Wireless Communications
Lecture 6

[Note]

$\overline{|ch|}$ and $\overline{|ch|^2}$ are both log-normal and $|ch|$ is Rayleigh.

$$\Rightarrow \overline{|ch|} = \sqrt{\frac{\pi}{2}}P, \quad P = \overline{|ch|^2}$$

[Wideband Channels]

The transmitted signal is

$$s(t) = \text{Re}\left\{u(t)e^{j2\pi f_c t}\right\}$$

The received signal is

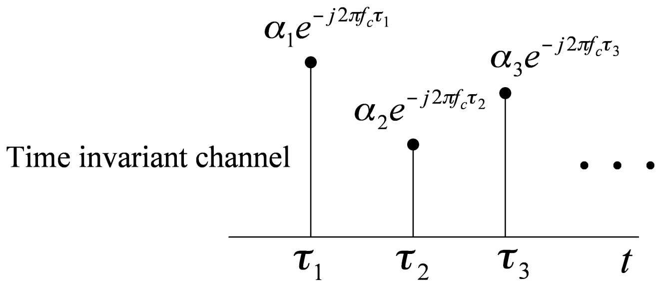
$$r(t) = \text{Re}\left\{\sum_{n=1}^{N(t)} \alpha_n(t)u(t-\tau_n(t))e^{j2\pi(f_c+\Delta f_n(t))t - j2\pi f_c \tau_n(t)}\right\}$$

Resolvable paths $u(t-\tau_n(t)) \ncong u(t)$

[Time-invariant case]

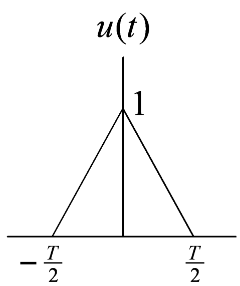
Assume fixed TX, RX and environments $\Rightarrow \Delta f_n = 0$.

$$\begin{aligned} r_{bb}(t) &= \sum_n \alpha_n u(t-\tau_n)e^{-j2\pi f_c \tau_n} \\ ch(t) &= \sum_n \alpha_n \delta(t-\tau_n)e^{-j2\pi f_c \tau_n} \end{aligned}$$

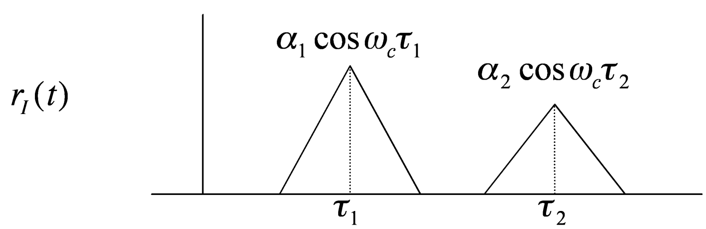


[Example] 2 paths $N = 2$, $|\tau_2 - \tau_1| > T$,

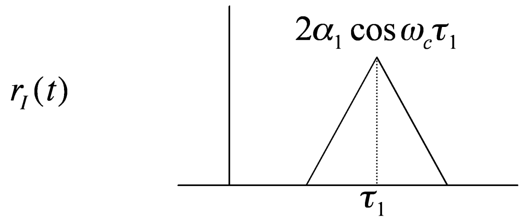
$$\omega_c(\tau_1 - \tau_2) = 2\pi k, \quad \alpha_1 > \alpha_2$$



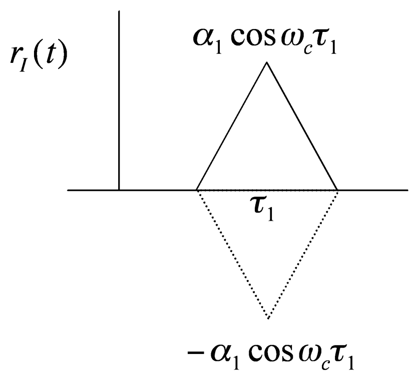
Plot In-phase part of the channel



[Example] $N = 2$, $\omega_c(\tau_1 - \tau_2) = 2\pi k$, $\alpha_1 = \alpha_2$, $|\tau_1 - \tau_2| \ll T$.

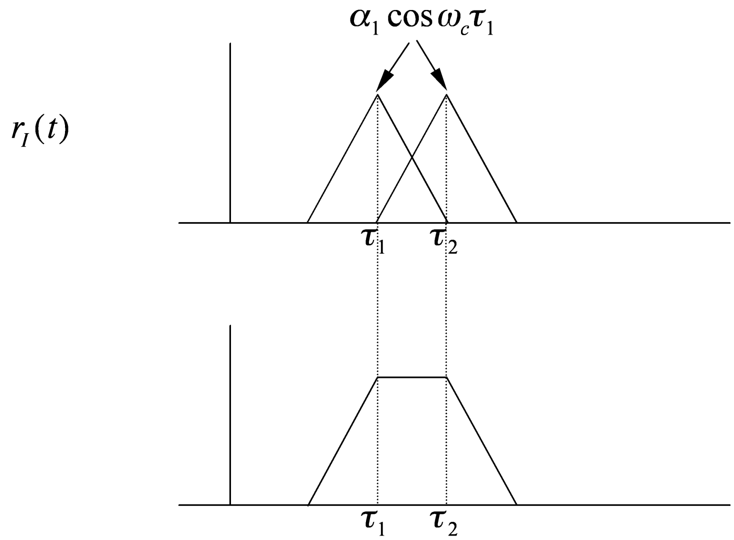


[Example] $N = 2$, $\alpha_1 = \alpha_2$, $\omega_c(\tau_1 - \tau_2) = 2\pi k + \pi$, $|\tau_1 - \tau_2| \ll T$.

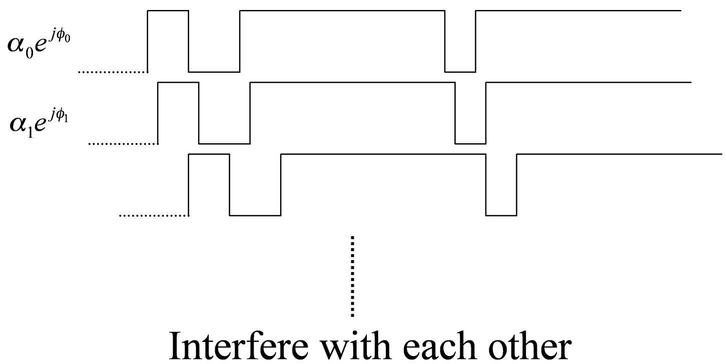


$\Rightarrow$ Zero

[Example] $N = 2$, $|\Delta\tau| = T/2$, $\omega_c \Delta\tau = 2\pi k$, $\alpha_1 = \alpha_2$.

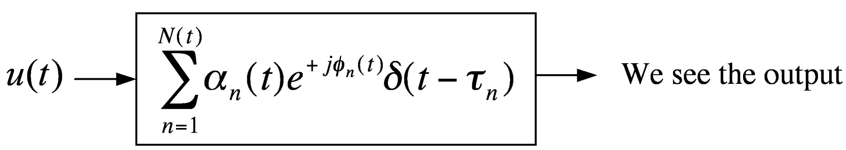


[Delay Spread] Send a signal with length T and channel $\tau_{\max} - \tau_{\min} = T_m$. The output length is $T + T_m$. It causes ISI.



[Remedies for ISI]

- *Equalizer*



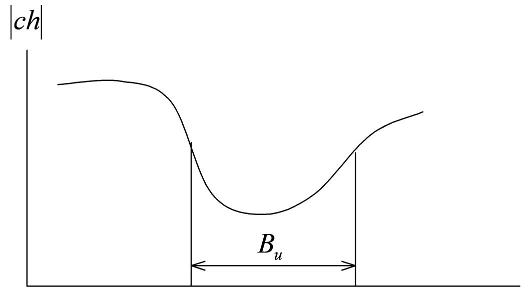
Learn the channel and undo its effect

- *Multicarrier (MC) modulation*
- *Spread Spectrum (SS)*

$T_m \ll T \Leftrightarrow B_u = T^{-1} \ll T_m^{-1} \Rightarrow$ no delay spread

[Narrowband channel]

$T_m \downarrow \Rightarrow$ larger correlation in frequency.
T_m^{-1} is a measure of coherence Bandwidth.



[Time-varying case]

$$r(t) = \mathrm{Re}\left\{ \sum_{n=1}^{N(t)} \alpha_n(t) e^{j\phi_n(t)} u(t - \tau_n(t)) e^{j2\pi f_c t} \right\}$$

$$\begin{aligned} r_{bb}(t) &= \sum_n \alpha_n(t) e^{j\phi_n(t)} u(t - \tau_n(t)) \\ &= \int_{-\infty}^{\infty} \underbrace{ch(\tau, t)}_{\text{time-varying}} \; u(t - \tau) d\tau \end{aligned}$$

$$\underbrace{ch(\tau, t)}_{\text{mobility induced time}} = \sum \alpha_n(t) e^{j\phi_n(t)} \delta(\tau - \tau_n(t)), \quad \tau : \text{delay}$$

Check:

$$\begin{aligned} r_{bb}(t) &= \int \sum \alpha_n(t) e^{j\phi_n(t)} u(t - \tau) \delta(\tau - \tau_n(t)) d\tau \\ &= \sum \alpha_n(t) e^{j\phi_n(t)} \int u(t - \tau) \delta(\tau - \tau_n(t)) d\tau \\ &= \sum \alpha_n(t) e^{j\phi_n(t)} u(t - \tau_n(t)) \\ &\Rightarrow \text{Confirmed} \end{aligned}$$

[Example] Ex. 3.1 of Textbook, Goldsmith.

[What is $ch(\tau, t)$?]

$$\delta(t-\tau) \Rightarrow ch(\tau, t)$$

Time-invariant case: $ch(\tau, t) = ch(\tau, t+\Delta t)$. This means response to impulse $\delta(t-\tau)$ is the same as response to the $\delta(t+\Delta t-\tau)$.

$ch(\tau, t) = ch(\tau, t+\Delta t) = ch(\tau, 0) = ch(\tau)$, $\quad ch(\tau, 0)$: response to $\delta(-\tau)$ at $t=0$, which is $\delta(\tau)$

[Statistical Characterization]

$$\begin{aligned}
A_c(\tau_1, \tau_2; t, t+\Delta t) &= \mathbf{E}[ch^*(\tau_1, t)ch(\tau_2, t+\Delta t)] \\
\text{Different paths are independent.} \Rightarrow & \\
&= \mathbf{E}[ch^*(\tau_1, t)ch(\tau_1, t+\Delta t)]\delta(\tau_1-\tau_2) \\
&= A_c(\tau_1; \Delta t)\delta(\tau_1-\tau_2), \quad \text{(WSS)} \\
&\triangleq A_c(\tau; \Delta t)
\end{aligned}$$

The *scattering function* for time-varying channels is defined as the Fourier transform of $A_c(\tau; \Delta t)$ with respect to the Δt:

$$S_c(\tau, \rho) = \mathcal{F}\{A_c(\tau; \Delta t)\} = \int_{-\infty}^{\infty} A_c(\tau; \Delta t)e^{-j2\pi\rho\Delta t} d\Delta t$$

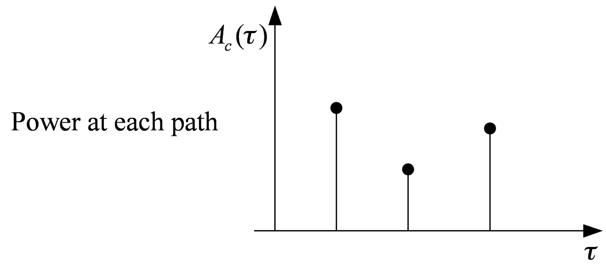
[Power delay Profile] The *power delay profile* $A_c(\tau)$ is defined as the autocorrelation $A_c(\tau; \Delta t)$ with $\Delta t = 0$.

The average and rms delay spread are defined as follows:

$$\mu_{T_m} = \frac{\int_0^\infty \tau A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}$$

and

$$\sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau-\mu_{T_m})^2 A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}}$$



[Example] Exponential profile

$$A_c(\tau) = \frac{1}{\bar{T}_m} e^{-\tau/\bar{T}_m} \quad \text{Normalized}$$

$$\sigma_{T_m} = \bar{T}_m$$

[Example]

$$A_c(\tau) = \begin{cases} e^{-\tau/10^{-5}} & 0 \le \tau \le 20\ \mu\text{s} \\ 0 & \text{else} \end{cases}$$

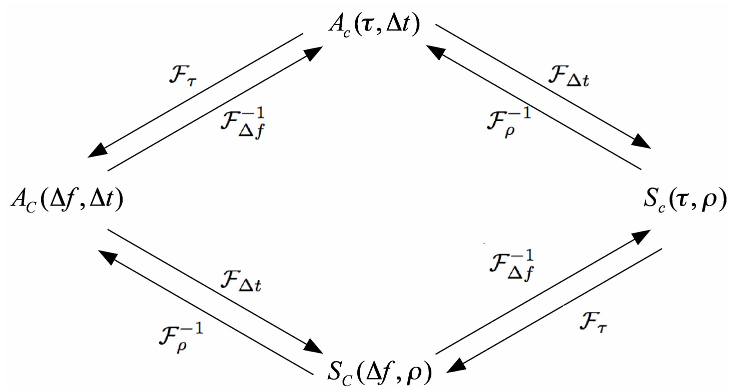
$$\mu_{T_m} = \frac{\int_0^{20} \tau e^{-\tau/10} d\tau}{\int_0^{20} e^{-\tau/10} d\tau} = \frac{-20e^{-2} + \int_0^{20} 10 e^{-\tau/10} d\tau}{1/10(1-e^{-2})} = 6.87\ \mu\text{s}$$

[Coherence Bandwidth]

$$C(f;t) = \int ch(\tau;t) e^{-j2\pi f \tau} d\tau$$

Autocorrelation in the frequency domain

$$\begin{aligned} A_C(f_1, f_2; \Delta t) &= \mathbf{E}[C^*(f_1, t) C(f_2, t+\Delta t)] \\ &= \int\int \overline{ch^*(\tau_1, t) ch(\tau_2, t+\Delta t)} e^{-j2\pi(-f_1\tau_1 + f_2\tau_2)} d\tau_1 d\tau_2 \\ &= \int_{-\infty}^{\infty} A_c(\tau, \Delta t) e^{-j2\pi(f_2 - f_1)\tau} d\tau \\ &= A_C(\Delta f; \Delta t) \end{aligned}$$



Fourier of Power delay profile is

$$A_C(\Delta f;0) = A_C(\Delta f) = \int_{-\infty}^{\infty} A_c(\tau)e^{-j2\pi\Delta f\tau}d\tau$$

Coherence BW is where $A_C(\Delta f) = 0$.

Coherence BW is proportional to 1/delay spread

The doppler effect can be characterized as

$$S_C(\Delta f;\rho) = \int_{-\infty}^{\infty} A_C(\Delta f;\Delta t)e^{-j2\pi\rho\Delta t}d\Delta t$$

Set $\Delta f = 0$

$$S_C(\rho) = \int_{-\infty}^{\infty} A_C(\Delta t)e^{-j2\pi\rho\Delta t}d\Delta t$$

Doppler spread $\propto$ 1/coherence time.

Coherence time C_T : $A_c(\tau,\Delta t)$ at each τ.

$$C_T = 0.4f_D \quad \text{for Bessel function}$$

Doppler spread $\propto 1/C_T$.