Wireless Communications
Lecture 11

[Review of Last lecture: Capacity of fading channels]

- Capacity of narrowband fading channels:

$$C = \max \int_0^\infty B \log_2 \left(1 + \frac{P(\gamma)\gamma}{\overline{P}}\right) p(\gamma) d\gamma$$

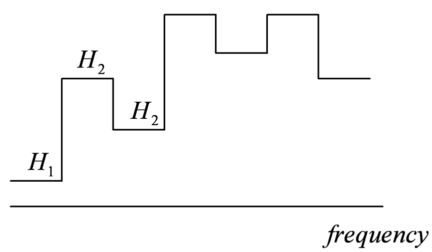
such that $\int P(\gamma)p(\gamma)d\gamma = \overline{P}$, note that $\gamma = \frac{\overline{P}|ch|^2}{N_o B}$ and received SNR when adapting $\frac{P(\gamma)|ch|^2}{N_o B}$

$\Rightarrow$ adapting the power transmitted to channel condition:

$$\frac{P(\gamma)}{\overline{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \text{else} \end{cases} \quad \Rightarrow \text{ find } \gamma_0 \text{ and } C$$

- Capacity of time-invarient wideband channels:

Each subchannel almost constant



Block frequency-selective fading

$$C = \max \sum_j B \log_2 \left(1 + \frac{|H_j|^2 P_j}{N_o B}\right)$$

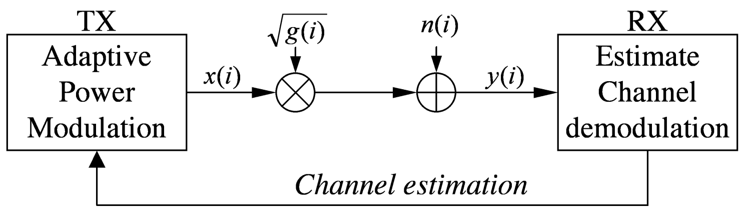
such that $\sum_j P_j \leq P$.

$$\frac{P_j}{\overline{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma_j} & \gamma_j \geq \gamma_0 \\ 0 & \text{else} \end{cases} \quad \text{with } \gamma_j = \frac{|H_j|^2 P_j}{N_o B} \Rightarrow \text{Optimum Power}$$

Adaptive Modulation: Optimizing power/rate with more practical goals that capacity

[Basic assumption of adaptive modulation]

- Channel can be estimated in the RX and is sent back to the transmitter before it changes
- If the channel is changing too fast, adapt to large-scale fading.
- Adaptive schemes are solutions to optimization problems, depending on the objective & the constraints.
- Note that the capacity achieving rate/power may not be the solution to other practical optimization scenarios.



System model with transmitter and receiver CSI

where

$$\begin{aligned} T_s &: \text{ symbol period} \\ B &: \text{ bandwidth } B = 1/T_s \\ g(i) &: \text{ square of amplitude of channel at time } i \\ n(i) &: \text{ AWGN} \end{aligned}$$

If $\overline{P}$ is the average transmitted power for the fixed TX power case, received SNR: $\gamma_i = \frac{\overline{P}g(i)}{N_0 B}$. For adaptive case, $P(i)$ is TX power and $R(i)$ is rate.

$$R(i) = \frac{\log_2 M(i)}{T_s}$$

where spectral efficiency=$\log_2 M(i)$ in bits/sec/Hz.

If $P(i) = P(\gamma_{(i)})$, then

$$\text{Received SNR} = \frac{P(i)g(i)}{N_o B} = \frac{P(i)\gamma_i}{\overline{P}}$$

Note: To change the rate, it is easier to adapt M. Adapting T_s is difficult due to the change of bandwidth.

[**Variable data rate case**] If we can only change the constellation size/type (M), it makes sense to use smaller M for worse channel conditions.

[**Example**] Consider an adaptive modulation scheme that uses either QPSK or 8PSK depending on channel condition. The target $P_b = 10^{-3}$. When should each modulation be used? What is the average spectral efficiency? (average received power = 20 dB).

For QPSK, $P_b = Q(\sqrt{\gamma})$ and for 8PSK, $P_b \approx 0.666Q(\sqrt{2\gamma}\sin(\pi/8))$. Then,

$$\begin{array}{ll} \text{QPSK} & \gamma \geq 10.35\text{dB for } P_b \leq 10^{-3} \\ \text{8PSK} & \gamma \geq 14.79\text{dB for } P_b \leq 10^{-3} \end{array}$$

$$\text{If} \begin{cases} \gamma \geq 14.79\text{dB} & \text{use 8PSK} \\ 10.35 \leq \gamma < 14.79\text{dB} & \text{use QPSK} \\ \text{else} & \text{no transmission} \end{cases}$$

$$\begin{aligned} \text{Average spectral efficiency} &= 3 \times prob\{\gamma \geq 14.79\text{dB}\} + 2 \times prob\{10.35\text{dB} \leq \gamma < 14.79\text{dB}\} \\ &= \frac{3}{100}\int_{30.1}^{\infty} e^{-\gamma/100} d\gamma + 2\int_{10.85}^{30.1} \frac{1}{100} e^{-\gamma/100} d\gamma \\ &= 2.534\text{bps/Hz} \end{aligned}$$

[**Variable-power case (fix rate)**]

The goal then would be to have a fixed RX power to maintain a fixed P_b

$$\frac{P(\gamma)}{\overline{P}}\gamma = \sigma(\text{fixed}) \Rightarrow \frac{P(\gamma)}{\overline{P}} = \frac{\sigma}{\gamma} \ \textbf{channel inversion}$$

σ is

$$\begin{aligned} \int \frac{P(\gamma)}{\overline{P}} p(\gamma) d\gamma &= 1 \\ \int \frac{\sigma P(\gamma)}{\gamma} d\gamma &= 1 \end{aligned}$$

$$\sigma = \frac{1}{\mathbf{E}(1/\gamma)}$$

Given a required P_b, if required σ is bigger than $\frac{1}{\mathbf{E}(1/\gamma)}$, it is not feasible.

[Example] γ is exponential distributed, $\mathbf{E}(1/\gamma) = \infty$ and $\sigma = 0$, therefore no constant P_b can be met.

[Inversion above a certain threshold]

$$\frac{P(\gamma)}{\overline{P}} = \begin{cases} \frac{\sigma}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \text{else} \end{cases}$$

Then,

$$P_{out} = prob\{\gamma < \gamma_0\} \Rightarrow \gamma_0$$

$$\int P(\gamma)p(\gamma)d\gamma = \overline{P}$$

$$\int_{\gamma_0}^{\infty} \frac{\sigma}{\gamma} p(\gamma)d\gamma = 1$$

$$\sigma = \frac{1}{\int_{\gamma_0}^{\infty} \frac{P(\gamma)}{\gamma} d\gamma}$$

As γ_0 increases, σ also increases.

[Example] For a fixed modulation $P_b = Q(\sqrt{2\text{SNR}_{rec}})$. It is desirable to maintain $P_b = 10^{-3}$ for a Rayleigh fading channel with $\overline{\gamma} = 10$ dB. What is the adaptive modulation scheme & outage probability?

$P_b = Q(\sqrt{2\sigma}) = 10^{-3}$, then $\sigma = 4.77$.

$$4.77 = \frac{1}{\int_{\gamma_0}^{\infty} \frac{e^{-\gamma/10}}{10\gamma} d\gamma}$$

Solve for γ_0 numerically $\gamma_0 = 0.7423$.

$$P_{outage} = prob\{\gamma < \gamma_0\} = 1 - e^{-\gamma_0/10} = 0.379$$

[**Adaptive rate & power scheme**]

In the BER for an AWGN channel with MQAM modulation, ideal coherent phase detection, and SNR γ is bounded by

$$P_b \quad \leq \quad 0.2e^{-1.5\gamma/(M-1)} \quad \text{for } M \geq 4 \text{ and } 0 \leq \gamma \leq 30 \text{ dB}$$

To reach a certain P_b

$$M(\gamma) = 1 + \frac{1.5\gamma P(\gamma)}{-\ln(5P_b)\overline{P}} = 1 + K\gamma\frac{P(\gamma)}{\overline{P}}$$

where

$$K = \frac{-1.5}{\ln(5P_b)} < 1$$

$$P_b \leq 0.2e^{-.15\text{SNR}_{rec}/(M-1)} \leq 0.2e^{-1.5/(M-1)}\Big|_{\text{SNR}_{rec}=0\text{dB}} \leq 0.2$$

Then, $\ln(5P_b) \leq 0 \Rightarrow K \geq 0$.

Maximizing average spectral efficiency

$$\mathbf{E}[\log_2 M(\gamma)] = \int_0^\infty \log_2\left(1 + K\gamma\frac{P(\gamma)}{\overline{P}}\right)p(\gamma)d\gamma$$

where $\int_0^\infty P(\gamma)p(\gamma)d\gamma = \overline{P}$.
Same as capacity achieving formulation except for K

$$\frac{P(\gamma)}{\overline{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma K} & \gamma \geq \frac{\gamma_0}{K} = \gamma_K \\ 0 & \text{else} \end{cases}$$

The corresponding average spectral efficiency is

$$\frac{R}{B} = \int_{\gamma_K}^\infty \log_2\left(\frac{\gamma}{\gamma_K}\right)p(\gamma)d\gamma$$

Compare to capacity of the same channel

$$\frac{C}{B} = \int_{\gamma_0}^\infty \log_2\left(\frac{\gamma}{\gamma_0}\right)p(\gamma)d\gamma$$

$\gamma_k > \gamma_0$.