Behrooz Parhami's ECE 252B Course Page for Spring 2016

Computer Arithmetic

Page last updated on 2016 June 09

Enrollment code: 12195
Prerequisite: ECE 154A (in lieu of ECE 152A and 152B)
Class meetings: MW 12:00-1:30, Phelps 1431
Instructor: Professor Behrooz Parhami
Open office hours: M 3:30-5:00, W 10:00-11:30, HFH 5155
Course announcements: Listed in reverse chronological order
Course calendar: Schedule of lectures, homework, and exams
Homework assignments: Four assignments, worth 25% in all
Exams: Closed-book midterm (25%); Open-book final (50%)
Research paper: Not applicable for spring 2016
Research paper guidelines: Brief guide to format and contents
Poster presentation tips: Brief guide to format and structure
Policy on academic integrity: Please read very carefully
Grade stats: Range, mean, etc. for homework and exam grades

References: Textbook and other information sources
Lecture slides: Available on the textbook's Web page
Miscellaneous information: Motivation, catalog entry, history

Course Announcements

2016/06/09: The spring 2016 offering of ECE 252B is officially over and course grades have been reported to the Registrar. Wishing you all a very pleasant summer!
2016/05/20: Homework 4 (last one for the course; due W 6/01, 12:00 noon) has been posted to the homework area of this page.
2016/05/06: Homework 3 (due W 5/18, 12:00 noon) has been posted to the homework area of this page.
2016/04/14: Homework 2 (due W 4/27, 12:00 noon) has been posted to the homework area of this page.
2016/04/01: This is no April Fools' Day joke! Homework 1 (due at 12:00 noon on W 4/13) has been posted to the homework area of this page.
2016/03/28: Please study chapters 1-2 of the textbook by the end of the first week of classes. Lectures begin with chapter 3, but I will review some key concepts from the first two chapters via slides 9 (overview of topics), 11 (dangers of arithmetic errors), 14 (representational trade-offs), 22 (dot notation), 32 (2's-complement number representation), 37 (sign-and-magnitude versus 2's-complement representation), 39 (a key property of 2's-complement numbers), and 43 (extended dot notation).
2016/03/22: Welcome to the Web page for the graduate course ECE 252B in spring 2016. Information on the spring 2015 and earlier offerings of the course is available via PDF files accessible under the "History" section at the end of this page. Throughout the current quarter, the "Course Announcements" section will alert you to significant additions or changes to this Web page. Please visit regularly.

Course Calendar

Course lectures, homework assignments, and exams have been scheduled as follows. This schedule will be strictly observed. Please review the first two chapters in the textbook (before the first class, if possible). These chapters contain material that you should already know. PowerPoint and pdf files of course lectures, including skipped material in Chapters 1-2, can be

Day & Date (book chapters) Lecture/discussion topic [Homework posted/due] {Special notes}
M 03/28 (ch. 3) Redundant number representation
W 03/30 (ch. 4) Residue number systems
M 04/04 (Ch. 5) Basic addition and counting [HW1 posted, ch. 1-7]
W 04/06 (ch. 6) Carry-lookahead adders
M 04/11 (ch. 7) Variations in fast adders
W 04/13 (ch. 8) Multioperand addition [HW1 due]
M 04/18 (ch. 9) Basic multiplication schemes [HW2 posted, ch. 8-11]
W 04/20 (ch. 10) High-radix multipliers
M 04/25 (ch. 11) Tree and array multipliers
W 04/27 (ch. 12) Variations in multipliers [HW2 due]
M 05/02 (ch. 1-12) Midterm exam, closed book, 12:00-1:50 {Note the extended time}
W 05/04 (ch. 13) Basic division and some speedup methods
M 05/09 (ch. 14) High-radix division [HW3 posted, ch. 12-16]
W 05/11 (ch. 15) Variations in dividers
M 05/16 (ch. 16) Division by convergence
W 05/18 (ch. 17-18) Floating-point numbers and operations [HW3 due]
M 05/23 (ch. 19-20) Errors, precision, and certifiability [HW4 posted, ch. 17-21]
W 05/25 (ch. 21) Square-rooting methods
M 05/30 (No lecture) Memorial Day observance
W 06/01 (ch. 22-23) CORDIC algorithms and function evaluation [HW4 due] {Instructor/course eval survey}
T 06/07 (ch. 3-21) Final exam, Open book, 12:00-3:00
W 06/15 {Course grades must be submitted by midnight}

Homework Assignments

- Turn in solutions in class before the lecture begins.
- Because solutions will be handed out on the due date, no extension can be granted.
- Use a cover page that includes your name, course name, and assignment number.
- Staple the sheets and write your name on top of each sheet in case they are separated.
- Although some cooperation is permitted, direct copying will have severe consequences.

**Homework 1:** Number systems, addition/subtraction (ch. 1-7, due W 2015/04/13, 12:00 noon)
Do the following problems from the textbook: 1.2, 2.10, 3.2, 4.11, 6.15, 7.6ab

**Homework 2:** Multioperand addition, multiplication (ch. 8-11, due W 2015/04/27, 12:00 noon)
Do the following problems from the textbook: 8.6, 8.29, 9.4d, 9.14abe, 10.9, 11.19

**Homework 3:** Variations in multiplication, division (ch. 12-16, due W 2015/05/18, 12:00 noon)
Do the following problems from the textbook: 12.9, 13.5ab, 13.12bc, 14.10, 15.13, 16.4

**Homework 4:** Floating-point arithmetic, square-rooting (ch. 17-21, due W 2015/06/01, 12:00 noon)
Do the following problems from the textbook: 17.7, 18.9, 18.15, 19.9, 20.25, 21.2a

Sample Exams and Study Guide

The following sample exams (from spring 2007 and 2008) are meant to indicate the types and levels of problems, rather than the coverage (which is outlined in the course calendar). Students are responsible for all sections and topics (in the textbook and class handouts) that are not explicitly excluded in the study guides that follow the sample exams, even if the material was not covered in class lectures.

Sample Midterm Exam (105 minutes)

Problem 1 [15 points] Defining concepts and terms. Define each of the following concepts/terms precisely and concisely within the space provided (about 1.5 inches per term) [3 points each]: Manchester carry chain; Multiplier recoding; $ulp$; Conditional-sum adder; Parallel prefix graph

Problem 2 [10 points] Number representation. Show that flipping (complementing) the sign bit of $k$-bit numbers in 2's-complement format results in biased representation and determine the bias amount that characterizes this new representation.

Problem 3 [20 points] Basic design concepts. Draw diagrams showing each of the following. No explanation is necessary; the diagrams should be self-explanatory.
   a. How an ordinary binary adder can be augmented to perform addition or subtraction of 2's-complement numbers under the control of an add'/sub signal (0 means "add", 1 means "subtract").
   b. How 2-bits-at-a-time or radix-4 sequential multiplication might be performed at high speed without Booth's recoding and without precomputing 3 times the multiplicand.

Problem 4 [15 points] Carry-skip addition.
   a. Show that the optimal block width $b$ in a fixed-block carry-skip adder is proportional to the square root of the word width $k$. [10 points]
   b. Briefly discuss why carry-skip adders are of interest at all, given that faster logarithmic-time adders are available. [5 points]

Problem 5 [15 points] Multioperand addition. The following describes a multioperand addition process in tabular form:

0 0 8 8 8 8 8 8 8
0 2 6 6 6 6 6 4
0 4 4 4 4 4 3 2
1 3 3 3 3 3 3 2 1
2 2 2 2 2 2 1 1

a. Explain the process described by this table. [5 points]
   b. In the hardware implementation implied by the table, what component types are used and how many of each? Be as precise as possible in specifying the components used. [10 points]

   a. Represent $x = 3$, $y = -3$, and $z = 5$ as 4-bit 2's-complement numbers. [5 points]
   b. Using the right-shift algorithm, perform $x$ times $z$, using the representations of part a, to get the 8-bit product $p = 15$. [10 points]
   c. Using the left-shift algorithm, perform $y$ times $z$, to get the 8-bit product $p' = -15$. [10 points]

Sample Final Exam (2.5 hours)

Do Problems 1-2, plus 5 of the remaining 6. If all optional problems are answered, the first 5 will be graded.

Problem 1 [15 points]
   a. The standard 2-way carry operator has two pairs of inputs and a pair of outputs. Present a suitable generalization to an $h$-way operator with $h$ pairs of inputs.
   b. Name and justify one, and only one, advantage of each of the following dividers over the other two: high-radix, array, convergence.
   c. Explain why square-rooting cannot be viewed as a special case of division, in the same way that squaring is a special case of multiplication.

Problem 2 [10 points] Problem 5.18c from the textbook.

Problem 3 [15 points] Problem 7.28 from the textbook.

Problem 4 [15 points] Problem 11.22 from the textbook.

Problem 5 [15 points] Problem 15.6 from the textbook.

Problem 6 [15 points] Problem 19.2 from the textbook.

Problem 7 [15 points] Problem 21.18 from the textbook.

Problem 8 [15 points] Problem 22.20abc from the textbook.
Midterm Exam Study Guide
The following textbook sections are excluded from the midterm exam: 3.4-3.6, 4.4-4.6, 6.3, 7.2, 10.5

Final Exam Study Guide
In addition to the midterm exclusions, the following textbook sections are excluded from the final exam covering Chapters 3-21: 15.4, 15.6, 19.4, 19.5, 19.6, 20.2, 21.4, 21.6

Research Paper and Presentation [Not applicable for spring 2016*]
Each student will review a subfield of computer arithmetic or do original research on a selected and approved topic. A list of research topics is provided below ("N/A" designates topics that are not available for the current quarter); however, students should feel free to propose their own topics for approval. A publishable report earns an "A" for the course, regardless of homework and midterm grades. See the course calendar for research milestones and due dates. Consult Research Paper Guidelines for formatting tips.

* Even though no research is required for the current offering of ECE 252B, you may take additional directed research units (ECE 596) during spring or summer if you find any of the following topics of interest.

Topics for Part I of the Textbook: Number Representation

01. Implementation of Arithmetic Operations in Mechanical Calculators (Assigned to: TBD)

02. The Need for, and Practicality of, Decimal Computer Arithmetic in Hardware (Assigned to: TBD)

03. Practical Implementations of Ternary Computer Arithmetic (Assigned to: TBD)

04. A Comparison of Carry-Save and Borrow-Save Number Systems and Arithmetic (Assigned to: TBD)

05. Modulo-(2^a+1) Number Representations and Arithmetic (Assigned to: TBD)

06. Number Representation with Discrete Logarithms (Assigned to: TBD)

Topics for Part II of the Textbook: Addition/Subtraction

07. Variable-Block Carry-Lookahead Adders (Assigned to: TBD)

08. Parallel-Prefix Ling Adders (Assigned to: TBD)

09. Design of Optimal Adders with Input Timing Profile (Assigned to: TBD)

10. Saturating Two-Operand and Multioperand Adders (Assigned to: TBD)

11. Saturating Parallel Counters and Compressors (Assigned to: TBD)

12. Nonbinary Parallel Counters: The Ternary Example (Assigned to: TBD)

13. Implementation of Parallel Counters by Means of Sorting Networks (Assigned to: TBD)

13a. Multiplexer-Based Designs for Parallel Counters and Compressors (Assigned to: TBD)


14. Counting Networks: Design Methods and Applications (Assigned to: TBD)

**Topics for Part III of the Textbook: Multiplication**

15. Trade-offs in Compensation Methods for Truncated Multipliers (Assigned to: TBD)


16. Truncated Squarers and Cubers (Assigned to: TBD)


17. Multimode Multiplication and Squaring Circuits (Assigned to: TBD)


18. Design of Cubing Circuits (Assigned to: TBD)

19. Generalized Recursive Multipliers Built of Possibly Nonsquare Modules (Assigned to: TBD)

20. Merged Arithmetic: The Case of Add-Multiply-Add Circuits (Assigned to: TBD)


21. A Comparison of Hardware Multipliers Used in Microprocessors (Assigned to: TBD)


22. A Survey of Multiplier Circuits in Digital Signal Processors (Assigned to: TBD)


**Topics for Part IV of the Textbook: Division**

23. Radix-16 SRT Division: Algorithm and Implementations (Assigned to: TBD)

[Intel's] New Radix-16 Divider

24. A Survey of the Applications of Reciprocation and Square-Rooting (Assigned to: TBD)

25. Combinational Circuits for Fast Approximate Reciprocation (Assigned to: TBD)


26. On-the-Fly Conversion of Redundant Quotients into Nonredundant Form (Assigned to: TBD)

27. Practical Hardware Implementation of Montgomery Modular Multiplication (Assigned to: TBD)
27a. Convergence Division with Faster-than-Quadratic Convergence (Assigned: TBD)

**Topics for Part V of the Textbook: Real Arithmetic**

28. History of Floating-Point Number Representation Formats and Associated Standards (Assigned to: TBD)

29. Sign-Logarithmic Arithmetic and the European Logarithmic Microprocessor (Assigned to: TBD)

30. Residue Logarithmic Number Representation and Arithmetic (Assigned to: TBD)

31. Accurate Summation of Sets of Floating-Point Numbers (Assigned to: TBD)

32. Cube Roots via Newton-Raphson Method

33. Argument Reduction for Faster, More Accurate Function Evaluation (Assigned to: TBD)

36. Function Evaluation by Piecewise Linear Approximation (Assigned to: TBD)

38. Smaller Lookup Tables by Exploiting Symmetry and Nonuniform Segmentation (Assigned to: TBD)

**Topics for Part VII of the Textbook: Implementation Topics**

39. Pipelined Arithmetic in Vector Supercomputers (Assigned to: TBD)

40. Online or Digit-Pipelined Arithmetic with Carry-Save Operands (Assigned to: TBD)
40a. On-the-Fly Arithmetic Converters as Finite Automata (Assigned to: TBD)

41. Low-Power Full-Adder Cells and Their Applications (Assigned to: TBD)

42. Low-Power Design Techniques for Multipliers (Assigned to: TBD)

43. Dedicated Hardware Multipliers on FPGA Chips (Assigned to: TBD)
Using Embedded Multipliers in Spartan-3 FPGAs

44. Design Methods for Implementing Wider Multipliers Using Embedded FPGA Multipliers (Assigned to: TBD)

45. Augmenting FPGAs for Faster Arithmetic Operations (Assigned to: TBD)

General Research Topics Spanning Multiple Parts

46. A Survey of Arithmetic Circuits in Electronic Scientific Calculators (Assigned to: TBD)
References TBD.

47. Arithmetic in Early Supercomputers: IBM System/360 Model 91 and CDC 6600 (Assigned to: TBD)

48. Arithmetic and Energy Economy Provisions in IBM Blue Gene/L Parallel Supercomputer (Assigned to: TBD)

49. Implementation of Arithmetic Operations in Graphics Processors (Assigned to: TBD)

50. Number Crunching for Computer Games: History and Techniques (Assigned to: TBD)

51. Arithmetic Operations for Elliptic Curve Cryptography (Assigned to: TBD)

52. Implementation of Ultrahigh-Precision Arithmetic on Parallel Computers (Assigned to: TBD)
53. A Comparison of Synchronous and Asynchronous Arithmetic Circuits (Assigned to: TBD)

54. Implementing Arithmetic Operations with Neuronlike Hardware Elements (Assigned to: TBD)

55. Computer Arithmetic with Emerging Technologies (Assigned to: TBD)


**Poster Presentation Tips**

Here are some guidelines for preparing your research poster. The idea of the poster is to present your research results and conclusions thus far, get oral feedback during the session from the instructor and your peers, and to provide the instructor with something to comment on before your final report is due. Please send a PDF copy of the poster via e-mail by midnight on the poster presentation day.

Posters prepared for conferences must be colorful and eye-catching, as they are typically competing with dozens of other posters for the attendees' attention. Here is an example of a conference poster. Such posters are often mounted on a colored cardboard base, even if the pages themselves are standard PowerPoint slides. In our case, you should aim for a "plain" poster (loose sheets, to be taped to the wall in our classroom) that conveys your message in a simple and direct way. Eight to 10 pages, each resembling a PowerPoint slide, would be an appropriate goal. You can organize the pages into 2 x 4 (2 columns, 4 rows), 2 x 5, or 3 x 3 array on the wall. The top two of these might contain the project title, your name, course name and number, and a very short (50-word) abstract. The final two can perhaps contain your conclusions and directions for further work (including work that does not appear in the poster, but will be included in your research report). The rest will contain brief description of ideas, with emphasis on diagrams, graphs, tables, and the like, rather than text which is very difficult to absorb for a visitor in a very limited time span.

**Grade Statistics**

All grades are in percent.

HW1 grades: Range = [35, 90], Mean = 62, Median = 61
HW2 grades: Range = [59, 100], Mean = 82, Median = 85
HW3 grades: Range = [70, 81], Mean = 76, Median = 77
HW4 grades: Range = [59, 91], Mean = 73, Median = 71
Midterm exam grades: Range = [43, 82], Mean = 61, Median = 59
Final exam grades: Range = [51, 84], Mean = 68, Median = 70

**References**

*Primary textbook (required):*

*Verilog descriptions of arithmetic circuits (recommended):*
This course does not involve a lab component or implementation projects. For those interested in pursuing practical circuit implementations, the following book may be useful:

*Other useful books (not required):*
Brent/Zimmermann, Modern Computer Arithmetic, Cambridge, 2011
Deschamps/Biou/Sutter, Synthesis of Arithmetic Circuits: ... , Wiley, 2006 (TK7895.A65D47)
Ercegovac/Lang, Digital Arithmetic, Morgan Kaufmann, 2004 (QA76.9.C62E73)
Ercegovac/Lang, Division and Square Root: ... , Kluwer, 1994 (QA76.9.C62E73)
Muller, Elementary Functions: Algorithms and Implementation, Birkhauser, 2006 (QA331.M866)
Muller et al., Handbook of Floating-Point Arithmetic, Birkhauser, 2010
Motivation: Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the architectural and algorithmic speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.


History: Professor Parhami took over the teaching of ECE 252B from the late Dr. James Howard in the winter quarter of 1989. By covering sequential machines, computer arithmetic, and advanced microprocessor-based design, the graduate course sequence ECE 252A/B/C was meant to provide a firm foundation in the theories and techniques of advanced digital design. During the first few offerings of ECE 252B, Professor Parhami gradually modified the content to increase both its coverage and research orientation (the now-discontinued ECE 252A & 252C underwent similar transformations by Professor Kwang-Ting Cheng and Professor Parhami, respectively). In 2000, based on a decade of experience in teaching computer arithmetic, Professor Parhami published a reference volume and graduate-level textbook, Computer Arithmetic: Algorithms and Hardware Designs (Oxford Univ. Press), which is being used at many universities worldwide. The 2nd edition of this textbook appeared in 2010.

Offering of ECE 252B in spring 2016 (PDF file)
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Offering of ECE 252B in spring 2010 (PDF file)
Offering of ECE 252B in spring 2009 (PDF file)
Offerings of ECE 252B from 2000 to 2008 (PDF file)