Robust Dynamical Model for Simultaneous Registration and Segmentation in a Variational Framework: A Bayesian Approach

Pratim Ghosh Mehmet Emre Sargin B.S. Manjunath

University of California, Santa Barbara, CA 93106-9560

Problem Definition and Contribution

The problem is to segment and track an object of interest in a given sequence of images. We assume that the initial condition of the object of interest is known *a priori*. The key components of our method are:

- A dynamical model using Bayesian formulation for the level-set based simultaneous segmentation and non-linear registration (SRS).
- Unsupervised shape model: No prior learning of the shape variation.
- Computationally efficient solution to the Bayesian problem with certain simplifying assumptions.

Simultaneous Registration and Segmentation, Ehrhardt et al. ICCV'07

- Registration part:

$$C_{reg}(\mathbf{u}) = \overbrace{\frac{1}{2}\int_\Omega (I(\mathbf{x},t) - I(\mathbf{T}(\mathbf{x}),t-1))^2 d\mathbf{x}}^{C^1_{reg}} + \alpha \overbrace{\frac{1}{2}\int_\Omega trace(\nabla \mathbf{u}^T \nabla \mathbf{u})\, d\mathbf{x}}^{C^2_{reg}}$$

where $\mathbf{T}(\mathbf{x}) = \mathbf{x} - \mathbf{u}$ and $\mathbf{u}(\mathbf{x},t-1,t) : \Omega \to \mathbb{R}^2$.

- Segmentation part ($\phi^o(\mathbf{x},t) : \Omega \to \mathbb{R}$): $C_{cv}(\phi^o(\mathbf{x},t)) =$

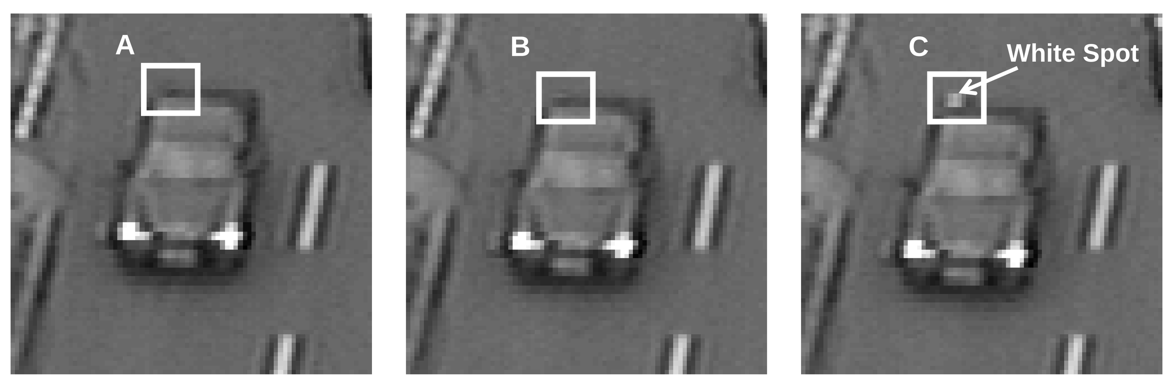
$$\overbrace{\int_\Omega \ln p(I|\theta_2) + H_\epsilon(\phi^o(\mathbf{x},t)) \ln \frac{p(I|\theta_2)}{p(I|\theta_1)} d\mathbf{x}}^{C^1_{cv}} + \nu \underbrace{\int_\Omega |\nabla H_\epsilon(\phi^o(\mathbf{x},t))| d\mathbf{x}}_{C^2_{cv}}.$$

- The combination of registration and segmentation:

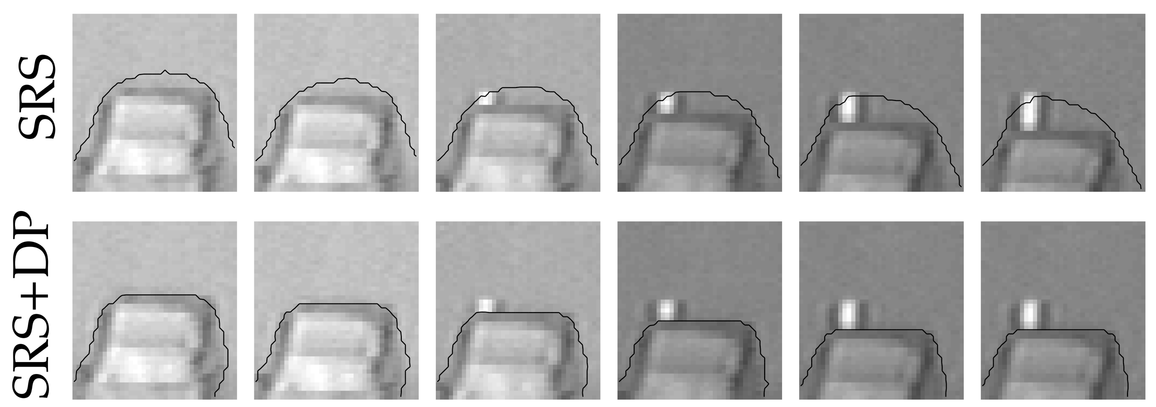
$$C_{comb}(\phi^o(\mathbf{x},t),\mathbf{u}) = \frac{1}{2}\int_\Omega N_\epsilon\Big(\phi^o(\mathbf{x},t),\phi^o(\mathbf{x},t-1)\Big)\Big(\phi^o(\mathbf{x},t) - \underbrace{\phi^o(\mathbf{T}(\mathbf{x}),t-1)}_{\text{prior}}\Big)^2 d\mathbf{x}$$

Drawback of SRS

- Example of a common source of error in the estimation of $\mathbf{u}$.



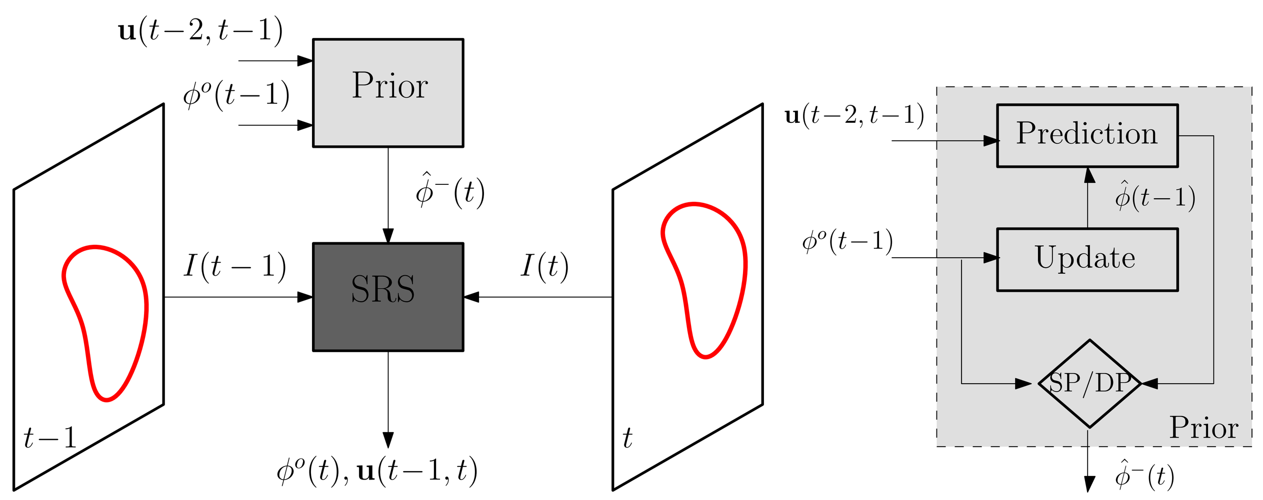
- Segmentation Result:



Design of a Dynamic Prior (SRS+DP)

We propose a dynamic prior term $\hat{\phi}^-(t)$ which:

- Maximizes the *a posteriori* probability given all the past observations.
- Avoids the dependence on the current estimate $\mathbf{u}(\mathbf{x},t-1,t)$ in the estimation of $\phi^o(\mathbf{x},t)$, thus providing robustness against noise/clutter.
- Allows an efficient computation scheme if the underlying process is assumed to be first order Markov.



Model Based Probabilistic Tracking

- Consider the Bayesian formulation:

$$\hat{\phi}(\mathbf{X},t) = \underset{\phi(\mathbf{X},t)}{\operatorname{argmax}}\, \mathcal{P}(\phi(\mathbf{X},t)|\phi^o(\mathbf{X},1\!:\!t)), \quad \text{where} \;\; \phi(\mathbf{X},t) = \{\phi(\mathbf{x},t)\}$$

$$\Longrightarrow \hat{\phi}(\mathbf{x},t) = \underset{\phi(\mathbf{x},t)}{\operatorname{argmax}}\, \mathcal{P}(\phi(\mathbf{x},t)|\phi^o(\mathbf{x},1\!:\!t)). \quad \text{(after simplification)}$$

- From the underlying SRS model (i.e. $\phi(\mathbf{x},t+1) \approx \phi(\mathbf{x} - \mathbf{u}(\mathbf{x},t,t+1),t)$), we compute a linear stochastic equation:

$$\phi(\mathbf{x},t) = \phi(\mathbf{x},t-1) - \mathbf{u}^T(\mathbf{x},t-2,t-1)\nabla\phi(\mathbf{x},t-1) + w$$

where $\mathcal{P}(w) \sim \mathcal{N}(0,Q)$ and the associated observation model is: $\phi^o(\mathbf{x},t) = \phi(\mathbf{x},t) + v$, where $\mathcal{P}(v) \sim \mathcal{N}(0,R)$.

Prediction and the new Observation model

- The dynamic prior maximizes: $\hat{\phi}^-(\mathbf{x},t) = \underset{\phi(\mathbf{x},t)}{\operatorname{argmax}}\, \mathcal{P}(\phi(\mathbf{x},t)|\phi^o(\mathbf{x},1\!:\!t-1))$, which simplifies to computing: $\hat{\phi}^-(\mathbf{x},t) = \hat{\phi}(\mathbf{x},t-1) - \mathbf{u}^T(\mathbf{x},t-2,t-1)\nabla\hat{\phi}(\mathbf{x},t-1)$.
- The modified segmentation problem using the dynamic prior reads:

$$C_{cv}(\phi^o(\mathbf{x},t)) = C^1_{cv}(\phi^o(\mathbf{x},t)) + \nu C^2_{cv}(\phi^o(\mathbf{x},t)) + \beta\frac{1}{2}\int_\Omega |\phi^o(\mathbf{x},t) - \hat{\phi}^-(\mathbf{x},t)|^2 d\mathbf{x}$$

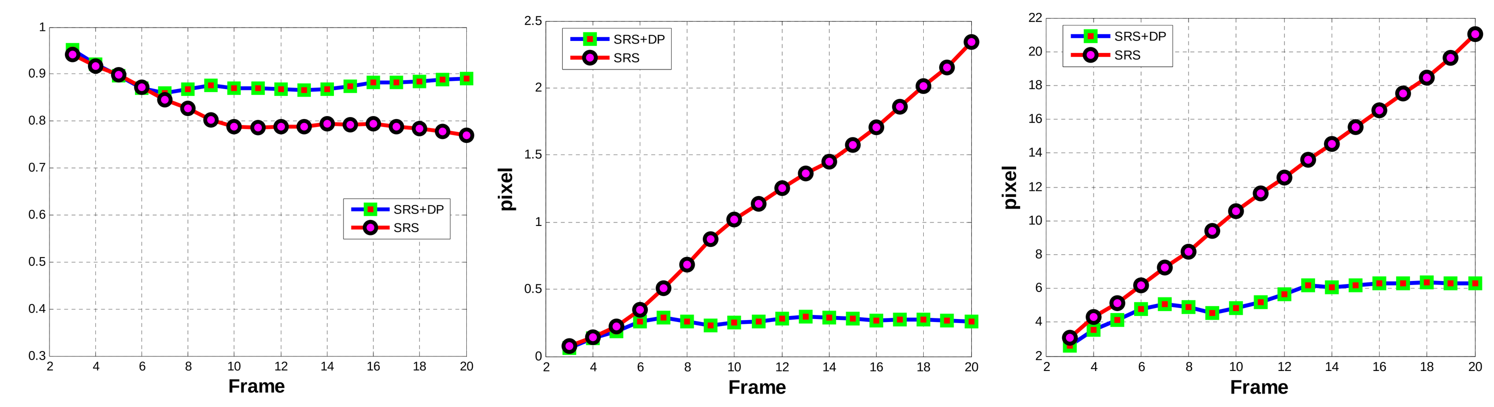
- We can now compute $\hat{\phi}(\mathbf{x},t)$ as: $\hat{\phi}(\mathbf{x},t) = (1-\kappa(t))\hat{\phi}^-(\mathbf{x},t) + \kappa(t)\phi^o(\mathbf{x},t)$, where $\kappa(t) = \frac{\sigma^-(t)}{\sigma^-(t)+R}$. Here, $\sigma^-(t)$ is the covariance of the prediction error and it can be recursively computed by $\sigma^-(t) = \sigma^-(t-1) + Q$.

Endocardium Tracking

- Evaluated over 32 (patients)×8 (slices)× 18 (frames)=4608 images.

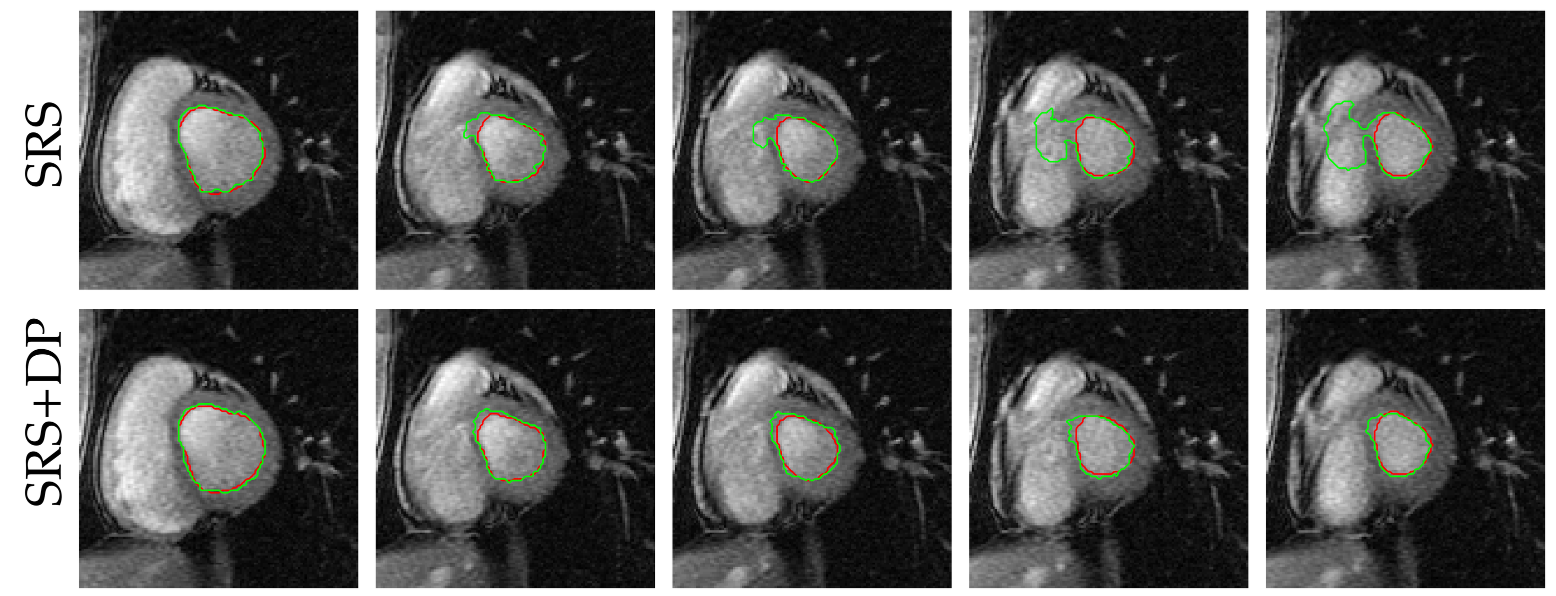
Error Metrics	SRS+DP	SRS
F-measure	0.8828 ± 0.0576	0.8196 ± 0.1397
MAD	0.2458 ± 0.2019	1.1519 ± 2.0956
HD	5.2929 ± 2.5605	11.9556 ± 13.1597

- Evaluated on 256 sequences:

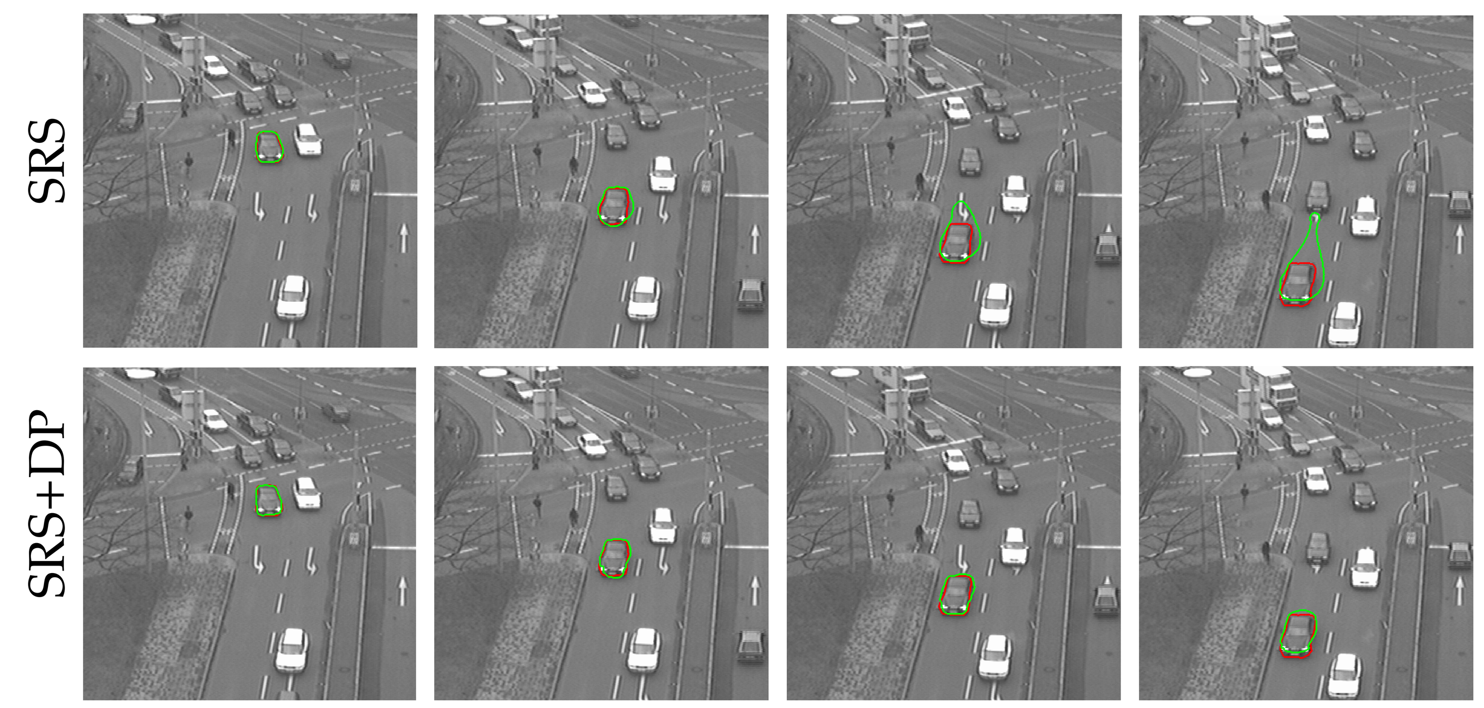


F-measure MAD-measure HD-measure

- Visual Results:



Vehicle tracking



This work was supported by the NSF ITR # 0331697.