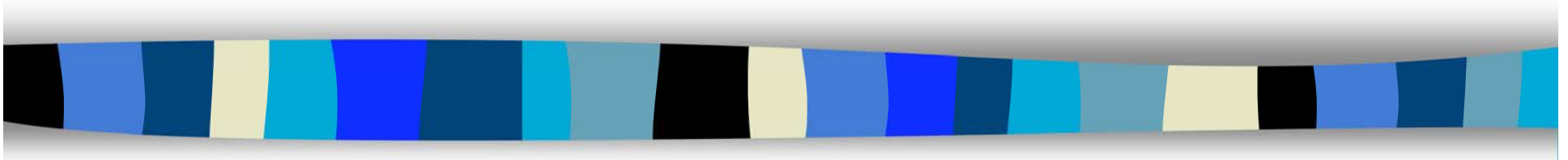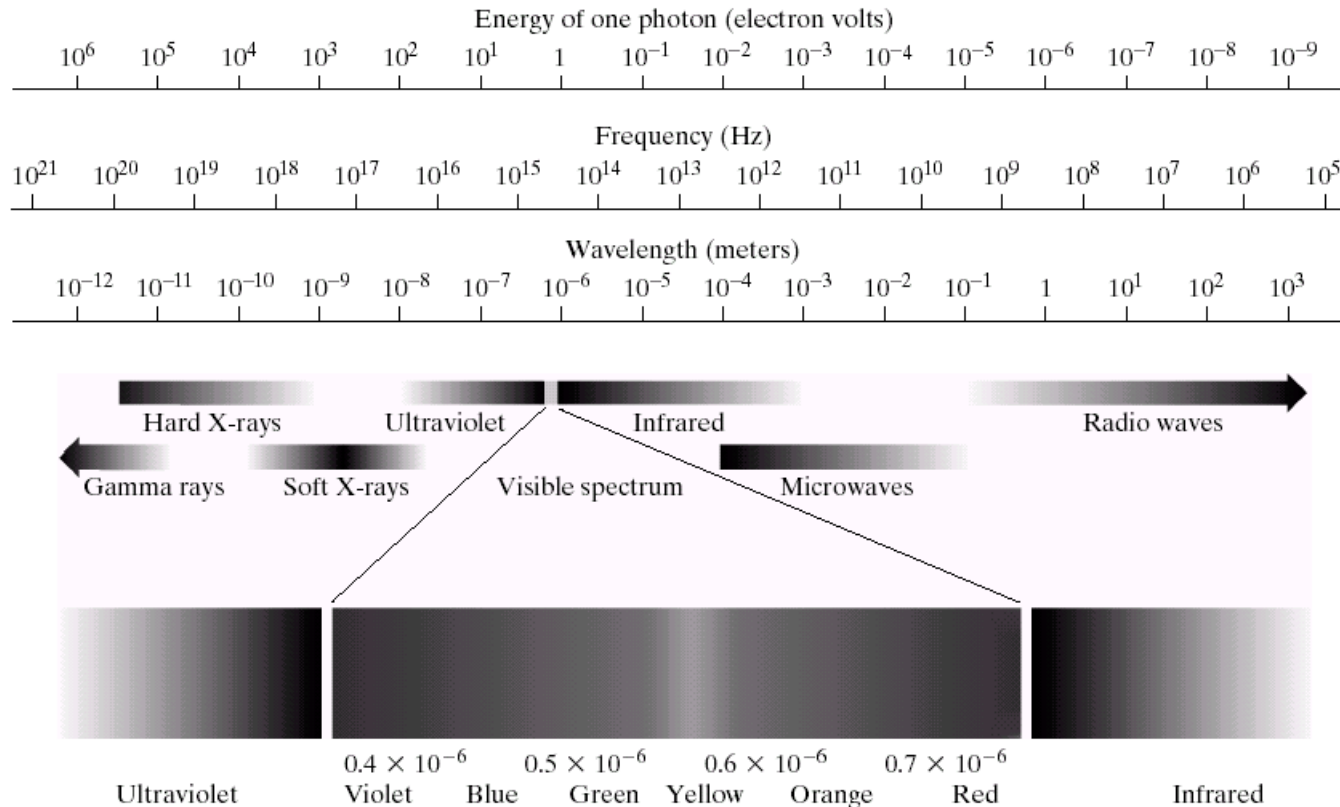# ECE 178: Introduction (contd.)

Lecture Notes #2: January 8, 2004

- Section 2.4 –sampling and quantization
- Section 2.5 –relationship between pixels, connectivity analysis

# Announcements (01/08/04)

- ■ 01/09/2004: Discussion sessions on Matlab
- ■ Note that the HW#1 due on Jan 16.
- ■ HW#2 will be due on Jan 23.
- ■ Today:
  - – A quick introduction to MATLAB
  - – Basic relationship between pixels (Section 2.5)
  - – Image sampling and quantization (Section 2.4, notes)
  - – Linear systems review (time permitting)

# Light and the EM Spectrum

Energy of one photon (electron volts)

| $10^6$ | $10^5$ | $10^4$ | $10^3$ | $10^2$ | $10^1$ | $1$ | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ | $10^{-7}$ | $10^{-8}$ | $10^{-9}$ |

Frequency (Hz)

| $10^{21}$ | $10^{20}$ | $10^{19}$ | $10^{18}$ | $10^{17}$ | $10^{16}$ | $10^{15}$ | $10^{14}$ | $10^{13}$ | $10^{12}$ | $10^{11}$ | $10^{10}$ | $10^9$ | $10^8$ | $10^7$ | $10^6$ | $10^5$ |

Wavelength (meters)

| $10^{-12}$ | $10^{-11}$ | $10^{-10}$ | $10^{-9}$ | $10^{-8}$ | $10^{-7}$ | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | $1$ | $10^1$ | $10^2$ | $10^3$ |

Hard X-rays    Ultraviolet    Infrared    Radio waves

Gamma rays    Soft X-rays    Visible spectrum    Microwaves

$0.4 \times 10^{-6}$    $0.5 \times 10^{-6}$    $0.6 \times 10^{-6}$    $0.7 \times 10^{-6}$

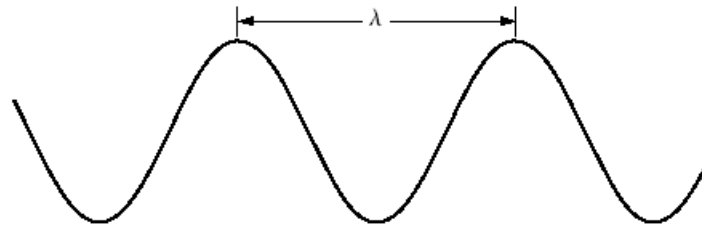Ultraviolet    Violet    Blue    Green    Yellow    Orange    Red    Infrared

**FIGURE 2.10** The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.
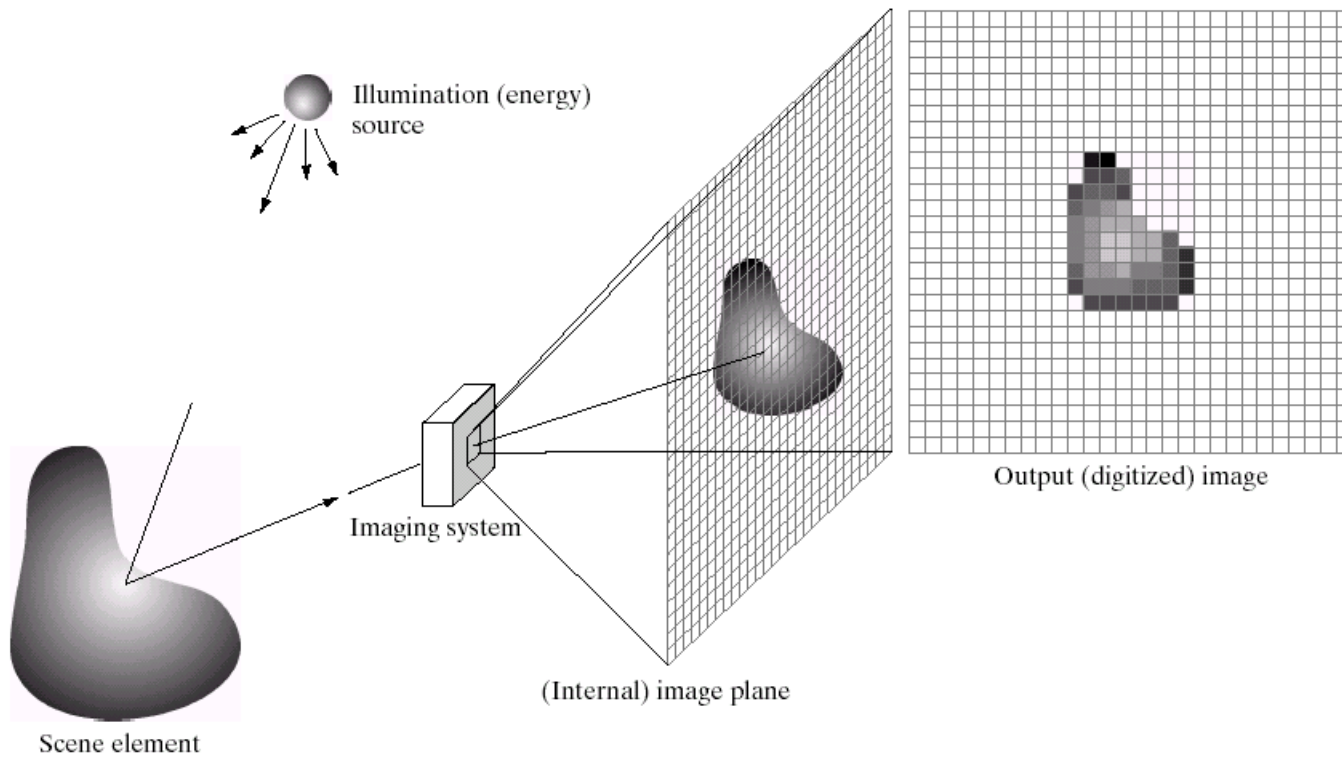
# Wavelength

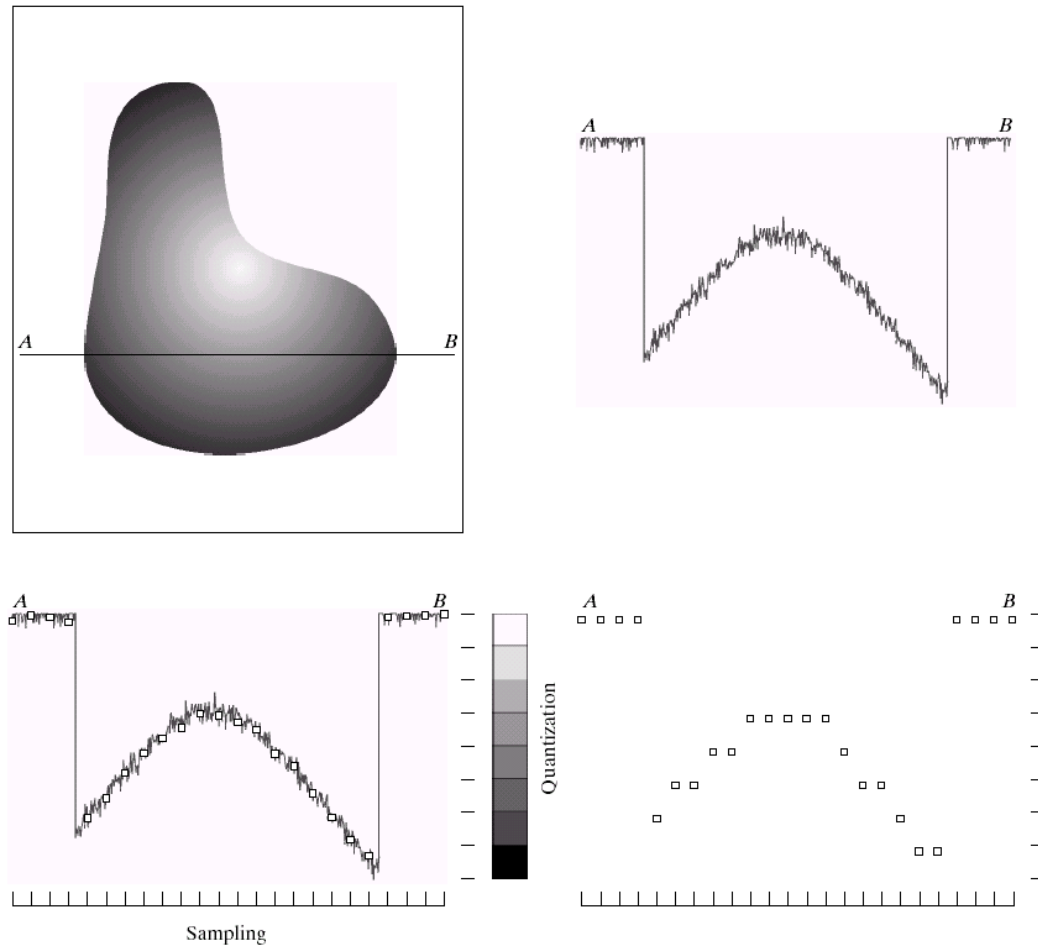**FIGURE 2.11**
Graphical
representation of
one wavelength.

# Digial Image Acquisition



Illumination (energy) source

Imaging system

(Internal) image plane

Output (digitized) image

Scene element

a
b  c  d  e

**FIGURE 2.15** An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.
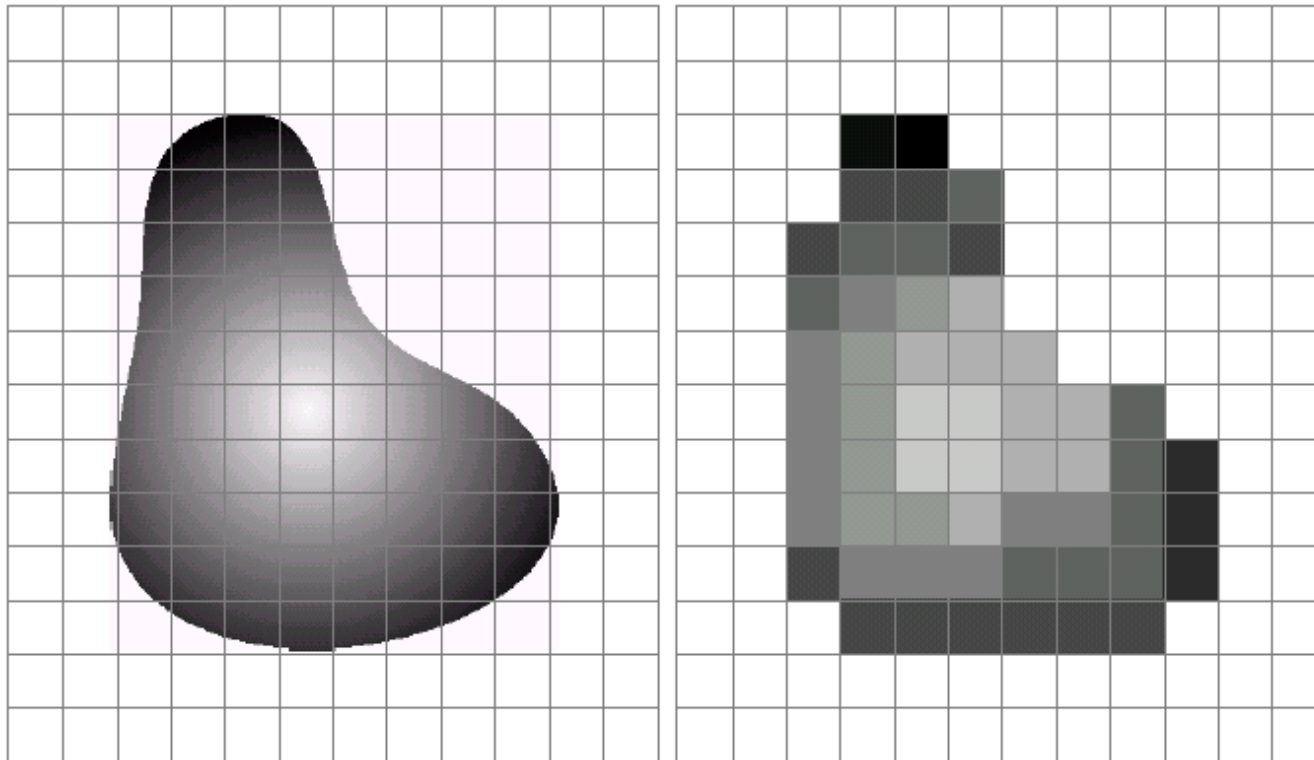
# Sampling and Quantization

a b
c d

**FIGURE 2.16** Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.
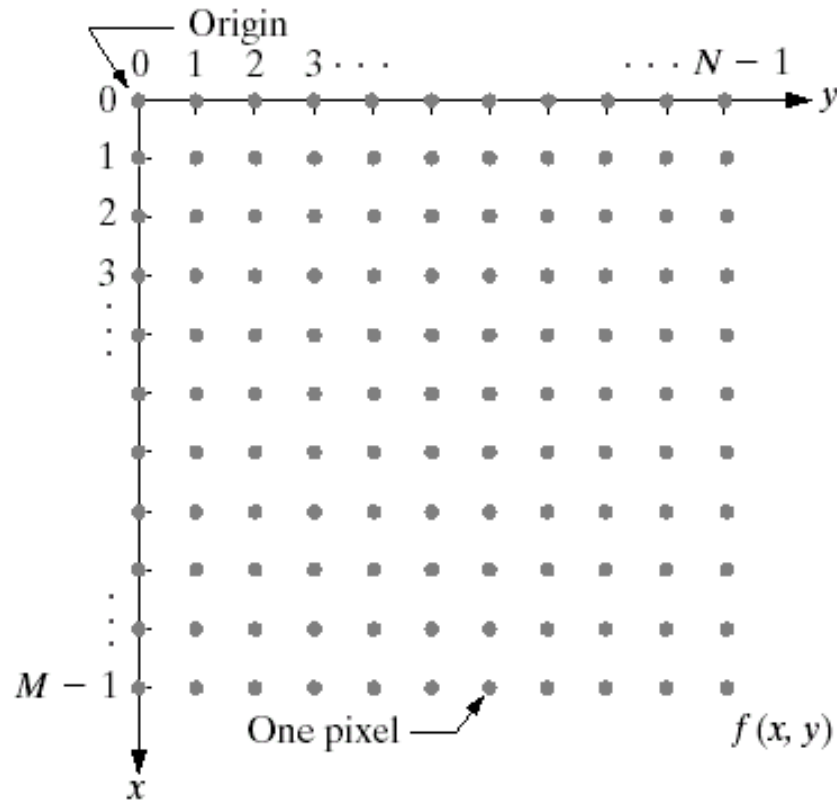
# Sampling & Quantization (contd.)



a b

**FIGURE 2.17** (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

# Digital Image: Representation



**FIGURE 2.18**
Coordinate convention used in this book to represent digital images.

# Storage Requirement

Image Dimension: NxN; k bits per pixel.

**TABLE 2.1**
Number of storage bits for various values of $N$ and $k$.

| $N/k$ | 1 ($L = 2$) | 2 ($L = 4$) | 3 ($L = 8$) | 4 ($L = 16$) | 5 ($L = 32$) | 6 ($L = 64$) | 7 ($L = 128$) | 8 ($L = 256$) |
|---|---|---|---|---|---|---|---|---|
| 32 | 1,024 | 2,048 | 3,072 | 4,096 | 5,120 | 6,144 | 7,168 | 8,192 |
| 64 | 4,096 | 8,192 | 12,288 | 16,384 | 20,480 | 24,576 | 28,672 | 32,768 |
| 128 | 16,384 | 32,768 | 49,152 | 65,536 | 81,920 | 98,304 | 114,688 | 131,072 |
| 256 | 65,536 | 131,072 | 196,608 | 262,144 | 327,680 | 393,216 | 458,752 | 524,288 |
| 512 | 262,144 | 524,288 | 786,432 | 1,048,576 | 1,310,720 | 1,572,864 | 1,835,008 | 2,097,152 |
| 1024 | 1,048,576 | 2,097,152 | 3,145,728 | 4,194,304 | 5,242,880 | 6,291,456 | 7,340,032 | 8,388,608 |
| 2048 | 4,194,304 | 8,388,608 | 12,582,912 | 16,777,216 | 20,971,520 | 25,165,824 | 29,369,128 | 33,554,432 |
| 4096 | 16,777,216 | 33,554,432 | 50,331,648 | 67,108,864 | 83,886,080 | 100,663,296 | 117,440,512 | 134,217,728 |
| 8192 | 67,108,864 | 134,217,728 | 201,326,592 | 268,435,456 | 335,544,320 | 402,653,184 | 469,762,048 | 536,870,912 |

# Spatial Resolution
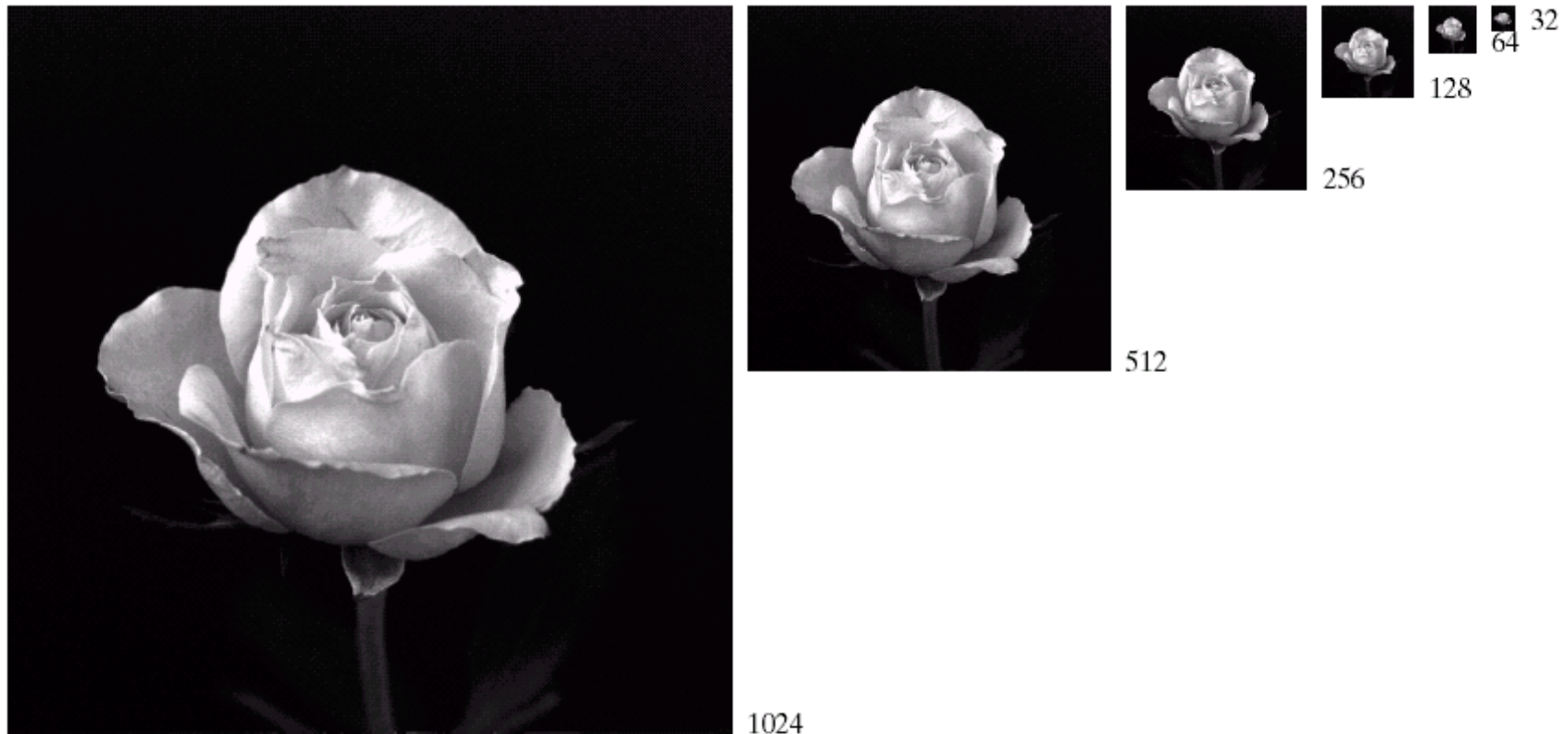


**FIGURE 2.19** A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

# Re-sampling...
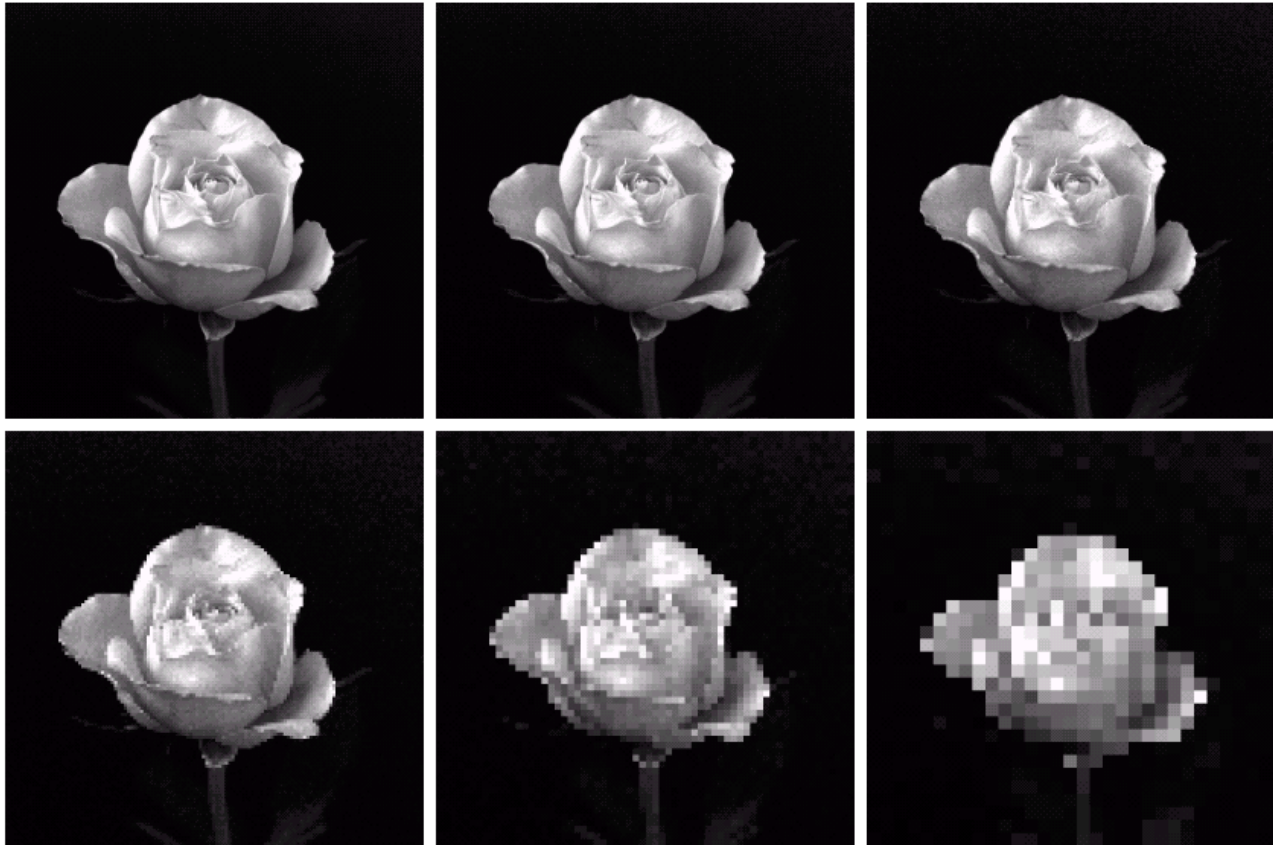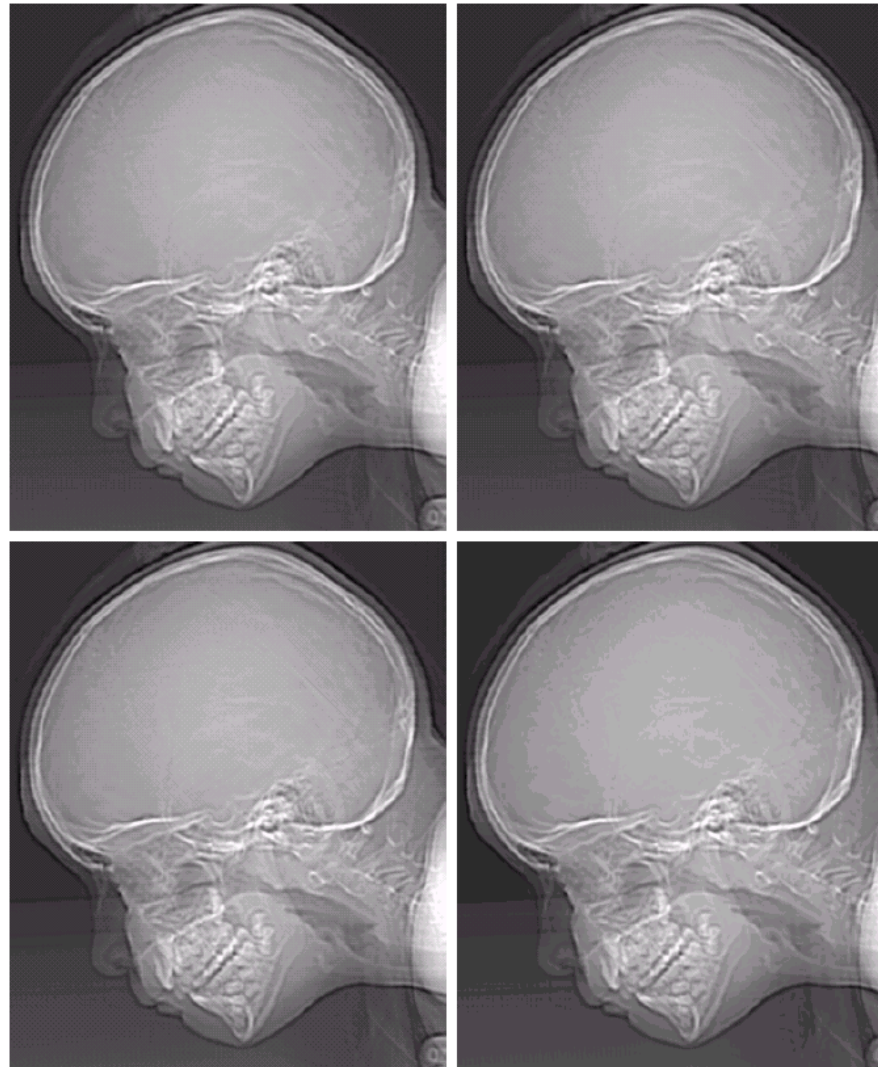


a b c
d e f

**FIGURE 2.20** (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

# Quantization: Gray-scale resolution



a b
c d

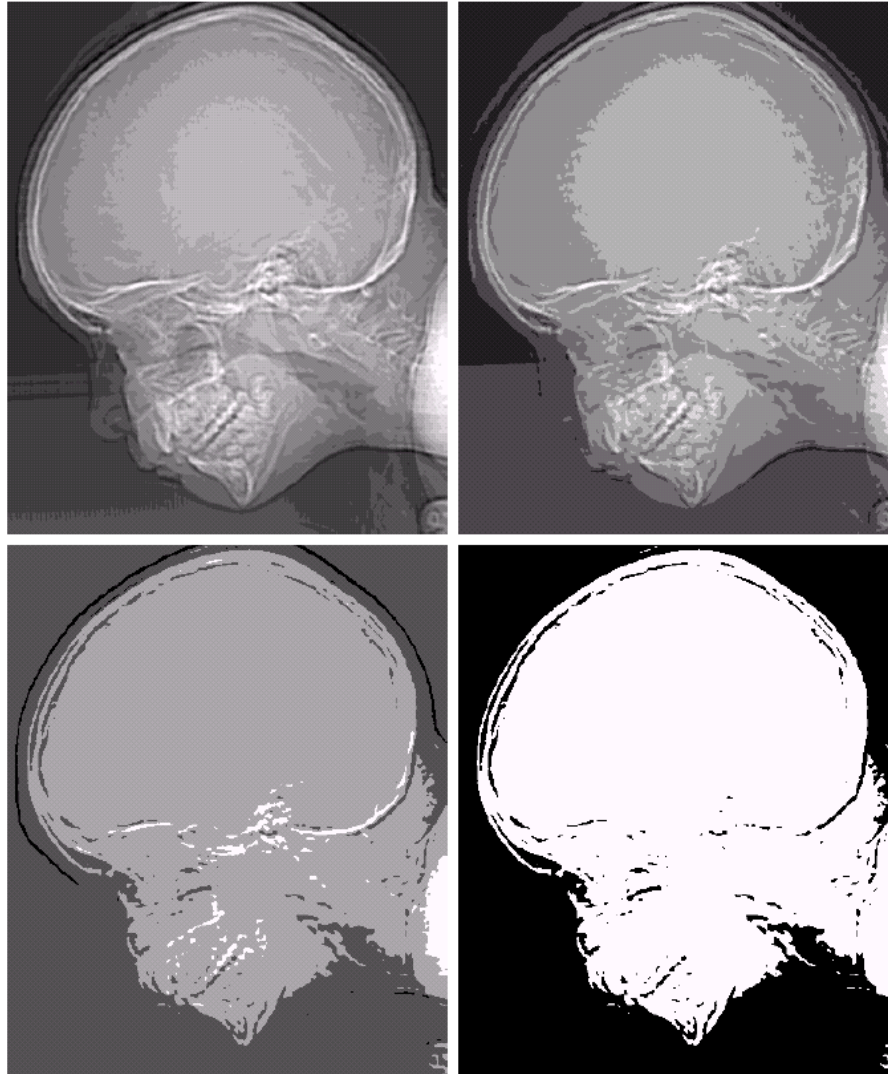**FIGURE 2.21**
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.
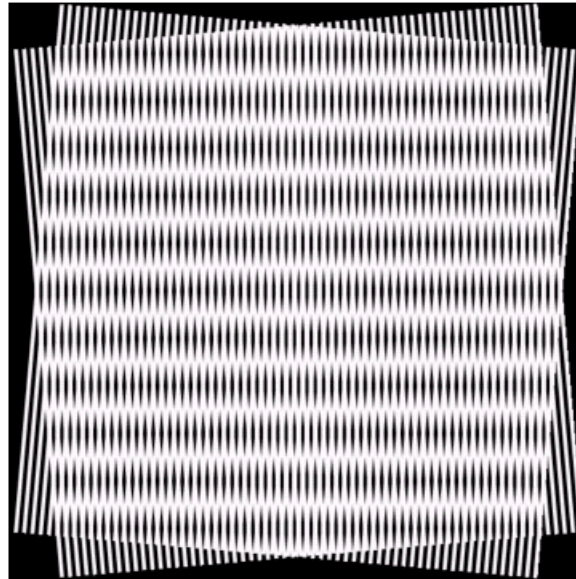
Jan 8

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# ...false contouring



e f
g h

**FIGURE 2.21**
*(Continued)*
(e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

# Sampling and Aliasing



**FIGURE 2.24** Illustration of the Moiré pattern effect.

# Additional Reading

- **Chapter 1, Introduction**
- **Chapter 2, Sections 2.1-2.4**
  - We will discuss sampling and quantization in detail later (Week 2)
- **Next:**
  - some basic relationships between pixels (Section 2.5)
  - MATLAB: an overview
  - A quick tour of linear systems (note, G&W additional reading)

# Relationship between pixels

- **Neighbors of a pixel**
    - 4-neighbors (N,S,W,E pixels) == $N_4(p)$. A pixel *p* at coordinates (x,y) has four horizontal and four vertical neighbors:
        - (x+1,y), (x-1, y), (x,y+1), (x, y-1)
    - You can add the four diagonal neighbors to give the 8-neighbor set. Diagonal neighbors == $N_D(p)$.
    - 8-neighbors: include diagonal pixels == $N_8(p)$.

# Pixel Connectivity

Connectivity -> to trace contours, define object boundaries,
segmentation.

In order for two pixels to be connected, they must be
"neighbors" sharing a common property—satisfy some
similarity criterion. For example, in a binary image with
pixel values "0" and "1", two neighboring pixels are said
to be connected if they have the same value.

Let V: Set of gray level values used to define
connectivity; e.g., $V=\{1\}$.

# Connectivity-contd.

- 4-adjacency: Two pixels p and q with values in V are 4-adjacent if q is in the set $N_4(p)$.

- 8-adjacency: q is in the set $N_8(p)$.

- m-adjacency: Modification of 8-A to eliminate multiple connections.
  - q is in $N_4(p)$ or
  - q in $N_D(p)$ and $N_4(p) \cap N_4(q)$ is empty.

# Connected components

- Let S represent a subset of pixels in an image.

- If p and q are in S, p is connected to q in S if there is a path from p to q entirely in S.

- Connected component: Set of pixels in S that are connected; There can be more than one such set within a given S.

# 4-connected components



p=0: no action;

p=1: check r and t.

- both r and t = 0; assign new label to p;
- only one of r and t is a 1. assign that label to p;
- both r and t are 1.
  - same label => assign it to p;
  - different label=> assign one of them to p and establish equivalence between labels (they are the same.)

*Second pass over the image to merge equivalent labels.*

# Exercise

**Develop a similar algorithm for 8-connectivity.**

# Problems with 4- and 8-connectivity

■ **Neither method is satisfactory.**

- – Why? A simple closed curve divides a plane into two simply connected regions.

- – However, neither 4-connectivity nor 8-connectivity can achieve this for discrete labelled components.

- – Give some examples..

# Related questions

- Can you "tile" a plane with a pentagon?

# Distance Measures

■ **What is a Distance Metric?**

For pixels *p,q,* and *z,* with coordinates (*x,y*), (*s,t*), and (*u,v*), respectively:

$$D(p,q) \geq 0 \quad (D(p,q) = 0 \text{ iff } p = q)$$

$$D(p,q) = D(q,p)$$

$$D(p,z) \leq D(p,q) + D(q,z)$$

# Distance Measures

- Euclidean

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

- City Block

$$D_4(p,q) = |x-s| + |y-t|$$

- Chessboard

$$D_8(p,q) = \max(|x-s|,|y-t|)$$

| 2 | 2 | 2 | 2 | 2 |
|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 2 |
| 2 | 1 | 0 | 1 | 2 |
| 2 | 1 | 1 | 1 | 2 |
| 2 | 2 | 2 | 2 | 2 |

# Matlab: a quick introduction

- http://varuna.ece.ucsb.edu/ece178/matlabip.htm
- A detailed document is available on-line
- More on MATLAB during the discussion session(s).