Wireless Communications
Lecture 14

OFDM systems, $Y_i = H_i X_i + W_i \quad 0 \le i \le N-1$.

[Channel Estimation]

Insert pilot tones, $X_{\text{pilot}}(l_i), \quad 0 \le i \le L-1$ and $l_i = \frac{N}{L} i$

$$\hat{H}_{l_i} = \frac{Y_{l_i}}{X_{\text{pilot}}(l_i)} = H_{l_i} + \frac{W_{l_i}}{X_{\text{pilot}}(l_i)}, \quad 0 \le i \le L-1$$

$$\begin{bmatrix} H_{l_0} \\ H_{l_1} \\ \vdots \\ H_{l_{L-1}} \end{bmatrix} = \begin{bmatrix} e^{-j2\pi\frac{0\times 0}{L}} & e^{-j2\pi\frac{0\times 1}{L}} & \cdots & e^{-j2\pi\frac{0\times (L-1)}{L}} \\ e^{-j2\pi\frac{1\times 0}{L}} & e^{-j2\pi\frac{1\times 1}{L}} & \cdots & e^{-j2\pi\frac{1\times (L-1)}{L}} \\ \vdots & \ddots & \ddots & \vdots \\ e^{-j2\pi\frac{(L-1)\times 0}{L}} & e^{-j2\pi\frac{(L-1)\times 1}{L}} & \cdots & e^{-j2\pi\frac{(L-1)\times (L-1)}{L}} \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L-1} \end{bmatrix}$$

$$H_i = \sum_{k=0}^{L-1} e^{-j2\pi ki/N} h_k$$

$$\begin{bmatrix} H_{l_0} \\ H_{l_1} \\ \vdots \\ H_{l_{L-1}} \end{bmatrix} \begin{matrix} \text{IFFT}_L \\ \longrightarrow \\ \longleftarrow \\ \text{FFT}_L \end{matrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L-1} \end{bmatrix}$$

Estimate at pilot tones $\longrightarrow$ IFFT in base L to get the time-domain channel

$$\underbrace{\hat{H}_i = \sum_{k=0}^{L-1} e^{-j2\pi ik/N} h_k}_{\text{zero padding and FFT in base } N}$$

[Frequency-domain interpolation]

$$\begin{aligned}
\hat{h}_k &= \frac{1}{L}\sum_{i=0}^{L-1}\hat{H}_{l_i}e^{j2\pi ki/L} \quad 0 \le k \le L-1 \\
\hat{H}_i &= \sum_{k=0}^{L-1}\hat{h}_k e^{-j2\pi ki/N} \quad 0 \le i \le N-1 \\
&= \frac{1}{L}\sum_{k=0}^{L-1}\sum_{z=0}^{L-1}\hat{H}_{l_z}e^{j2\pi kz/L-j2\pi ki/N} \\
&= \frac{1}{L}\sum_{z=0}^{L-1}\hat{H}_{l_z}\sum_{k=0}^{L-1}e^{-j2\pi k(i/N-z/L)} \\
&= \frac{1}{L}\sum_{z=0}^{L-1}\hat{H}_{l_z}\frac{1-e^{-j2\pi Li/N}}{1-e^{-j2\pi(i/N-z/L)}} \\
&= \frac{1}{L}\sum_{z=0}^{L-1}\hat{H}_{l_z}\frac{(1-e^{-j2\pi Li/N})}{e^{-j\pi(i/N-z/L)}2j\sin(\pi i/N-\pi z/L)} \\
&= \sum_{z=0}^{L-1}\hat{H}_{l_z}\frac{(1-e^{-j2\pi Li/N})}{2jL}\frac{\cos(\pi(i/N-z/L))+j\sin(\pi(i/N-z/L))}{\sin(\pi i/N-\pi z/L)} \\
&= \sum_{z=0}^{L-1}\underbrace{\frac{(1-e^{-j2\pi Li/N})}{2jL}\Big[j+\cot\big(\pi(i/N-z/L)\big)\Big]\hat{H}_{l_z}}_{\text{Frequency domain interpolation}}
\end{aligned}$$

Other simpler interpolations possible: linear quadratic, $\cdots$

[Impact of mobility]

$$y_i = \sum_{k=0}^{L-1} h_k^{(i)} x((i-k))_N + w_i \quad 0 \le i \le N-1$$

$$\begin{aligned} Y_k &= \sum_{i=0}^{N-1} y_i e^{-j2\pi ki/N} = \sum_{i=0}^{N-1}\sum_{k'=0}^{L-1} h_{k'}^{(i)} x((i-k'))_N e^{-j2\pi ki/N} \\ &= \frac{1}{N}\sum_{i=0}^{N-1}\sum_{k'=0}^{L-1} h_{k'}^{(i)} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi l(i-k')}{N} - j2\pi\frac{ki}{N}} \\ &= \frac{1}{N}\sum_{l=0}^{N-1}\sum_{k'=0}^{N-1} X_l e^{-j2\pi\frac{lk'}{N}} \sum_{i=0}^{N-1} h_{k'}^{(i)} e^{\frac{-j2\pi i(k-l)}{N}} \end{aligned}$$

$$F_{k'}(z) = \sum_{i=0}^{N-1} h_{k'}^{(i)} e^{-j2\pi iz/N}$$

$$Y_k = \frac{1}{N}\sum_{l=0}^{N-1}\sum_{k'=0}^{N-1} X_l F_{k'}[k-l] e^{-j2\pi lk'/N}$$

Define $H_{k',u} = \frac{1}{N}\sum_{k'=0}^{N-1} F_{k'}(u) e^{-j2\pi\frac{(k-u)k'}{N}}$

$$Y_k = \sum_{l=0}^{N-1} x_l H_{k,k-l} \quad \text{adding the noise}$$

$$Y_k = H_{k,0}X_k + \underbrace{\sum_{l\neq k} X_l H_{k,k-l}}_{\text{Inter Carrier Interference}} + W_k$$

Need to mitigate ICI.

One possibility : Linear approximation of channel time - domain variations