Wireless Communications
Lecture 3

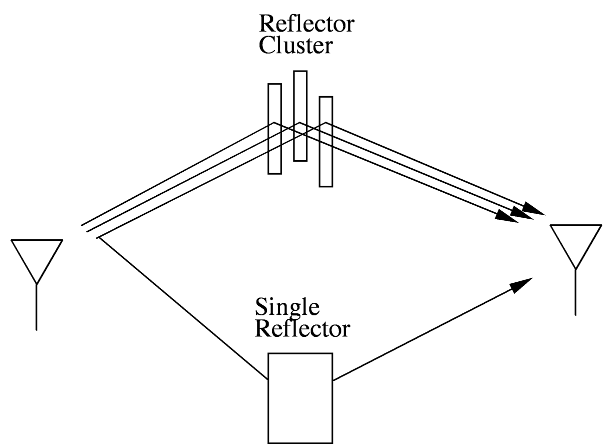
[Time-varying Channel Impulse Response]

The transmitted signal is

$$s(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\} = \mathrm{Re}\{u(t)\}\cos(2\pi f_c t) - \mathrm{Im}\{u(t)\}\sin(2\pi f_c t)$$

The received signal is the sum of all multipath components:

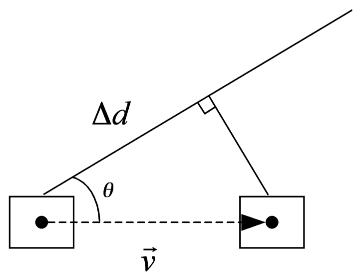
$$r(t) = \mathrm{Re}\left\{\sum_{n=1}^{N(t)} \alpha_n(t)u(t-\tau_n(t))e^{j2\pi[f_c+\Delta f_n(t)]t - j2\pi f_c \tau_n(t)}\right\}$$



[Doppler Shift] $f_c \to f_c + \frac{v}{c} f_c \cos\theta$

Proof: Due to the movement, The wave will get there $\Delta d/c$ sec earlier

$$\begin{aligned} t + \Delta d &= t + \frac{vt\cos\theta}{c} \\ \cos\left(2\pi f_c\left(t + \frac{vt\cos\theta}{c}\right)\right) &= \cos\left(2\pi\left(f_c + \frac{v}{c}f_c\cos\theta\right)t\right) \end{aligned}$$

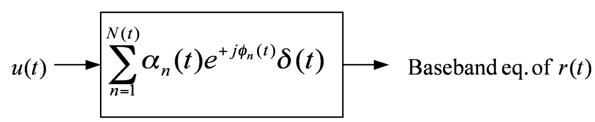


[Resolvable & Nonresolvable path]

Assume $\tau_{\max} - \tau_{\min} \ll B_u^{-1}$, *Narrowband channel*
The received signal is

$$\begin{aligned} r(t) &= \mathrm{Re}\left\{ \sum_{n=1}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} u(t) e^{j2\pi f_c t} \right\} \\ \phi_n(t) &= 2\pi \Delta f_n(t) t - 2\pi f_c \tau_n(t) \end{aligned}$$

The baseband equivalent channel is



$$ch_{BB_{eq}} = \sum_{n=1}^{N(t)} \alpha_n(t) e^{+j\phi_n(t)}, \quad \alpha_n(t) \text{ and } \phi_n(t) \text{ are independent}$$

The in-phase and quadrature components are given by

$$\begin{aligned} ch_I &= \sum_{n=1}^{N(t)} \alpha_n(t) \cos(\phi_n(t)) \\ ch_Q &= \sum_{n=1}^{N(t)} \alpha_n(t) \sin(\phi_n(t)) \end{aligned}$$

By Central Limit Theorem(CLT) ch_I and ch_Q are Gaussian random processes. In the adsence of a LOS path, $\phi_n(t)$ can be approximated by uniform. Thus

$$\mathbf{E}[ch_I] = \sum_n \overline{\alpha_n(t)} \quad \overline{\cos(\phi_n(t))} = 0 = \mathbf{E}[ch_Q]$$

By the independence of α_n and ϕ_n, the independence of ϕ_n and ϕ_m ($n \neq m$) and the uniform distribution of ϕ_n,

$$\begin{aligned}
\mathbf{E}[ch_I ch_Q] &= \mathbf{E}\left[\sum_n \alpha_n(t)\cos(\phi_n(t))\sum_m \alpha_m(t)\sin(\phi_m(t))\right] \\
&= \sum_n\sum_m \mathbf{E}[\alpha_n(t)\alpha_m(t)]\mathbf{E}[\cos(\phi_n(t))\sin(\phi_m(t)] \\
&= \sum_n \overline{\alpha_n^2(t)} \quad \overline{\cos(\phi_n(t))\sin(\phi_n(t))} \\
&= 0
\end{aligned}$$

The autocorrelation of ch_I is

$$\begin{aligned}
A_{r_I}(t,\tau) &= \mathbf{E}[ch_I(t)ch_I(t+\tau)] \\
&= \sum \overline{\alpha_n(t)\alpha_n(t+\tau)} \quad \overline{\cos(\phi_n(t))\cos(\phi_n(t+\tau))}
\end{aligned}$$

assume α_n is not a function of t and $\cos\theta_n$. Therefore Δf_n is not a function of t

$$= \sum \overline{\alpha_n^2(t)}\frac{1}{2}\overline{\cos(2\pi\Delta f_n\tau)}, \quad ch_I \text{ is WSS}$$

The autocorrelation of ch_Q is

$$A_{r_Q}(t,\tau) = \mathbf{E}[ch_Q(t)ch_Q(t+\tau)] = \sum \overline{\alpha_n^2}\frac{1}{2}\overline{\cos(2\pi\Delta f_n\tau)} = A_{r_I}(t,\tau)$$

The cross-correlation is given by

$$A_{r_I,r_Q}(t,\tau) = \mathbf{E}[ch_I(t)ch_Q(t+\tau)] = \sum \overline{\alpha_n^2(t)} \quad \overline{\sin(2\pi\Delta f_n\tau)} = -A_{r_Q,r_I}(t,\tau)$$

[Jakes model] Assume uniform angle of arrival, i.e., θ_n is uniform

$$\overline{\cos\left(2\pi\frac{v}{c}f_c\tau\cos\theta_n\right)} = J_0\left(2\pi\frac{v}{c}f_c\tau\right), \quad \frac{v}{c}f_c = f_D$$

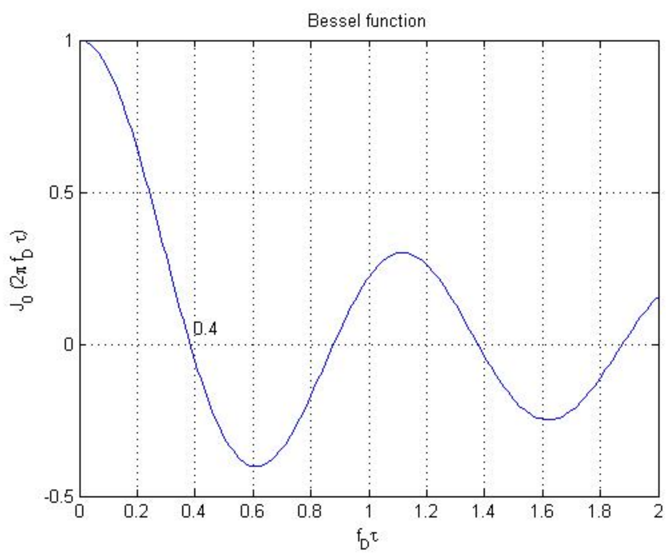
$A_{r_I}(\tau)$ becomes

$$A_{r_I}(\tau) = \frac{\sum\alpha_n^2}{2}J_0(2\pi f_D\tau) = A_{r_Q}(\tau), \quad \Omega_p = \sum\alpha_n^2$$

where

$$J_o(x) = \frac{1}{\pi}\int_0^\pi e^{-jx\cos\theta}d\theta$$

$A_{r_I,r_Q}(\tau) = 0.$

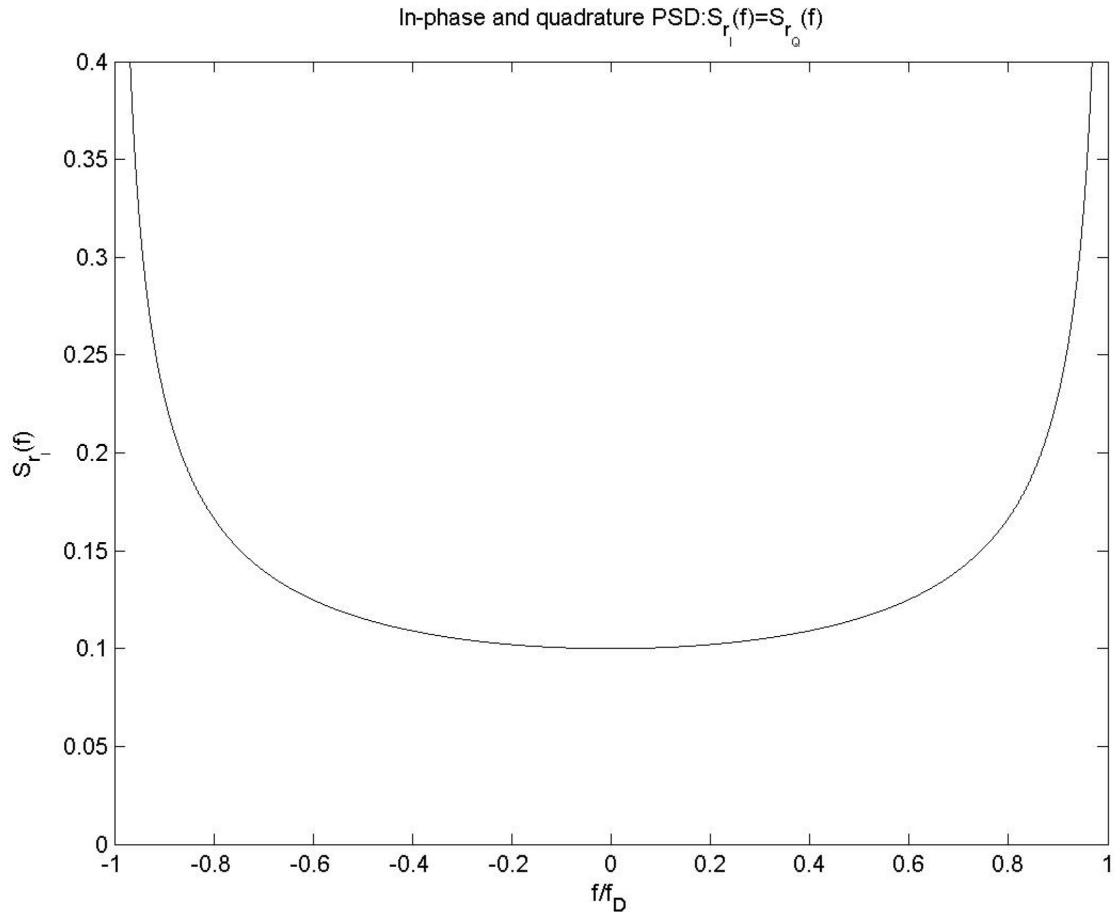


Uncorrelated when

$$f_D\tau = 0.4 \Rightarrow \frac{v}{c}f_c\frac{\text{distance}}{v} = 0.4 \Rightarrow \frac{\text{distance}}{\lambda} = 0.4$$

[Jakes Spectrum] The power spectral densities (PSDs) of $r_I(t)$ and $r_Q(t)$

$$S_{r_I}(f) = S_{r_Q}(f) = \mathcal{F}\{A_{r_I}(\tau)\} = \begin{cases} \frac{\Omega_p}{2\pi f_D\sqrt{1-(f/f_D)^2}} & |f| \le f_D \\ 0 & \text{else} \end{cases}$$



[Example] Send a $\cos(2\pi f_c t)$ through the channel. Then the received signal is

$$r(t) = \mathrm{Re}\left\{ch_{bb}(t)e^{j2\pi f_c t}\right\} = ch_I(t)\cos(2\pi f_c t) - ch_Q(t)\sin(2\pi f_c t)$$

The autocorrelation of $r(t)$ becomes

$$A_r(\tau) = \frac{1}{2}A_{r_I}(\tau)\cos(2\pi f_c \tau) + \frac{1}{2}A_{r_Q}(\tau)\cos(2\pi f_c \tau) = A_{r_I}(\tau)\cos(2\pi f_c \tau)$$

0.5 of PSD of A_{r_I} is shifted to $-f_c$ and f_c.

[Envelop and Power distribution]

$|ch(t)| = \sqrt{ch_I^2(t) + ch_Q^2(t)}$ is Rayleigh distributed with $\mathbf{E}[|ch(t)|^2] = \Omega_p$:

$$\begin{aligned} P_X(x) &= \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, \quad \text{for } x \geq 0 \\ & \mathbf{E}[x^2] = 2\sigma^2 \end{aligned}$$

The distribution of power of the channel, $|ch(t)|^2$, is exponential:

$$P_X(x) = \lambda e^{-\lambda x}, \quad \mathbf{E}[x] = \frac{1}{\lambda} \Rightarrow \lambda = 1/\Omega_p$$

[Example] (3.2 of Andrea Goldsmith's book) $r_{bb}(t) = u(t)h_{bb}(t)$.

$$\begin{aligned} prob\{|r_{bb}(t)|^2 < Z\} &= prob\left\{|h_{bb}(t)|^2 P_t < Z\right\} \\ &= \int_0^{Z/P_t} \lambda e^{-\lambda x} dx \\ &= 1 - e^{-\lambda Z/P_t} \end{aligned}$$

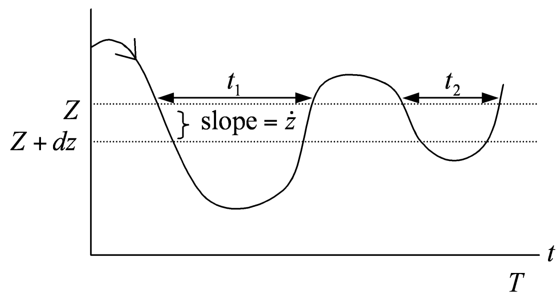
$$\lambda = \frac{1}{\overline{|h_{bb}(t)|^2}} = \frac{P_t}{\overline{|r_{bb}(t)|^2}} = \frac{P_t}{0.1}$$

Then,

$$\begin{aligned} prob\{|r_{bb}(t)|^2 < Z\} &= 1 - e^{-0.01/0.1} \\ &= 0.095 \end{aligned}$$

[Level Crossing Rate]

The level crossing rate is



$$L_Z = \int_{-\infty}^{0} \dot{Z} p(Z, \dot{z}) d\dot{z}$$

For a Rayleigh signal

$$L_Z = \sqrt{2\pi} f_D \rho e^{-\rho^2}, \quad \rho = Z/\sqrt{\bar{P}_r}$$

[Average Fade Duration]

$$\begin{aligned} \bar{t}_Z &= \frac{1}{N_t} \sum_{i=1}^{N_t} t_i, \quad L_Z = \frac{N_t}{T} \\ &= \frac{1}{L_Z T} \sum_{i=1}^{N_t} t_i, \end{aligned}$$

Since

$$\frac{1}{T} \sum_{i=1}^{N_t} t_i = prob\{|\text{signal}| < Z\}$$

Then,

$$\bar{t}_Z = \frac{1 - e^{-Z^2/\Omega_p}}{\sqrt{2\pi} f_D Z/\sqrt{\Omega_p} e^{-Z^2/\Omega_p}}$$