Wireless Communications
Lecture 2

[Trigonometry relationship]

$$
\begin{aligned}
\sin A \cos B &= \frac{\sin(A+B) + \sin(A-B)}{2} \\
\sin A \sin B &= -\frac{\cos(A+B) - \cos(A-B)}{2} \\
\cos A \cos B &= \frac{\cos(A+B) + \cos(A-B)}{2}
\end{aligned}
$$

$$
\begin{aligned}
\sin(A+B) &= \sin A \cos B + \cos A \sin B \\
\cos(A+B) &= \cos A \cos B - \sin A \sin B
\end{aligned}
$$

$$
\begin{aligned}
\sin(2A) &= 2 \sin A \cos A \\
\cos(2A) &= \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1
\end{aligned}
$$

[Fourier Transform]

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$
\begin{aligned}
x(t) &\longrightarrow X(f) \\
x^*(t) &\longrightarrow X^*(-f)
\end{aligned}
$$

[Relating energy of $s(t)$ to its baseband equivalent $u(t)$]

$|S(f)|$ symmetric around zero?

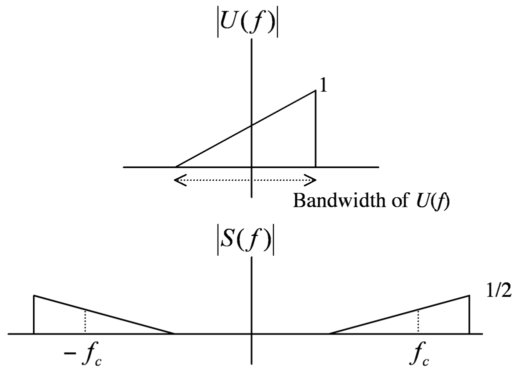
Finding $S(f)$

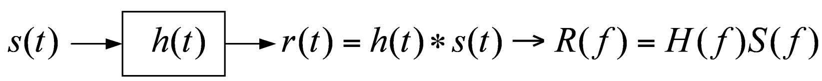
$$\mathcal{F}\{u(t)e^{j2\pi f_c t}\} = U(f - f_c)$$

$$\mathrm{Re}\{x(t)\} = \frac{x(t)+x^*(t)}{2}$$

$$\mathcal{F}\{u(t)e^{j2\pi f_c t}\} = \frac{U(f-f_c)+U^*(-f-f_c)}{2}$$

[example]





$$S(f) = \frac{1}{2}\left[U(f-f_c)+U^*(-f-f_c)\right]$$

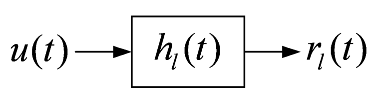
$$H(f) = \left[H_l(f-f_c)+H_l^*(-f-f_c)\right]$$

$$R(f) = H(f)S(f)$$

$$= \frac{1}{2}\left[U(f-f_c)H_l(f-f_c)+U^*(-f-f_c)H_l^*(-f-f_c)\right]$$

$$r(t) = \mathrm{Re}\{z(t)e^{j2\pi f_c t}\}, \quad z(t) = \mathcal{F}^{-1}\{U(f)H_l(f)\}$$

$$z(t) = u(t) * h_l(t) \Rightarrow r_l(t) = z(t)$$



[Two ray model, proof of large d]

$$r(t) = \text{Re}\left\{u\left(t - \frac{l}{c}\right)e^{j2\pi f_c(t-l/c)}\right\}\frac{\lambda\sqrt{G_tG_r}}{4\pi l} + \text{Re}\left\{u\left(t - \frac{x+x'}{c}\right)e^{j2\pi f_c\left(t-\frac{(x+x')}{c}\right)}\right\}\frac{\lambda\sqrt{G_tG_r}}{4\pi(x+x')}R$$

where R is the ground reflection coefficient.

Assuming wide enough time-duration for $u(t)$, such that $u\left(t - \frac{l}{c}\right)$ and $u\left(t - \frac{x+x'}{c}\right) \simeq u(t)$ and taking the time that the direct path arrived at origin:

$$r(t) = \text{Re}\left\{u(t)e^{j2\pi f_c(t)}\right\}\frac{\lambda\sqrt{G_tG_r}}{4\pi l} + \text{Re}\left\{u(t)e^{j2\pi f_c\left(t+\frac{l}{c}-\frac{(x+x')}{c}\right)}\right\}\frac{\lambda\sqrt{G_tG_r}}{4\pi(x+x')}R$$

The received power is

$$P_r = \left(\frac{\lambda}{4\pi}\right)^2 G_tG_r\overline{\left|\frac{u(t)}{l} + \frac{u(t)}{x+x'}e^{-j\Delta\phi}R\right|^2} \times \frac{1}{2}$$

Transmitted power $P_t = P_u/2$

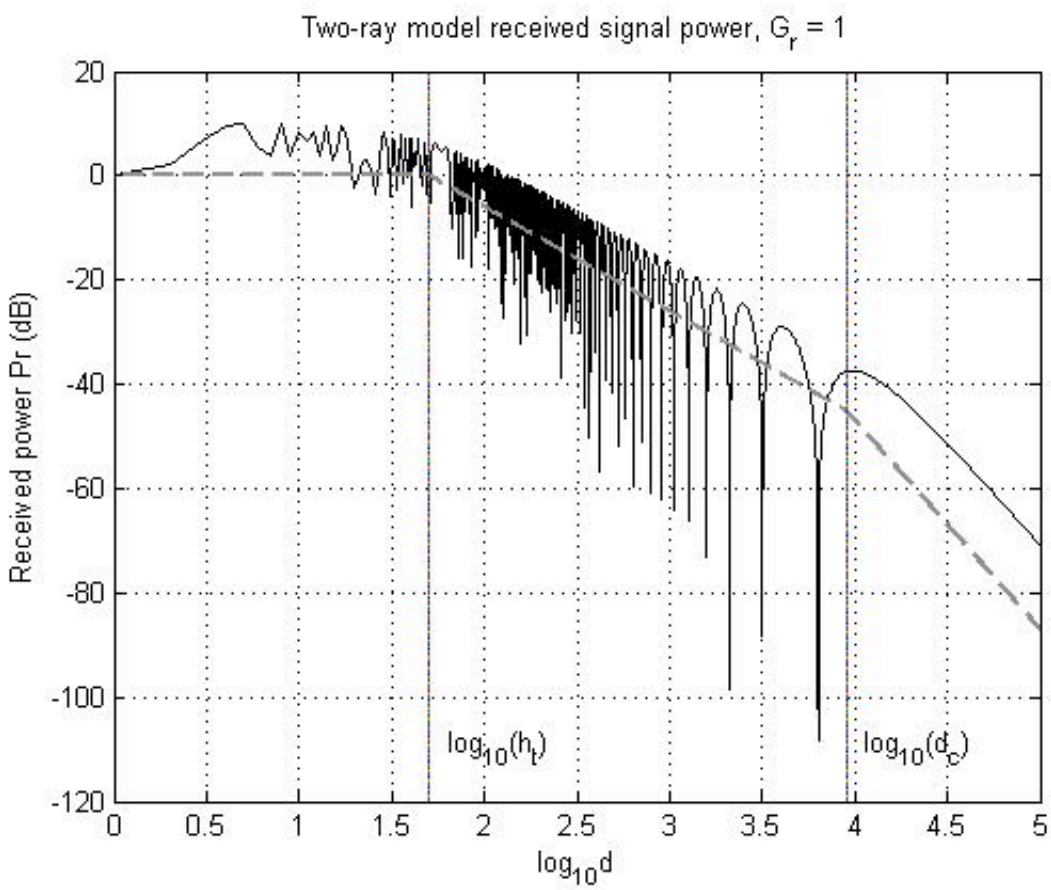
$$P_r = \left(\frac{\lambda}{4\pi}\right)^2 G_tG_rP_t\left|\frac{1}{l} + \frac{e^{-j\Delta\phi}R}{x+x'}\right|^2$$

If d denotes the horizontal separation of the antennas, h_t the transmitter height, and h_r the receiver height

$$\begin{aligned}
\Delta\phi &= -2\pi f_c\left(\frac{l}{c} - \frac{(x+x')}{c}\right) \\
&= -2\pi\left(\frac{l}{\lambda} - \frac{(x+x')}{\lambda}\right), \quad \lambda = \frac{c}{f_c} \\
(x+x') - l &= \sqrt{(h_t+h_r)^2 + d^2} - \sqrt{(h_t-h_r)^2 + d^2} \\
d &\gg h_t + h_r \Rightarrow \Delta\phi \simeq \frac{4\pi h_th_r}{\lambda d} \quad \text{using Taylor series approximation} \\
\sqrt{1+x^2} &\simeq 1 + \frac{x^2}{2} \\
R &= \frac{\sin\theta - Z}{\sin\theta + Z}, \quad \text{for large } d,\ \theta \simeq 0, R \simeq -1, x + x' \simeq l \simeq d, \Delta\phi \text{ is small}
\end{aligned}$$

The the received power is

$$
\begin{aligned}
P_r &= \left(\frac{\lambda}{4\pi}\right)^2 G_t G_r P_t \left|\frac{1}{l}\right|^2 \left|1 - e^{-j\Delta\phi}\right|^2 \\
&= \left(\frac{\lambda}{4\pi d}\right)^2 G_t G_r P_t \left|\Delta\phi\right|^2 \\
&= G_t G_r \left[\frac{h_t h_r}{d^2}\right]^2 P_t
\end{aligned}
$$



$f = 900$ MHz, $R = -1$, $h_t = 50$m, $h_r = 2$ m, $G_l = 1$,and $G_r = 1$