Wireless Communications
Lecture 5

[Review of log-normal fading] $\psi_{\text{dB}} = 10 \log \overline{|ch|^2}$,

$$P_{\psi_{\text{dB}}} = \frac{1}{\sqrt{2\pi}\sigma_{\text{dB}}} e^{\frac{-(\psi_{\text{dB}} - m_{\text{dB}})^2}{2\sigma_{\text{dB}}^2}}, \quad m_{\text{dB}} = \alpha_{\text{dB}} - 10n \log d$$

[Example] Best path loss fit is -31.54 dB $- 37.1 \log d$ for the data below. Find σ_{dB}^2, variance of log-normal fading

d	P_r/P_t
10 m	-70 dB
20 m	-75 dB
50 m	-90 dB
100 m	-110 dB
300 m	-125 dB

[Solution] Take the average out and average the rest for variance:

$$\begin{aligned} \sigma_{\text{dB}}^2 &= \frac{1}{5}[(-70 + 31.54 + 37.1 \log 10)^2 + \\ &\quad + (-75 + 31.54 + 37.1 \log 20)^2 + \cdots \\ &= 13.29 \\ \sigma_{\text{dB}} &= 3.65 \end{aligned}$$

[Outage Probability] $prob\{P_{r_{\text{dB}}} \leq P_{\min}\}$

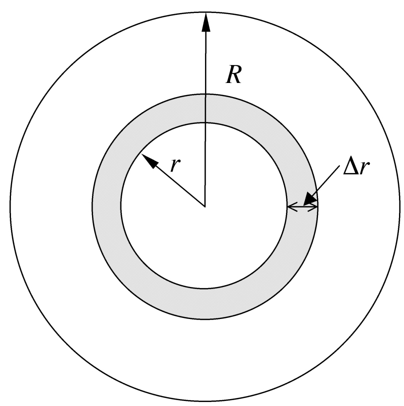
$$\begin{aligned} P_{r_{\text{dB}}} &= P_{T_{\text{dB}}} + \psi_{\text{dB}} \quad \text{(Normal distribution)} \\ prob\{P_{r_{\text{dB}}} \leq P_{\min}\} &= 1 - Q\left(\frac{P_{\min} - \overbrace{(P_{T_{\text{dB}}} + K_{\text{dB}} - 10\gamma \log d)}^{\alpha_{\text{dB}} - 10n \log d}}{\sigma_{\psi_{\text{dB}}}}\right) \\ Q(z) &= \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-x^2/2} dx \end{aligned}$$

[Example 2.5 of Goldsmith's] Find outage probability at 150 m, $P_{\min} = -110.5$ dBm, $P_t =$ 10mW for previous example.

[solution] We have $K = -31.54, \gamma = 3.71$. Therefore,

$$P_{\text{outage}} = 1 - Q\left(\frac{-110.5 - (10 - 31.54 - 37.11\log(150))}{3.65}\right) = 0.0121$$

[Cell Coverage] Find what percentage of the cell is above certain level?



We know $prob\,\{P_r(r) \geq P_{\min}\}$, then:

$$\begin{aligned}
C_{r,r+\Delta r} &= \text{area in the ring above } P_{\min} \\
&= prob\,\{P_r(r) \geq P_{\min}\}\,[\pi(r+\Delta r)^2 - \pi r^2] \\
&= prob\,\{P_r(r) \geq P_{\min}\}\,2\pi r \Delta r \\
C_{0,R} &= 2\pi \int_0^R r \underbrace{prob\,\{P_r(r) \geq P_{\min}\}}_{Q\left(\frac{P_{\min} - (P_{T_{\text{dB}}} + K_{\text{dB}} - 10\gamma\log r)}{\sigma_{\text{dB}}}\right)} dr \\
C &= \frac{C_{0,R}}{\pi R^2} = \text{percentage of area above } P_{\min}
\end{aligned}$$

Calculation involves $C = \frac{2}{R^2}\int_0^R rQ(a + b\ln r)dr$, $dr/r = du \to \ln r = u, \ln 0 = -\infty$ and integration by parts

$$\begin{aligned}
\int_{-\infty}^{\ln R} e^{2u}Q(a+bu)du &= \frac{1}{2}e^{2u}Q(a+bu)|_{-\infty}^{\ln R} + \int_{-\infty}^{\ln R} \frac{(1/2e^{2u})b}{\sqrt{2\pi}} e^{-\frac{(a+bu)^2}{2}} du \\
&= \frac{1}{2}e^{2u}Q(a+bu)|_{-\infty}^{\ln R} + \frac{b}{2\sqrt{2\pi}} \int_{-\infty}^{\ln R} e^{2u-\frac{(a+bu)^2}{2}} du \\
&= \cdots \\
&= Q(a) + e^{\frac{2-2ab}{b^2}} Q\left(\frac{2-ab}{b}\right)
\end{aligned}$$

[Rician channel] $ch(t) = ch_I(t) + jch_Q(t)$

We assume uniform angle of arrival $\Rightarrow ch_I(t)$ and $ch_Q(t)$ are zero mean. If a LOS path exists: separate that part

$$ch(t) = \underbrace{z_0(t)}_{\text{LOS}} + \underbrace{z(t)}_{\text{zero mean}}$$

$r(t) = |ch(t)|$ has a Rician distribution

$$P(r) = \frac{r}{\sigma^2} e^{\frac{-(r^2+s^2)}{2\sigma^2}} I_0\left(\frac{rs}{\sigma^2}\right), \quad r \geq 0, I_0(x) : \text{modified Bessel function}$$

where $2\sigma^2 = \sum_{n \neq \text{LOS path}} \overline{\alpha_n^2}$ is the average power in the non-LOS multipath components and $s^2 = |z_0|^2$ is the power in the LOS component.

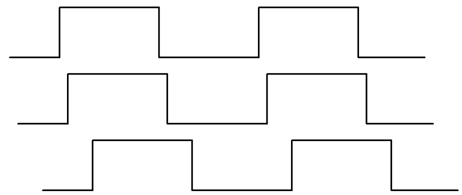
$$\mathbf{E}[r^2(t)] = s^2 + 2\sigma^2$$

The fading parameter K is defined by

$$K = \frac{s^2}{2\sigma^2}$$

K is a measure of the severity of the fading.

Resolvable Paths:



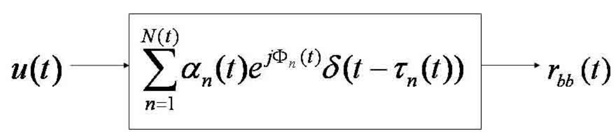
[Wideband Channels] The received signal is

$$r(t) = \text{Re}\left\{ \sum_{n=1}^{N(t)} \alpha_n(t) u(t - \tau_n(t)) e^{j2\pi(f_c + \Delta f_n(t))t - j2\pi f_c \tau_n(t)} \right\}$$

- The different multipath components can be resolved.
- This results in *Intersymbol Interference* (ISI)
- The approximation $u(t - \tau_n(t)) \approx u(t)$ is no longer valid.

$$r(t) = \text{Re}\left\{ \sum_{n=1}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} u(t - \tau_n(t)) e^{j2\pi f_c t} \right\}$$

$$r_{bb}(t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} u(t - \tau_n(t))$$



[Note]: Two time progression

1. distance R approach.

$$h(t, R) = \sum \alpha_n(R) e^{j\phi_n(R)} \delta(t - \tau_n(R))$$

2. t, τ approach

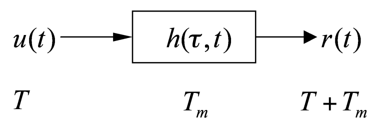
$$h(\tau, t) = \sum \alpha_n(t) e^{j\phi_n(t)} \delta(\tau - \tau_n(t))$$

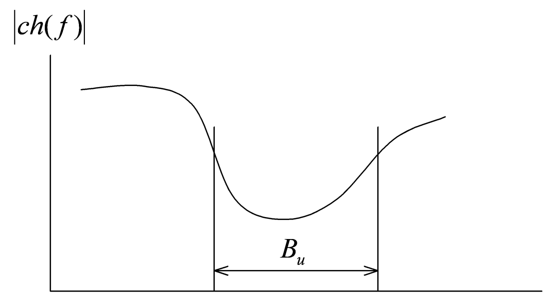
[Example 3.1 of Goldsmith's] Consider a TX in a factory near a conveyer belt. LOS path with α_0, ϕ_0. Every T_0 second a metal item comes down the belt with α_1, ϕ_1, τ_1. τ_1 is delay difference. Characterize the channel.

$$h(\tau, t) = \begin{cases} \alpha_0 e^{j\phi_0}\delta(\tau) & t \neq nT_0 \\ \alpha_0 e^{j\phi_0}\delta(\tau) + \alpha_1 e^{j\phi_1}\delta(\tau - \tau_1) & t = nT_0 \end{cases}$$

[Increase in time length]

$T_m \gg T \Rightarrow$ resolvable. *Equivalently,* $B_u \ll T_m^{-1}$ *which is called is wideband channel.*





$\Rightarrow$ wideband channel or frequency selective fading.

[Example]