Wireless Communications
Lecture 18

[MUD: Multi-User Detection]

For uplink, to separate all the users jointly
Consider 2 users (synchronous)

$$r(t) = \alpha_1 b_1 S_{c_1}(t) + \alpha_2 b_2 S_{c_2}(t) + \text{noise}$$

where α_i denotes channel (no delay spread), b_1 denotes bit of the first user over a given period and b_2 denotes bit of the second user over a given period

$$\begin{aligned} r_1(t) &= \int r(t) S_{c_1}(t) dt = \alpha_1 b_1 + \alpha_2 b_2 \rho_{12}(0) \\ r_2(t) &= \int r(t) S_{c_2}(t) dt = \alpha_1 b_1 \rho_{12}(0) + \alpha_2 b_2 \end{aligned}$$

$$\begin{aligned} \vec{\mathbf{r}} &= \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \rho_{12}(0) \\ \rho_{12}(0) & 1 \end{bmatrix}}_{\mathbf{R}} \underbrace{\begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{\vec{\mathbf{b}}} + \text{noise} \\ \hat{\mathbf{b}}^* &= \mathbf{R}^{-1} \vec{\mathbf{r}} = \left(\mathbf{A} \vec{\mathbf{b}} + \text{noise} \right) \end{aligned}$$

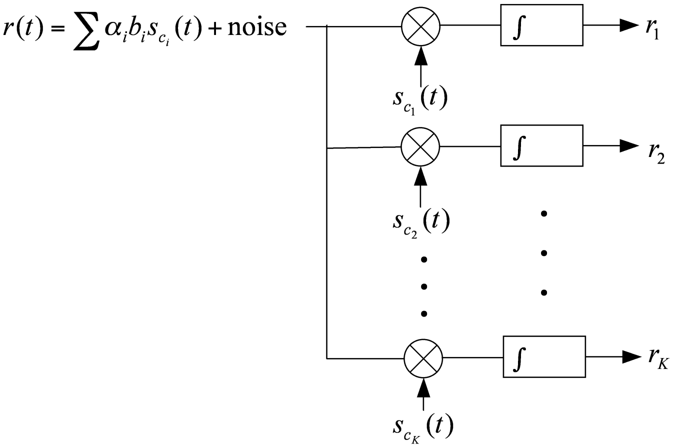
Consider K users.

$$\vec{\mathbf{r}} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1K} \\ \rho_{21} & 1 & \rho_{23} & \cdots & \rho_{2K} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_{K1} & \rho_{K2} & \rho_{K3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \alpha_K \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix} + \text{noise}$$

Note : Typically users won't be synchronous and there is delay spread. It makes MUD more difficult.

[**Another possibility: Successive interference cancellation**]

For uplink, first decode the strongest signal, remove it and proceed to the 2nd strongest signal and so on.



choose the largest r_i, then decode b_i.
Form $\hat{r}(t) = r(t) - \hat{\alpha}_i \hat{b}_i S_{c_1}(t)$
Repeat the process with $\hat{r}(t)$.

Consider 2 users, $\alpha_1 b_1 s_{c_1}(t) + \alpha_2 b_2 s_{c_2}(t)$. If $\alpha_1 \gg \alpha_2$,

$$\begin{aligned} r_1 &= \alpha_1 b_1 + \underbrace{\alpha_2 b_2 \rho_{12}}_{\text{treat like noise}} \simeq \alpha_1 b_1 \\ r_2 &= \underbrace{\alpha_1 b_1 \rho_{12}}_{\text{could be comparable to } \alpha_2 b_2} + \alpha_2 b_2 \end{aligned}$$

Decode $r_1 \rightarrow \hat{b}_1$ to form

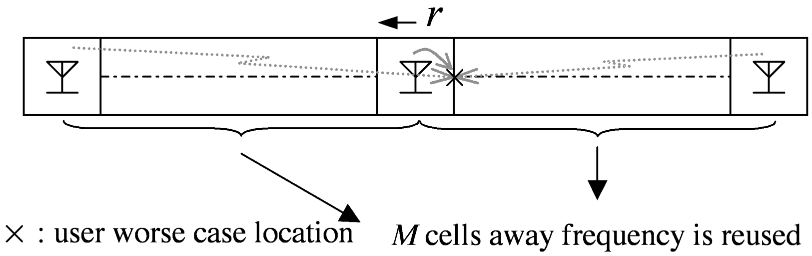
$$\begin{aligned} \hat{r}(t) &= r(t) - \hat{\alpha}_1 \hat{b}_1 s_{c_1}(t) \\ &= \alpha_2 b_2 s_{c_2}(t) \end{aligned}$$

In the ideal case, $\int \hat{r}(t) s_{c_2}(t) dt$ to get b_2.
Problem: This works if some users are much stronger than the rest.

[Multiple Access Techniques] FDMA, frequency reuse key to cellular system.

Consider a 1D case:

- *For downlink case*



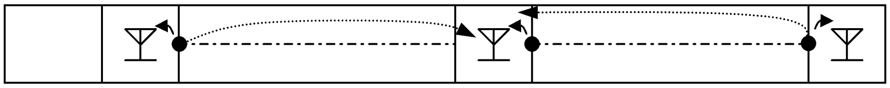
S_p is defined as signal power at X, $S_p = K/r^n$.

$$\begin{aligned} I_{p_1} &= \frac{K}{(2Mr-r)^n} \quad \text{near interference} \\ I_{p_2} &= \frac{K}{(2Mr+r)^n} \quad \text{far base station} \end{aligned}$$

$$\begin{aligned} \text{SIR} &= \frac{1/r^n}{\frac{1}{(2Mr-r)^n} + \frac{1}{(2Mr+r)^n}} \\ &= \frac{1}{(2M-1)^{-n} + (2M+1)^{-n}} \end{aligned}$$

This is considering only the first set of interfering base stations.

- *For uplink case*



$$\begin{aligned} S_p &= K/r^n \\ I_{p_1} &= \frac{2K}{(2M-1)^n r^n} \Rightarrow \text{SIR}_{\text{close}} = \frac{(2M-1)^n}{2} \\ I_{p_2} &= \frac{2K}{(2M+1)^n r^n} \Rightarrow \text{SIR}_{\text{far}} = \frac{(2M+1)^n}{2} \end{aligned}$$

Uplink interference is harder to analyze since it is also a function of the positions of the interferences