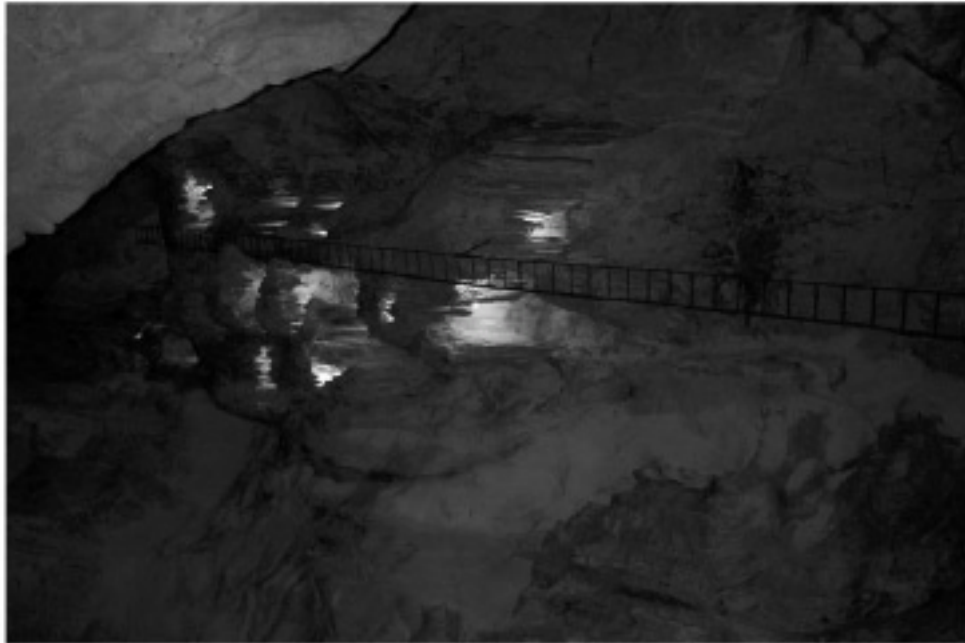
# Histogram Matching: example

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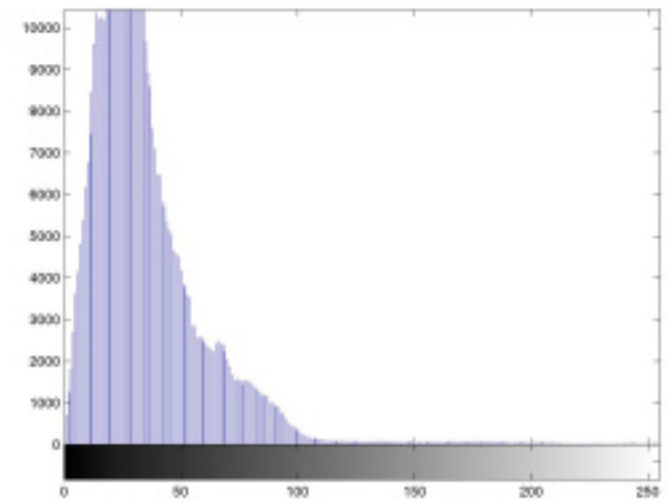


Original image  
(Jenolan caves,  
blue mountain,  
Sydney, Australia)

## Color to grayscale

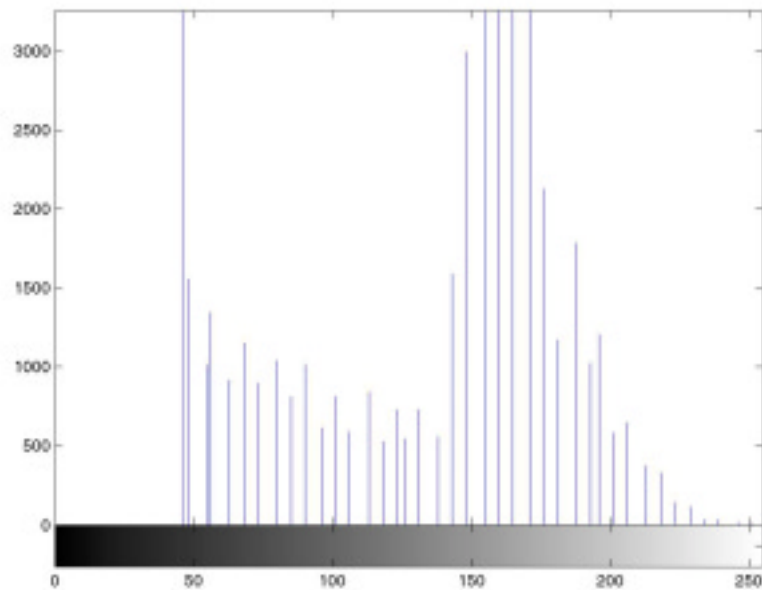


```
I=imread('sydney1.jpg');  
I1=rgb2gray(I);  
I1=imresize(I1,0.5);  
Imhist(I1);
```



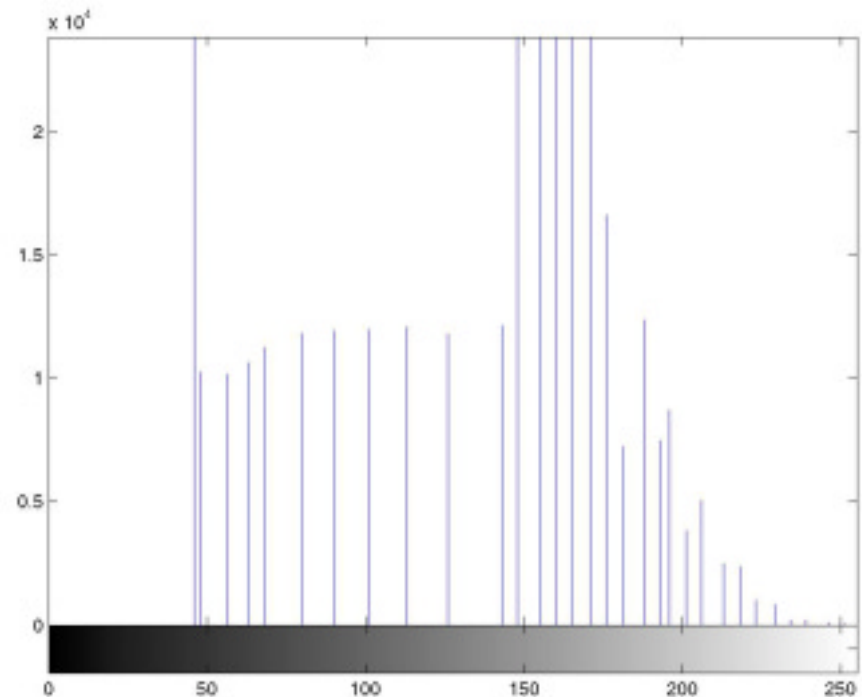


# Desired & modified histograms



Imhist(J)  
J=some image

```
I2=histeq(I1,imhist(J));  
Imhist(I2);
```

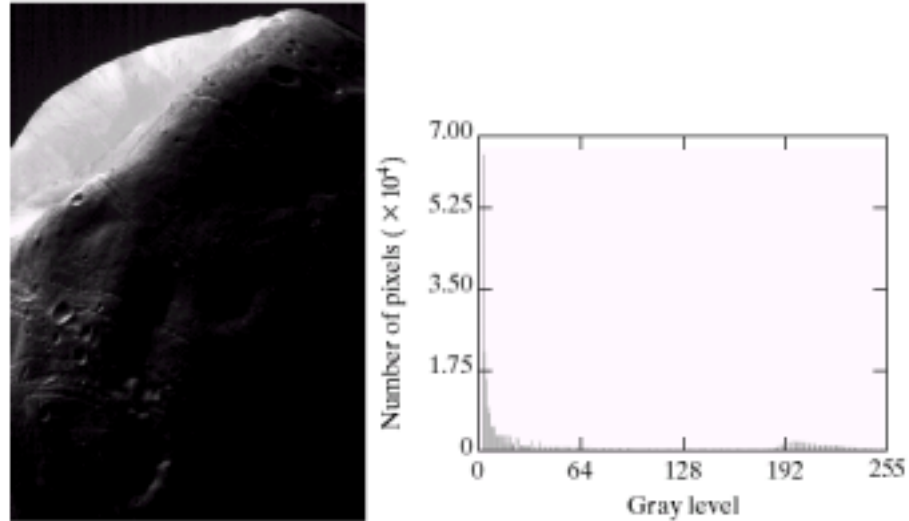


# Histogram modified image

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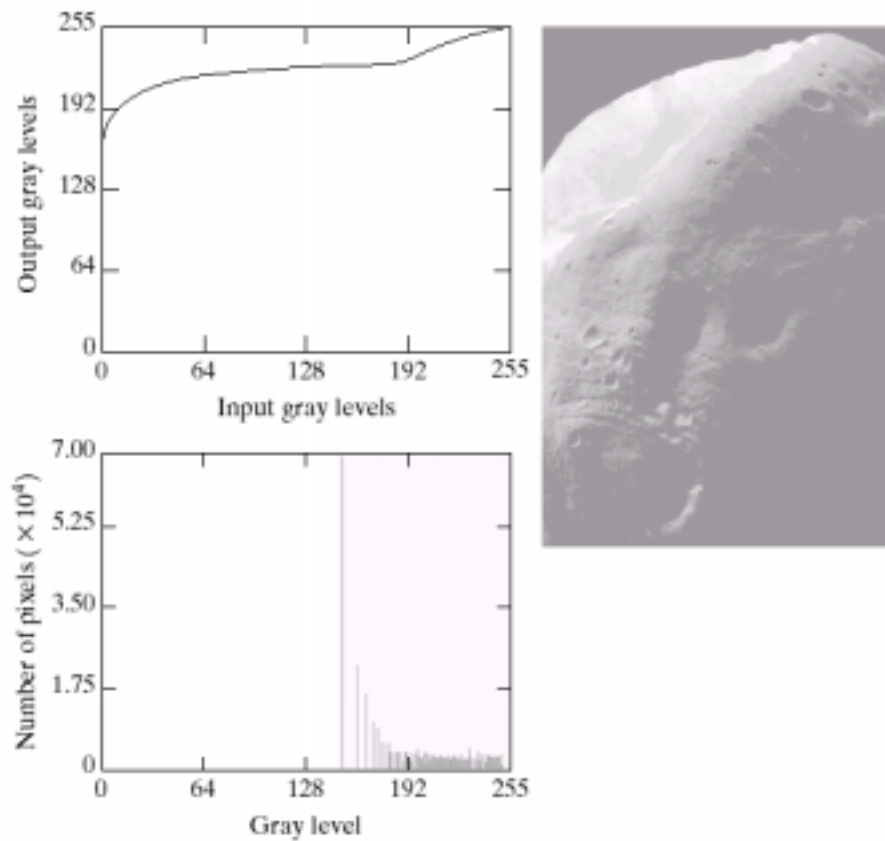
# Fig 3.20: Another example



a b

**FIGURE 3.20** (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

# Fig 3.21



a b  
c

**FIGURE 3.21**  
(a) Transformation function for histogram equalization.  
(b) Histogram-equalized image (note the washed-out appearance).  
(c) Histogram of (b).

# Fig 3.22

a c  
b  
d

**FIGURE 3.22**  
(a) Specified histogram.  
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).  
(c) Enhanced image using mappings from curve (2).  
(d) Histogram of (c).

