Wireless Communications
Lecture 17

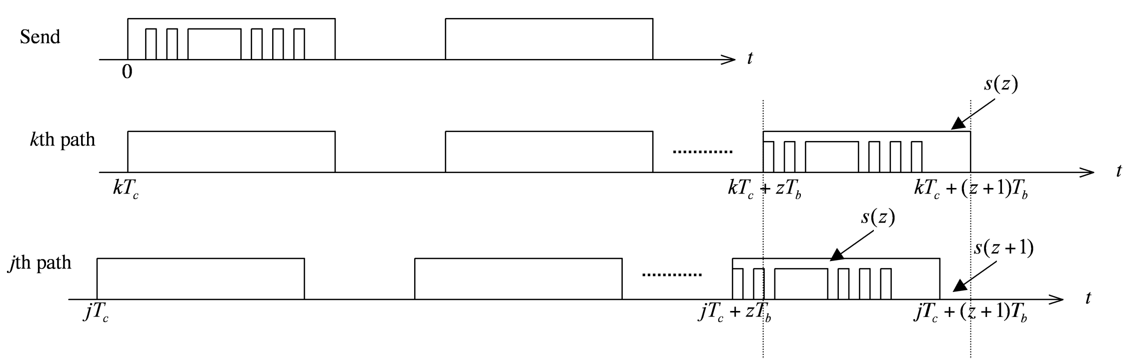
[Rake Receiver Performance] TX sends $s(t)$. It spreads with $s_c(t)$. Assume a channel model with impulse response $h(t) = \sum_{j=1}^{J} \alpha_j \delta(t - jT_c)$. Rake receiver has fingers at kT_c.

For the kth finger of Rake receiver, the input signal is

$$\sum_{j=1}^{J} \alpha_j s(t - jT_c) s_c(t - jT_c)$$

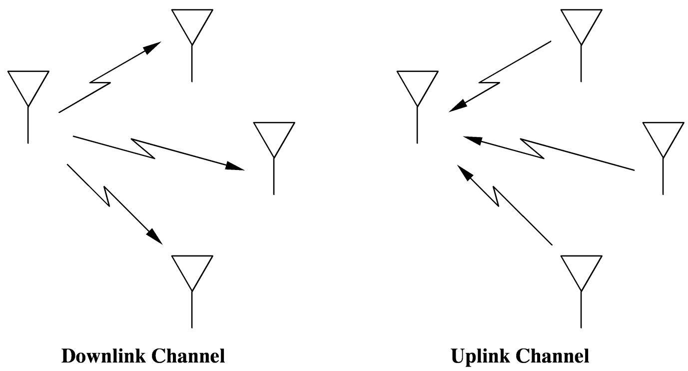
where α_j is the gain associated with the jth multipath component. The output of Rake receiver is

$$\begin{aligned}
\text{Output} &= \int_{zT_b+kT_c}^{(z+1)T_b+kT_c} \sum_{j=1}^{J} \alpha_j s(t - jT_c) s_c(t - jT_c) s_c(t - kT_c) dt \\
&= \sum_{j=1}^{J} \alpha_j \int_{zT_b+kT_c}^{(z+1)T_b+kT_c} s(t - jT_c) s_c(t - jT_c) s_c(t - kT_c) dt \\
&= \alpha_k s(z) + \sum_{j \neq k} \alpha_j \int_{zT_b+kT_c}^{(z+1)T_b+kT_c} s(t - jT_c) s_c(t - jT_c) s_c(t - kT_c) dt
\end{aligned}$$



The integral will have the impact of both $s(z)$ and $s(z+1)$. If they are the same, it becomes $\rho_c((j-k)T_c)$.

[Multiuser DSSS systems] Separate different users with codes



- Spreading Codes for Multiuser DSSS

  Each user is assigned a unique spreading code sequence $s_{c_i}(t)$. $s_{c_i}(t)$ and $s_{c_j}(t)$ for users i and j,

$$\rho_{ij}(\tau) = \frac{1}{T}\int_0^T s_{c_i}(t)s_{c_j}(t-\tau)dt$$

  $s_{c_i}(t)$ and $s_{c_i}(t)$ are periodic codes.

  For synchronous users, their signals arrive at the receiver aligned in time, so $\tau = 0$ and the cross correlation becomes

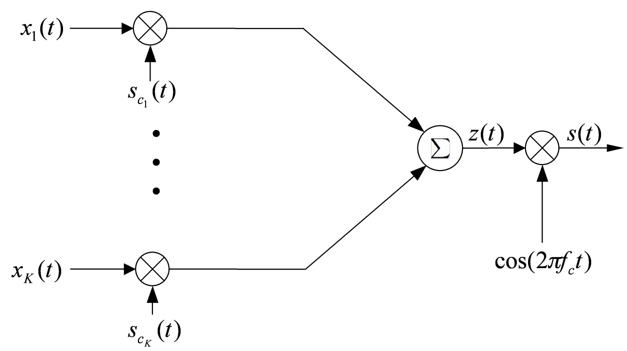
$$\rho_{ij}(0) = \frac{1}{T}\int_0^T s_{c_i}(t)s_{c_j}(t)dt$$

  we want $\underbrace{\rho_{ij}(\tau) = 0 \quad \forall\tau}_{\text{orthogonal codes}}$.

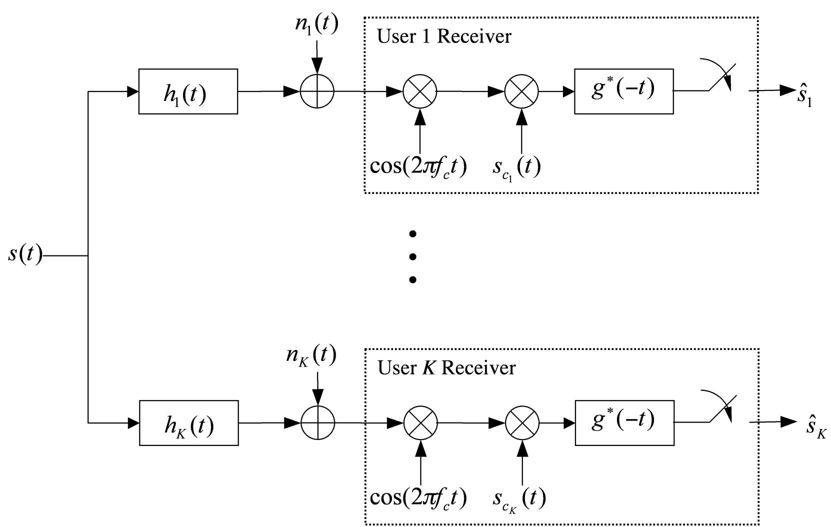
  - If all codes arrive at the same time $\Rightarrow$ synchronous case ($\rho_{ij}(0)$)
  - Asynchronous case: they arrive at different times.
  - For asynchronous case: not possible to design orthogonal codes.
  - For synchronous case: only can design a finite number given N(spreading gain)
  - Maximal linear codes are good for auto-correlation properties but not so good for cross-correlation properties.
  - Gold codes have good cross-correlation properties

$$\rho_{ij}(\tau) = -\frac{1}{N}$$

- Downlink case:



Downlink transmitter



Downlink channel and receiver

The transmitter consists of K branches, where kth branch multiplies the kth user's signal $x_k(t)$ with the spreading code $s_{c_k}(t)$. The branches are summed together, resulting in the baseband multiuser signal.

$$z(t) = \sum_{k=1}^{K} x_k(t) s_{c_k}(t)$$

Let's assume $h_k(t) = \alpha_k \delta(t)$. Consider ith receiver, $\hat{s}_i$

$$\begin{aligned}
\hat{s}_i &= \int_{jT_b}^{jT_b+T_b} \alpha_i \sum_{k=1}^{K} x_k(t) s_{c_k}(t) s_{c_i}(t) dt \\
&= \alpha_i \sum_{k=1}^{K} \int_{jT_b}^{jT_b+T_b} x_k(t) s_{c_k}(t) s_{c_i}(t) dt \\
&= \alpha_i x_{ij} + \alpha_i \sum_{k \neq i} \int_{jT_b}^{jT_b+T_b} \underbrace{x_k(t)}_{\text{constant}} s_{c_k}(t) s_{c_i}(t) dt \\
&= \alpha_i x_{ij} + \alpha_i \sum_{k \neq i} x_{kj} \int_{jT_b}^{jT_b+T_b} s_{c_k}(t) s_{c_i}(t) dt \\
&= \alpha_i x_{ij} + \underbrace{\alpha_i \sum_{k \neq i} x_{kj} \rho_{ki}(0)}_{\text{interference}} + \text{noise term}
\end{aligned}$$

The **SIR** for the kth user is defined as the ratio of power associated with the kth user's signal over the average power associated with the multiuser interference alone.

$$\text{SIR} = \frac{\alpha_i^2 \sigma_x^2}{\alpha_i^2 \sigma_x^2 \frac{K-1}{N}} = \frac{N}{K-1}$$

The **SNIR** for the kth user is defined as the ratio of power associated with the kth user's signal over the average power associated with the multiuser interference and noise at the demodulator output.

$$\text{SNIR} = \frac{\alpha_i^2 \sigma_x^2}{\alpha_i^2 \sigma_x^2 \frac{(K-1)}{N} + \sigma_n^2} = \left[\frac{K-1}{N} + \frac{1}{\text{SNR}_{\text{rec}}} \right]^{-1}$$

waterfall effect