IMAGE COMPRESSION-II

Week IX

IMAGE COMPRESSION

- Data redundancy
- Self-information and Entropy
- Error-free and lossy compression
- Huffman coding
- Predictive coding
- Transform coding

Data Redundancy

- **CODING**: Fewer bits to represent frequent symbols.
- **INTERPIXEL / INTERFRAME**: Neighboring pixels have similar values.
- **PSYCHOVISUAL**: Human visual system can not simultaneously distinguish all colors.

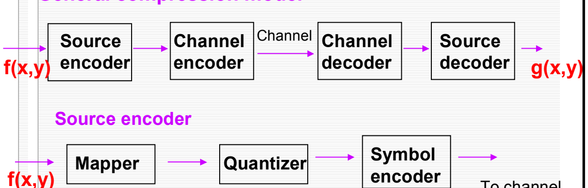
Coding Redundancy (contd.)

- Consider equation (A): It makes sense to assign fewer bits to those r_k for which $p_r(r_k)$ are large in order to reduce the sum.
- this achieves data compression and results in a variable length code.
- More probable gray levels will have fewer # of bits.

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)\, p_r(r_k) \quad \rightarrow (A)$$

General Model

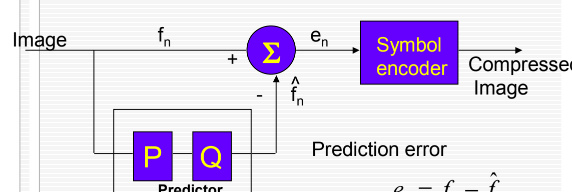
General compression model



Predictive coding

To reduce / eliminate interpixel redundancies

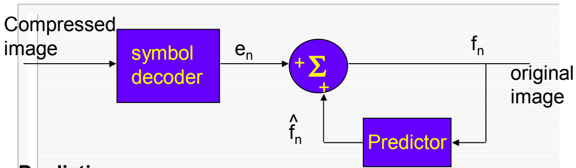
Lossless predictive coding: ENCODER



Prediction error

$$e_n = f_n - \hat{f}_n$$

Decoder



Prediction error:

$e_n = f_n - \hat{f}_n$

e_n is coded using a variable length code (symbol encoder)

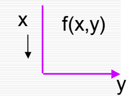
$f_n = e_n + \hat{f}_n$

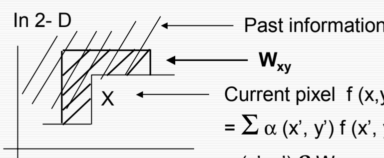
Example

Example 1: $\hat{f}_n = \text{Int}\left(\sum_{i=1}^{m} \alpha_i f_{n-i}\right)$

→ Linear predictor ; m = order of predictor

Example 2: $\hat{f}_n(x,y) = \text{Int}\left(\sum_{i=1}^{m} \alpha_i f(x, y-i)\right)$



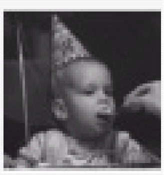


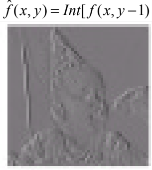
$= \sum \alpha(x', y') f(x', y')$

$(x', y') \in W_{xy}$

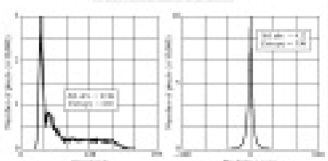
An example of first order predictor

$\hat{f}(x,y) = Int[f(x, y-1)]$





Prediction error image

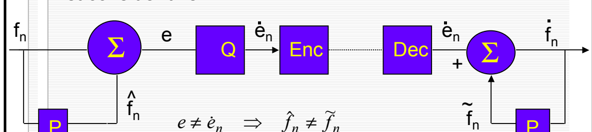


Histograms of original and error images.

Lossy Compression (Section 8.5)

Lossy compression: uses a quantizer to compress further the number of bits required to encode the 'error'.

First consider this:



$e \neq \dot{e}_n \Rightarrow \hat{f}_n \neq \tilde{f}_n$

Notice that, unlike in the case of loss-less prediction, in lossy prediction the predictors P "see" different inputs at the encoder and decoder

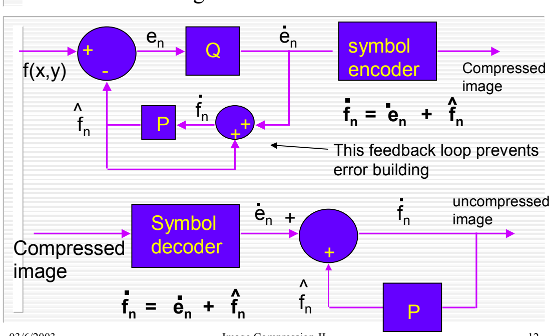
Quantization error

This results in a gradual buildup of error which is due to the quantization error at the encoder site.

In order to minimize this buildup of error due to quantization we should ensure that 'Ps' have the same input in both the cases.

f_n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\hat{f}_n = f_{n-1}$															
$e_n =$	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\dot{e}_n =$	0	2	2	2	2 .	.	.	.	.						
$\dot{f}_n =$	0	2	4	6	8	10	.	.	.	.	.	.			

Predictive Coding With Feedback



Example

Example:

$$\hat{f}_n = \alpha \dot{f}_{n-1}$$

and $\dot{e}_n = \begin{cases} +\xi & e_n > 0 \\ -\xi & e_n < 0 \end{cases}$

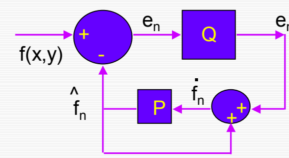
$0 < \alpha < 1$
prediction coefficient

$$\dot{f}_n = \dot{e}_n + \hat{f}_n = \dot{e}_n + \alpha \dot{f}_{n-1}$$

Example

with feedback

f_n =	0	1	2	3	4
e_n =	_	1	2	1	2
$\dot{e}_n$ =	_	0	2	0	2
$\dot{f}_n$ =	0	0	2	2	4
$\hat{f}_n$ =	_	0	0	2	2



Note: The quantizer used here is-- floor $(e_n/2)*2$. This is different from the one used in the earlier example. Note that this would result in a worse response if used without Feedback (output will be flat at "0").

Another example

{14, 15, 14, 15, 13, 15, 15, 14, 20, 26, 27, 28, 27, 27, 29, 37, 37, 62, 75, 77, 78, 79, 80, 81, 81, 82, 82}

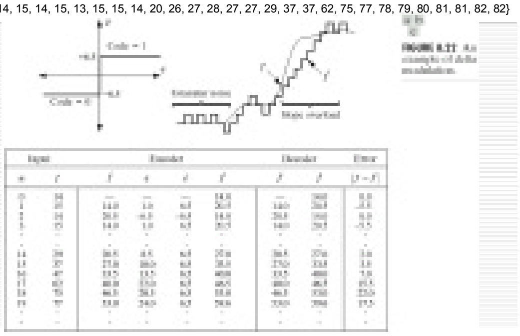


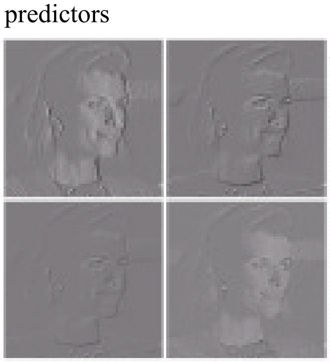
FIGURE 8.22 An example of delta modulation.

A comparison (Fig 8.23)

FIGURE 8.23 A 512 × 512 8-bit monochrome image.

Four linear predictors

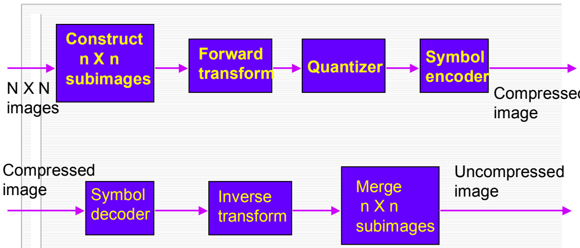
FIGURE 8.24 A comparison of four linear prediction techniques.



Transform Coding

Feb 28, 2002

Transform coding



Blocking artifact: boundaries between subimages become visible

Transform Selection

- DFT
- Discrete Cosine Transform (DCT)
- Wavelet transform
- Karhunen-Loeve Transform (KLT)
- ….

Fig 8.31

DFT, Hadamard and DCT

Discrete Cosine Transform

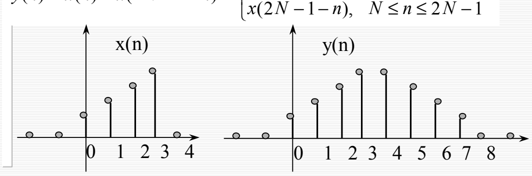
Ahmed, Natarajan, and Rao, IEEE T-Computers, pp. 90-93, 1974.

1-D Case: Extended 2N Point Sequence

Consider 1-D first; Let x(n) be a N point sequence $0 \le n \le N-1$.

$$x(n) \leftrightarrow \underset{y(n)}{\text{2-N point}} \overset{DFT}{\leftrightarrow} \underset{Y(u)}{2-N\text{ point}} \leftrightarrow \underset{C(u)}{N-\text{point}}$$

$$y(n) = x(n) + x(2N-1-n) = \begin{cases} x(n), & 0 \le n \le N-1 \\ x(2N-1-n), & N \le n \le 2N-1 \end{cases}$$



DCT & DFT

$$\begin{aligned}
Y(u) &= \sum_{n=0}^{2N-1} y(n)\exp\left(-j\frac{2\pi}{2N}un\right) \\
&= \sum_{n=0}^{N-1} x(n)\exp\left(-j\frac{2\pi}{2N}un\right) + \sum_{n=N}^{2N-1} x(2N-1-n)\exp\left(-j\frac{2\pi}{2N}un\right) \\
&= \sum_{n=0}^{N-1} x(n)\exp\left(-j\frac{2\pi}{2N}un\right) + \sum_{m=0}^{N-1} x(m)\exp\left(-j\frac{2\pi}{2N}u(2N-1-m)\right) \\
&= \exp\left(j\frac{\pi}{2N}u\right)\sum_{n=0}^{N-1} x(n)\exp\left(-j\frac{\pi}{2N}u - j\frac{2\pi}{2N}un\right) \\
&\quad + \exp\left(j\frac{\pi}{2N}u\right)\sum_{n=0}^{N-1} x(n)\exp\left(j\frac{\pi}{2N}u + j\frac{2\pi}{2N}un\right) \\
&= \exp\left(j\frac{\pi}{2N}u\right)\sum_{n=0}^{N-1} 2x(n)\cos\left(\frac{\pi}{2N}u(2n+1)\right).
\end{aligned}$$

DCT

The N - point DCTof x(n), C(u), is given by

$$C(u) = \begin{cases} \exp\left(-j\frac{\pi}{2N}u\right)Y(u), & 0 \le u \le N-1 \\ 0 & \text{otherwise.} \end{cases}$$

The unitary DCT transformations are:

$$F(u) = \alpha(u)\sum_{n=0}^{N-1} f(n)\cos\left(\frac{\pi}{2N}(2n+1)u\right), \quad 0 \le u \le N-1, \text{ where}$$

$$\alpha(0) = \frac{1}{\sqrt{N}}, \quad \alpha(u) = \sqrt{\frac{2}{N}} \text{ for } 1 \le k \le N-1. \text{ The inverse transformation is}$$

$$f(n) = \sum_{n=0}^{N-1} \alpha(u)F(u)\cos\left(\frac{\pi}{2N}(2n+1)u\right), \quad 0 \le u \le N-1.$$

Discrete Cosine Transform—in 2-D

$$C(u,v) = \alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\tfrac{(2x+1)u\pi}{2N}\right]\cos\left[\tfrac{(2y+1)v\pi}{2N}\right]$$

for $u, v = 0,1,2,\ldots,N-1$, where

$$\alpha(u) = \begin{cases} \sqrt{\tfrac{1}{N}} & \text{for } u = 0 \\ \sqrt{\tfrac{2}{N}} & \text{for } u = 1,2,..,N-1. \end{cases} \quad \text{and}$$

$$f(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \alpha(u)\alpha(v)\,C(u,v)\cos\left[\tfrac{(2x+1)u\pi}{2N}\right]\cos\left[\tfrac{(2y+1)v\pi}{2N}\right]$$

for $x, y = 0,1,2,\ldots,N-1$

DCT Basis functions

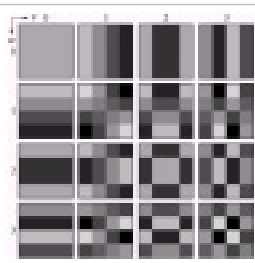


FIGURE 8.30 Discrete-cosine basis functions for N = 4. The origin of each block is at its top left.

Implicit Periodicity-DFT vs DCT (Fig 8.32)

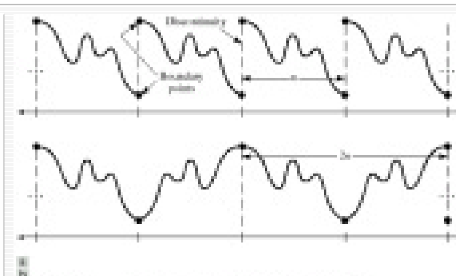


FIGURE 8.32 The periodicity implicit in the 1-D (a) DFT and (b) DCT.

Why DCT?

- Blocking artifacts less pronounced in DCT than in DFT.
- Good approximation to the Karhunen-Loeve Transform (KLT) but with basis vectors fixed.
- DCT is used in JPEG image compression standard.

Karhunen-Loeve Transform

- READ pp. 476 and Section 11.4 Text (if you are working on the face recognition project, you must!)
- Also called the Hotelling Transform.
- Transform is data dependent. Let ***X*** denote the random data, and let C be the covariance matrix of X, i.e.

$\boldsymbol{C} = E\{(\boldsymbol{X}-\boldsymbol{m})(\boldsymbol{X}-\boldsymbol{m})^T\}$

where ***m*** is the mean vector of the data.

The matrix *C* is real and symmetric, and hence can be diagonalized using its eigenvectors.

What are eigenvectors?

Let $C = \mathrm{E}[(x-m_x)(x-m_x)^T]$
where C is the $N \times N$ covariance matrix, x is an $N-$dimensional vector, and m_x is the mean vector of the samples.
The eigenvectors e_i of C are given by
$Ce_i = \lambda_i e_i$
where λ_i are the corresponding eigenvalues.
Now consider a matrix A whose columns correspond to the eigenvectors of C, arranged such that the first column corresponds to the eigenvector with the largest eigenvalue, and second with the second largest and so on.
Then, consider a transformation on the vectors x such that
$y = A^T(x-m_x)$. Note that y is zero mean, and its covariance matrix is
$C_y = \mathrm{E}(yy^T) = \mathrm{E}[A^T(x-m_x)(x-m_x)^T A] = A^T C A = \Lambda$
$=$ diagnoal matrix of eigenvalues, in decreasing order.
Note that the elements of the transformed vector are uncorrelated (non-diagnoal elements are zero).

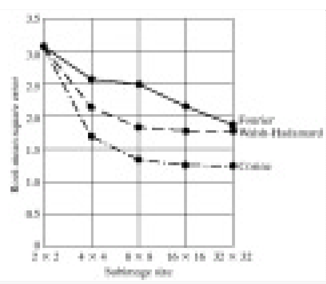
What is KLT

- The transformation from X to Y is called the KLT. In computing the transformation matrix A, we assume that the columns are made up of orthonormal eigenvectors (i.e., the inverse of A is also its transpose.)
- Thus the basis vectors of the KLT are the orthonormal eigenvectors of the covriance matrix C.
- The KLT yields uncorrelated coefficients.
- For compression, we only keep the top K coefficients corresponding to the K largest eigenvectors.
- Then the mean squared error in reconstruction is given by

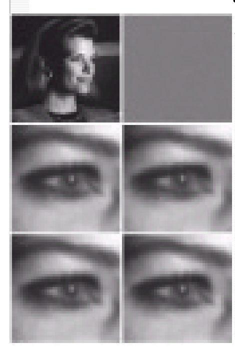
$$MSE = \sum_{j=K+1}^{N} \lambda_j$$

Sub-image size selection

FIGURE 8.23 Reconstruction error versus subimage size.



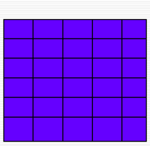
Different sub-image sizes



Bit Allocation/Threshold Coding

- # of coefficients to keep
- How to quantize them
  - Threshold coding
  - Zonal coding

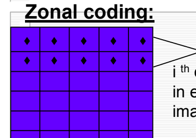
Threshold coding



For each subimage i
- Arrange the transform coefficients in decreasing order of magnitude
- Keep only the top X% of the coefficients and discard rest.
- Code the retained coefficient using variable length code.

Zonal Coding

Zonal coding:



i th coefficient in each sub image

(1) Compute the variance of each of the transform coeff; use the subimages to compute this.
(2) Keep X% of their coeff. which have maximum variance.
(3) Variable length coding (proportional to variance)

Bit allocation: In general, let the number of bits allocated be made proportional to the variance of the coefficients. Suppose the total number of bits per block is B. Let the number of retained coefficients be M. Let v(i) be variance of the i-th coefficient. Then

$$b(i) = \frac{B}{M} + \frac{1}{2}\log_2 v(i) - \frac{1}{2M}\sum_{i=1}^{M}\log_2 v(i)$$

Zonal Mask & bit allocation: an example

1	1	1	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8	7	6	4	3	0	0	0
7	6	5	4	0	0	0	0
6	5	4	0	0	0	0	0
4	4	0	0	0	0	0	0
3	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Typical Masks (Fig 8.36)

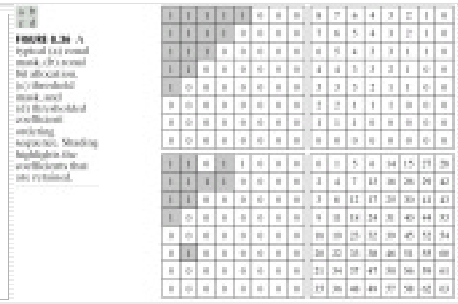


Image Approximations

The JPEG standard

- The following is mostly from Tekalp's book.
- Digital Video Processing by M. Tekalp (Prentice Hall).
- MPEG Video compression standard, edited by Mitchell, Pennebaker, Fogg and LeGall (Chapman and Hall).
- For the new JPEG-2000 check out the web site www.jpeg.org.

JPEG (contd.)

- JPEG is a lossy compression standard using DCT.
- Activities started in 1986 and the ISO in 1992.
- Four modes of operation: Sequential (basline), hierarchical, progressive, and lossless.
- Arbitrary image sizes; DCT mode 8-12 bits/sample. Luminance and chrominance channels are separately encoded.
- We will only discuss the baseline method.

JPEG-baseline.

- DCT: The image is divided into 8x8 blocks. Each pixel is level shifted by 2^{n-1} where 2^n is the maximum number of gray levels in the image. Thus for 8 bit images, you subtract 128. Then the 2-D DCT of each block is computed. For the baseline system, the input and output data precision is restricted to 8 bits and the DCT values are restricted to 11 bits.
- Quantization: the DCT coefficients are threshold coded using a quantization matrix, and then reordered using zig-zag scanning to form a 1-D sequence.
- The non-zero AC coefficients are Huffman coded. The DC coefficients of each block are DPCM coded relative to the DC coefficient of the previous block.

JPEG -color image

- RGB to Y-Cr-Cb space
  - Y = 0.3R + 0.6G + 0.1B
  - Cr = 0.5 (B-Y) + 0.5
  - Cb = (1/1.6) (R – Y) + 0.5
- Chrominance samples are sub-sampled by 2 in both directions.

Y1	Y2	Y3	Y4
Y5	Y6	Y7	Y8
Y9	Y10	Y11	Y12
Y13	Y14	Y15	Y16

Cr1	Cr2
Cr3	Cr4

Cb1	Cb2
Cb3	Cb4

Non-Interleaved
Scan 1:Y1,Y2,…,Y16
Scan 2: Cr1, Cr2, Cr3, Cr4
Scan 3: Cb1, Cb2, Cb3, Cb4

Interleaved: Y1, Y2, Y3, Y4, Cr1, Cb1,Y5,Y6,Y7,Y8,Cr2,Cb2,…

JPEG – quantization matrices

- Check out the matlab workspace (dctex.mat).
- Quantization table for the luminance channel.
- Quantization table for the chrominance channel.
- JPEG baseline method
  - Consider the 8x8 image (matlab: array s.)
  - Level shifted (s-128=sd).
  - 2d-DCT: dct2(sd)= dcts
  - After dividing by quantiization matrix qmat: dctshat.
  - Zigzag scan as in threshold coding.

  [20, 5, -3, -1, -2,-3, 1, 1, -1, -1, 0, 0, 1, 2, 3, -2, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, EOB].

An 8x8 sub-image (s)

s = (8x8block)

183	160	94	153	194	163	132	165
183	153	116	176	187	166	130	169
179	168	171	182	179	170	131	167
177	177	179	177	179	165	131	167
178	178	179	176	182	164	130	171
179	180	180	179	183	164	130	171
179	179	180	182	183	170	129	173
180	179	181	179	181	170	130	169

sd =(level shifted)

55	32	-34	25	66	35	4	37
55	25	-12	48	59	38	2	41
51	40	43	54	51	42	3	39
49	49	51	49	51	37	3	39
50	50	51	48	54	36	2	43
51	52	52	51	55	36	2	43
51	51	52	54	55	42	1	45
52	51	53	51	53	42	2	41

2D DCT (dcts) and the quantization matrix (qmat)

dcts=

312	56	-27	17	79	-60	26	-26
-38	-28	13	45	31	-1	-24	-10
-20	-18	10	33	21	-6	-16	-9
-11	-7	9	15	10	-11	-13	1
-6	1	6	5	-4	-7	-5	5
3	3	0	-2	-7	-4	1	2
3	5	0	-4	-8	-1	2	4
3	1	-1	-2	-3	-1	4	1

qmat=

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Division by qmat (dctshat)=dcts/qmat

dcthat=

20	5	-3	1	3	-2	1	0
-3	-2	1	2	1	0	0	0
-1	-1	1	1	1	0	0	0
-1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

dcts=

312	56	-27	17	79	-60	26	-26
-38	-28	13	45	31	-1	-24	-10
-20	-18	10	33	21	-6	-16	-9
-11	-7	9	15	10	-11	-13	1
-6	1	6	5	-4	-7	-5	5
3	3	0	-2	-7	-4	1	2
3	5	0	-4	-8	-1	2	4
3	1	-1	-2	-3	-1	4	1

Zig-zag scan of dcthat

dcthat=

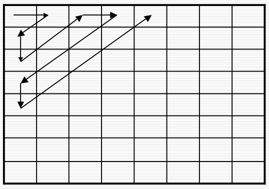
20	5	-3	1	3	-2	1	0
-3	-2	1	2	1	0	0	0
-1	-1	1	1	1	0	0	0
-1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Zigzag scan as in threshold coding.

[20, 5, -3, -1, -2,-3, 1, 1, -1, -1, 0, 0, 1, 2, 3, -2, 1, 1, 0, 0, 0, 0, 0,1, 1, 0, 1, EOB].

Threshold coding -revisited

Zig-zag scanning of the coefficients.



The coefficients along the zig-zag scan lines are mapped into [run,level] where the *level* is the value of non-zero coefficient, and *run* is the number of zero coeff. preceding it. The DC coefficients are usually coded separately from the rest.

JPEG – baseline method example

Zigzag scan as in threshold coding.
[20, 5, -3, -1, -2,-3, 1, 1, -1, -1, 0, 0, 1, 2, 3, -2, 1, 1, 0, 0, 0, 0, 0, 0,1, 1, 0, 1, EOB].

- The DC coefficient is DPCM coded (difference between the DC coefficient of the previous block and the current block.)
- The AC coef. are mapped to run-level pairs.
  (0,5), (0,-3), (0, -1), (0,-2),(0,-3),(0,1),(0,1),(0,-1),(0,-1), (2,1), (0,2), (0,3), (0,-2),(0,1),(0,1),(6,1),(0,1),(1,1),EOB.
- These are then Huffman coded (codes are specified in the JPEG scheme.)
- The decoder follows an inverse sequence of operations. The received coefficients are first multiplied by the same quantization matrix.
  (recddctshat=dctshat.*qmat.)
- Compute the inverse 2-D dct. (recdsd=idct2(recddctshat); add 128 back. (recds=recdsd+128.)

Decoder

Recddcthat=dcthat*qmat

320	55	-30	16	72	-80	51	0
-36	-24	14	38	26	0	0	0
-14	-13	16	24	40	0	0	0
-14	0	0	29	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Recdsd=

67	12	-9	20	69	43	-8	42
58	25	15	30	65	40	-4	47
46	41	44	40	59	38	0	49
41	52	59	43	57	42	3	42
44	54	58	40	58	47	3	33
49	52	53	40	61	47	1	33
53	50	53	46	63	41	0	45
55	50	56	53	64	34	-1	57

Received signal

Reconstructed S=

195	140	119	148	197	171	120	170
186	153	143	158	193	168	124	175
174	169	172	168	187	166	128	177
169	180	187	171	185	170	131	170
172	182	186	168	186	175	131	161
177	180	181	168	189	175	129	161
181	178	181	174	191	169	128	173
183	178	184	181	192	162	127	185

s = (8x8block)

183	160	94	153	194	163	132	165
183	153	116	176	187	166	130	169
179	168	171	182	179	170	131	167
177	177	179	177	179	165	131	167
178	178	179	176	182	164	130	171
179	180	180	179	183	164	130	171
179	179	180	182	183	170	129	173
180	179	181	179	181	170	130	169

Example

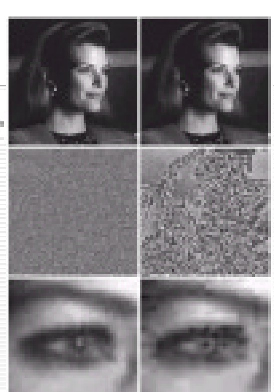


Image Compression: Summary

- Data redundancy
- Self-information and Entropy
- Error-free compression
  - Huffman coding, Arithmetic coding, LZW coding, Run-length encoding
  - Predictive coding
- Lossy coding techniques
  - Predictive coing (Lossy)
  - Transform coding
    - DCT, DFT, KLT, …
- JPEG image compression standard