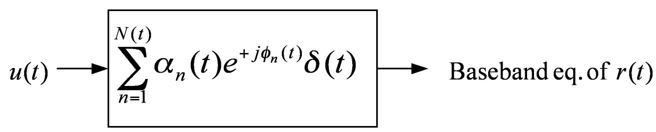
Wireless Communications
Lecture 4

[Review of previous lecture]

- Small-scale fading

$$s(t) = \text{Re}\left\{u(t)e^{j2\pi f_c t}\right\}$$



Send a cosine, e.g. $u(t) = 1$

$$\begin{aligned} r(t) &= \text{Re}\left\{\sum \alpha_n(t) e^{j\phi_n(t)} e^{j2\pi f_c t}\right\} \\ &= \underbrace{\sum \alpha_n(t)\cos(\phi_n(t))\cos(\omega_c t)}_{\text{In-phase:zero-mean Gaussian}} - \underbrace{\sum \alpha_n(t)\sin(\phi_n(t))\sin(\omega_c t)}_{\text{Quadrature:zero-mean Gaussian}} \\ ch(t) &= ch_I(t) + j ch_Q(t) \quad \text{Complex Gaussian random variable} \end{aligned}$$

The autocorrelation and cross-correlation of ch_I and ch_Q are

$$\begin{aligned} A_{ch_I}(\tau) &= A_{ch_Q}(\tau) \\ &= \frac{1}{2}\sum \alpha_n^2(t)\overline{\cos(2\pi\Delta f_n \tau)} \\ &= \frac{1}{2}\sum \alpha_n^2(t) J_0(2\pi f_D \tau) \\ A_{I,Q}(\tau) &= \frac{1}{2}\sum \alpha_n^2(t)\overline{\sin(2\pi\Delta f_n \tau)} \\ &= 0 \end{aligned}$$

- Power Spectrum

$$\mathcal{F}\{A_I(\tau)\} = S_I(f) = \frac{\Omega_p}{2\pi f_D\sqrt{1 - \left(\frac{f}{f_D}\right)^2}}, \quad |f| < f_D$$

What is $\int S_I(f)df$?

To get uncorrelated, $f_D\tau = 0.4\lambda$.

- Amplitude of channel, $|ch(t)|$ is Rayleigh distributed. $|ch(t)|^2$ is exponential distributed.

- Jakes Spectrum

$$A_{|ch|}(\tau) = \frac{\pi}{8} P J_0^2(2\pi f_D \tau) + \frac{\pi}{2} P$$

$\mathbf{E}[|ch|^2] = 2P = \Omega_p$, $\mathbf{E}^2[|ch|] = \frac{\pi}{2} P$. (*See Jakes for proof*)

[Deriving PSD in another way]

Let's assume

- large number of arriving paths.
- can account for antenna gain and non-uniform distribution of received paths.
- the paths are planar paths.

Let $p(\alpha)$ be probability distribution of path power/angle.

$$\begin{aligned} \int p(\alpha)d\alpha &= 1 \\ \text{Total power} &= P\int p(\alpha)G(\alpha)d\alpha \end{aligned}$$

where P is total power for isotropic antenna $G = 1$. In frequency domain:

$$S(f)|df| = Pp(\alpha)G(\alpha)|d\alpha| + P(p(-\alpha)G(-\alpha)|d\alpha| \quad 0 < \alpha < \pi$$

where $S(f)$ denotes power spectrum.

$$\begin{aligned} f &= f_c + \underbrace{\frac{v}{c}f_c}_{f_D}\cos\alpha \\ df &= -f_D \sin\alpha d\alpha \\ |df| &= f_D|\sin\alpha||d\alpha| \end{aligned}$$

Is it valid to do the change of variable and $|df|$ small enough?

$$\begin{aligned} S(f) &= \frac{P[p(\alpha)G(\alpha) + p(-\alpha)G(-\alpha)]}{f_D \sin\alpha} \quad \alpha = \cos^{-1}\left(\frac{f-f_c}{f_D}\right), 0 < \alpha < \pi \\ &= \frac{P[p(\alpha)G(\alpha) + p(-\alpha)G(-\alpha)]}{f_D\sqrt{1-\left(\frac{f-f_c}{f_D}\right)^2}}, \quad 0 \le \alpha \le \pi \Rightarrow |\frac{f-f_c}{f_D}| < 1 \end{aligned}$$

Therefore, to translate it to all f, 1/2 of $S(f)$ will be in $f > 0$ and 1/2 in $f < 0$.

[Example] For $p(\alpha) = 1/2\pi, G(\alpha) = 1$

$$S(f) = \frac{2P}{2\pi f_D\sqrt{1-\left(\frac{f-f_c}{f_D}\right)^2}}$$

[Log-normal Shadowing] $|ch|$ is Rayleigh but $\overline{|ch|}$ is log-normal.

- We have $u = 10 \log \overline{|ch|^2}$,

$$P_U(u) = \frac{1}{\sqrt{2\pi}\sigma_u} e^{\frac{-(u-m_u)^2}{2\sigma_u^2}}, \quad m_u = 10\log\alpha - n10\log d \text{ (Path Loss)}$$

$$\mathbf{E}[\overline{|ch|^2}] = \mathbf{E}[10^{u/10}]$$

$$\begin{aligned} 10^{\alpha u} &= e^{\beta u} \\ \alpha u \ln 10 &= \beta u \\ \beta &= \alpha \ln 10 \end{aligned}$$

$$\begin{aligned} \mathbf{E}[e^{\frac{\alpha \ln 10}{10}}] &= \cdots \\ &= \exp\left\{\frac{m_u}{g} + \frac{\sigma_u^2}{2g^2}\right\}, \quad g = \frac{10}{\ln 10} \end{aligned}$$

- Distribution of $\psi = \overline{|ch|}^2$,

$$P(\psi) = \frac{\partial}{\partial \psi} P_U(u = f(\psi)), \quad u = 10 \log \psi$$

$$\begin{aligned} \psi &= 10^{u/10} \\ &= e^{\ln 10/10^u} \end{aligned}$$

$$\frac{\partial \psi}{\partial u} = \frac{\ln 10}{10}\psi$$

$$P(\psi) = \frac{10}{\ln 10 \psi} \frac{1}{\sqrt{2\pi}\sigma_u} e^{\frac{-(10\log\psi - m_u)^2}{\sigma_u^2}}$$