

Homework 2: Image Rectification

I. INTRODUCTION

The goal of this project is to explore some fundamental concepts of projective geometry to rectify the image of one of the facades of the Bren School of Environmental Science & Management at UCSB (see Figure 1). This handout will guide you through the theoretical and practical steps needed to construct the final rectified image.



Fig. 1. The left images shows one of the almost planar facades of the Bren School with a large perspective distortion. The goal is to construct a projective transformation that rectifies the image producing a result similar to the right image.

II. CALCULATING THE HOMOGRAPHY

A transformation from the projective space \mathbb{P}^3 to itself is called homography. An homography is represented by a 3×3 matrix with 8 degrees of freedom (scale as usual does not matter):

$$\begin{bmatrix} x^C \\ y^C \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} x^R \\ y^R \\ 1 \end{bmatrix} \quad (1)$$

To estimate H we need to estimate 8 variables. To this purpose we need to establish the correspondences between at least 4 non aligned points (since each point pair contributes for two equations). In this homework we want to find the homography that maps the facade of the Bren School of Environmental Science in image Bren_R.jpg to the view we would obtain with a virtual camera having the image plane parallel to the facade plane (i.e. resembling the viewing conditions of Bren_C.jpg).

Question 1: Pick 4 corresponding coplanar points in the images Bren_R.jpg and Bren_C.jpg and compute their image matrix coordinates.

Suggestion: You may find useful to check the Matlab function `ginput`.

Question 2: Calculate the matrix A that relates the two sets of point coordinates according to:

$$\begin{bmatrix} x_{p,1}^C \\ y_{p,1}^C \\ \vdots \\ x_{p,4}^C \\ y_{p,4}^C \end{bmatrix} = A_p \mathbf{h}_p$$

where $\mathbf{h} = [h_{11} \ h_{12} \ h_{13} \ h_{21} \ h_{22} \ h_{23} \ h_{31} \ h_{32}]^T$. Find a solution for the linear system you obtained using `Matlab`. Write explicitly the homography H .

Suggestion: You may find useful to read Example 1.12 in the second handout. Note that the `Matlab` command `h = A\b` solves a linear system $A*\mathbf{h}=\mathbf{b}$ by performing Gaussian elimination (it also returns the solution in the least squares sense to an under or overdetermined system of equations).

Answer 1: From equation (1) we can write:

$$x_i^C = \frac{h_{11}x_i^R + h_{12}y_i^R + h_{13}}{h_{31}x_i^R + h_{32}y_i^R + 1}$$

$$y_i^C = \frac{h_{21}x_i^R + h_{22}y_i^R + h_{23}}{h_{31}x_i^R + h_{32}y_i^R + 1}$$

or equivalently:

$$x_{p,i}^C(h_{31}x_{p,i}^R + h_{32}y_{p,i}^R + 1) = h_{11}x_{p,i}^R + h_{12}y_{p,i}^R + h_{13}$$

$$y_{p,i}^C(h_{31}x_{p,i}^R + h_{32}y_{p,i}^R + 1) = h_{21}x_{p,i}^R + h_{22}y_{p,i}^R + h_{23}$$

These equations can be written in matrix form as:

$$\begin{bmatrix} x_{p,i}^R & y_{p,i}^R & 1 & 0 & 0 & 0 & -x_{p,i}^R x_{p,i}^C & -y_{p,i}^R x_{p,i}^C \\ 0 & 0 & 0 & x_{p,i}^R & y_{p,i}^R & 1 & -x_{p,i}^R y_{p,i}^C & -y_{p,i}^R y_{p,i}^C \end{bmatrix} \mathbf{h} = \begin{bmatrix} x_{p,i}^C \\ y_{p,i}^C \end{bmatrix}$$

Stacking the previous matrix equations one on top of the other for $1 \leq i \leq 4$ we obtain the desired linear system of 8 equations in 8 unknowns. Its solution allows us to recover the parameters of the homography.

Question 3: Now pick more than 4 corresponding points in the image (say for example 8) and repeat Question 2. This time you have an overdetermined system of linear equations: solve it in a least squares sense. How does the matrix H you just obtained compare to the one calculated in Question 1?

III. IMAGE RECTIFICATION

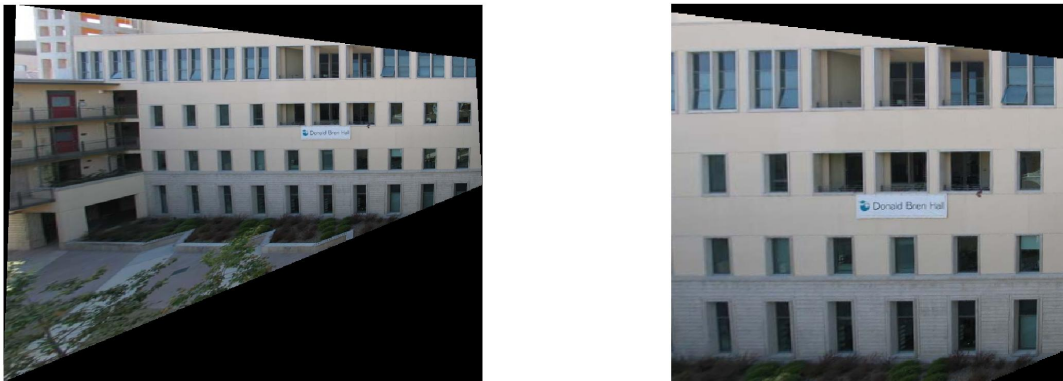


Fig. 2. The left images shows the rectification of and the right image shows a detail.

The last part of the project consists in warping the image `Bren_R.jpg` according to the homography calculated in Question 3. At least locally the result should look like `Bren_R.jpg`.

Question 4: Using the function `imTrans` (written and kindly provided by prof. Peter Kovesi), rectify the image `Bren_R.jpg` using the homography computed in Question 2. Note that because of the conventions used in `imTrans` you need to pass the matrix H^{-1} . Figure 2 shows an example of rectification.

Question 5: Describe the type of distortions you obtained after rectifying the image. What is their origin?

Suggestion We built an homography that maps a plane into a plane...